Last chapter we did (one lump sum). If I put $1000 in bank, what do I have in 4 years?

\[ FV = -1000 \quad \text{Adding interest in} \quad \uparrow \]

\[ PV = 1000 \quad i = 10\% = 0.10 \]
\[ n = 2 \quad \frac{n}{2} = 10\%/2 = 5\% = 0.05 \]
\[ x = 4 \quad n \times x = 4 \times 2 = 8 \]

\[ FV = PV \times \left(1 + \frac{i}{n}\right)^{n \times x} \]
\[ FV = 1000 \times \left(1 + \frac{0.1}{2}\right)^{2 \times 4} \]
\[ FV = 1477.46 \]

If we need $1477.46 in 4 years how much do we need to put in bank today?

\[ FV = 1477.6 \quad \text{Taking interest out} \quad \downarrow \]

\[ PV = \frac{FV}{\left(1 + \frac{i}{n}\right)^{n \times x}} \]
\[ PV = \frac{1477.6}{\left(1 + \frac{0.1}{2}\right)^{2 \times 4}} \]
\[ PV = $1000 \]
1. \[ PV = \$1000 \]
   \[ \dot{n} = 10\% \]
   \[ n = 2 \]
   \[ FV = ? \]
   \[ x = 4 \]

Add all interest in

\[ FV = \$1,477.6 \]

Solve for this

\[ FV(5\%, 8, -1000) = \$1,477.6 \]

2. \[ \dot{n} = 10\% \]
   \[ \frac{\dot{n}}{n} = \frac{10\%}{2} = 5\% \]
   \[ n \times x = 4 \times 2 = 8 \]

\[ FV = 1,447.6 \]

Solve for this

\[ PV = 1000 \]

Take all interest out

\[ PV(0.05, 8, 1447.6) = -\$1000 \]

In Excel Minus Because Excel knows Cash Flow
Chapter 4: What is value of one lump sum cash amount?

Chapter 5: What is value of multiple cash amounts (cash flows)

3 Example:

Invest in bank: (Savings plan)

<table>
<thead>
<tr>
<th>Time (n)</th>
<th>Amount Invested</th>
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<tbody>
<tr>
<td>0</td>
<td>$1000 invest</td>
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<tr>
<td>4</td>
<td>$2000 invest</td>
</tr>
<tr>
<td>6</td>
<td>$6000 invest</td>
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<tr>
<td>10</td>
<td>$10,602.12</td>
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Adding interest: solve for this

FV = $1000 \times \left(1 + \frac{0.10}{2}\right)^{2 \times 5}

FV = 1000 \times (1 + 0.05)^{10 - 0}

FV = 2000 \times (1 + 0.05)^{10 - 4}

FV = 6000 \times (1 + 0.05)^{10 - 6}

FV total = $11,602.12

Adding all the interest

Only 4 periods left

Only 6 periods left
Example: we want to withdraw these cash flows in future. How much do we have to put in Bank today?

\[ i = 10\% \]
\[ n = 2 \]
\[ \frac{i}{n} = \frac{10\%}{2} = 5\% \]
\[ X = 5 \]
\[ n \times X = 5 \times 2 = 10 \]

\[ PV = \text{Time 0} = \text{How much to put in Bank?} \]

\[ PV = \frac{FV}{(1 + \frac{i}{n})^n} \]
\[ PV = \frac{1000}{(1 + 0.05)^4} \]
\[ PV = \frac{2000}{(1 + 0.05)^6} \]
\[ PV = \frac{6000}{(1 + 0.05)^{10}} \]

Taking all Interest Out

\[ PV = 5998.6 \]

\[ 922.70 \]
\[ 1492.43 \]
\[ 3683.48 \]

Withdraw:
\[ \text{Time 4} = \$1000 \]
\[ \text{Time 6} = \$2000 \]
\[ \text{Time 10} = \$6000 \]
Example:  

Price of Machine = $100,000

What if you plan to buy a machine that you have estimated will yield these cash flows:

Time 0 = Price
Time 1 = 50,000
Time 2 = 50,000
Time 3 = 40,000
Time 4 = 30,000

Assumed Discount Rate = n = 15%

n = 1
x = 4

Future Positive Cash Flows

Q: Should we buy machine?

By taking future cash flows & discounting them back, we can compare future income to price:

- 43,478.26
- 37,807.18
- 26,380.65
- 17,152.60

\[ -124,738.69 \]

A: If you are willing to payout -124,738.69 for the cash flows, is -100,000, worth it? A: Yes!!!
Interest Rates

1. **Period Rate**
   
   Period interest rate = \[ \frac{i}{n} \]
   
   Example 6:
   
   \( \dot{i} = 12\% \text{ or } 0.12 \)
   \( n = 12 \)
   \( \{ \text{Period Rate} \} = \frac{\dot{i}}{n} = \frac{0.12}{12} = 0.01 \text{ or } 1\% \)

2. **APR (Annual Percentage Rate)**
   
   Also known as: Stated Rate, Quote Rate, Annual Interest Rate, Nominal Rate.

   Examples:
   
   10% compounded monthly
   12% compounded semiannually
   5.55% compounded daily

   \[ \text{APR} = \text{period rate} \times \# \text{ of compounding periods per year} = \frac{\dot{i}}{n} \times n = \dot{i} \]

   Truth-in-lending laws in USA require lenders to prominently display the APR on loan documents.

   Example 7:
   
   If the monthly rate is 0.5%, what is APR?
   
   \( \frac{\dot{i}}{n} = 0.5\% \text{ or } 0.005 \)
   \( n = 12 \)
   \( \dot{i} = \frac{\dot{i}}{n} \times n = 0.005 \times 12 = 0.06 \text{ or } 6\% \text{ compounded monthly} \)

   Note: If you are given an APR of 6% compounded monthly, you cannot find out what the semiannual rate might be!!! We could figure out what the monthly rate is but not a semiannual or daily or quarterly or any other period rate.
Anytime we have # of compounding periods per year that is greater than 1, we must figure out what the real annual rate is. This is called the Effective Annual Rate:

**EAR (Effective Annual Rate)**

EAR = Annual Interest Rate expressed as if it were compounded once per year

\[
EAR = \left(1 + \frac{\hat{r}}{n}\right)^n - 1
\]

(look familiar?)

**Excel:***

\[
= \text{EFFECT} (\text{Nominal Rate}, \text{m/p/y})
\]

\[
= \text{EFFECT} (\hat{r}, n) \quad \text{* must be integer}
\]

Example 8:

If the APR is 18% compounded monthly, is 18% APR the same as the EAR (Effective Annual Rate)?

\[
\hat{r} = 0.18 \\
n = 12
\]

Answer: NO!

\[
EAR = \left(1 + \frac{0.18}{12}\right)^{12} - 1 = \left(1.015\right)^{12} - 1 = 1.19561817 - 1
\]

\[
EAR = 0.19561817 \Rightarrow 19.56\%
\]

Notes:
1. Never divide \( \frac{\text{EAR}}{n} \) to get period rate! Only \( \frac{\hat{r}}{n} \) is used to find period rate (\( \frac{\hat{r}}{n} \)).
2. Does APR = EAR? Answer: only when \( n = 1 \)
3. EAR > APR when \( n > 1 \)

Example 9: which APR yields more interest: 11% compounded quarterly or 10.75% compounded yearly?

\[
\text{EAR}_1 = \left(1 + \frac{0.11}{4}\right)^4 - 1 = 0.114621259
\]

\[
\text{EAR}_2 = \left(1 + \frac{0.1075}{365}\right)^{365} - 1 = 0.113473238
\]

Answer: 11% compounded quarterly earns more interest.
Example 10:

MoneyTreeLoaning will give you $200 today, if you pay them $250 in 25 days. What is APR? What is EAR?

- **Period length in days**: 25
- **n**: # of periods per year = \( \frac{365}{25} = 14.6 \)
- \( \frac{i}{n} = \text{period rate} = \frac{\text{part}}{\text{whole}} = \frac{\text{Interest Paid}}{\text{Original Loan}} = \frac{250-200}{200} = \frac{50}{200} = \frac{5}{20} \)
- \( \frac{i}{n} = \frac{1}{4} = 0.25 \Rightarrow 25\% \)
- \( i = \text{APR} = \frac{i}{n} \times n = 0.25 \times 14.6 = 3.65 \Rightarrow 365\% \)
- \( \text{EAR} = (1 + \frac{3.65}{14.6})^{14.6} - 1 = (1.25)^{14.6} - 1 = 25.99478 - 
  = 24.99478 \Rightarrow 2499.48\% \)

Example 11:

- **IF EAR is 14.5% and n=2, what is the APR?**
- \( \text{APR} = (\text{EAR} + i)^{\frac{n}{2}} - 1 \)
  - \( \text{APR} = (0.145 + 1)^{\frac{1}{2}} - 1 \)
  - \( \text{APR} = 0.140093456 \) or 14.01%
What is an Annuity?

1. Pay an equal amount of cash each period
   Example: Mortgage Loan Payment.

OR

2. Receive an equal amount of cash each period
   Example: Receive Retirement check each month.

Define Annuity:

1. All cash flow payments are equal in amount
2. The time between each payment is equal (Total time fixed)

Types of Annuities:

1. Ordinary Annuity:
   Payments are made at the END of each period.
   Example: Monthly Mortgage Loan Payment.

2. Annuity Due:
   Payments are made at the BEGINNING of each period.
   Example: Lease payments.

Relationship between Annuity Due & Ordinary Annuity:

Annuity Due Value = Ordinary Annuity \times (1 + \frac{r}{n})

Note: All annuities are ordinary unless otherwise stated (for this class).
Future Value of Annuity (Savings Plan)

Math

\[ FV_{\text{Annuity}} = PMT \times \left[ \frac{(1 + \frac{i}{n})^{nx} - 1}{\frac{i}{n}} \right] \]

Excel

\[ \text{rate} = \frac{i}{n} = \text{period Rate} = \frac{\text{APR}}{n} \]

\[ n_{\text{per}} = nx = \text{Total # of periods} \]

\[ FV = \text{FV} = \text{Future value of all cash flows} \]

\[ PV = \text{PV} = \text{Annuity Due} = \text{Beg} = 1 \text{; ordinary} = \text{End} = 0 \text{ or } \text{Leave Blank} \]

\[ = \text{FV} \left( \text{rate}, n_{\text{per}}, \right) \]

\[ = \text{FV} \left( \text{rate}, n_{\text{per}}, \text{PMT}, PV, \text{type} \right) \]

\[ \text{type} \]

\[ \text{period Rate} \rightarrow \text{total periods} \rightarrow \text{payment} \rightarrow \text{If there is an amount at time zero put here.} \]

*Sign of cash flow matters to Excel functions.
Example 12: If you deposit $50 at the end of each year for the next 3 years and you earn 12% compounded yearly, what is the future value?

Annual Interest Rate = APR = i = 0.12 or 12%

n = # compounding periods per year = n = 1
years = x = 3

Equal payments made at equal time intervals = PMT = 50

Future value = FV = ?

We could use our old formula:

\[
FV = PV \times \left(1 + \frac{i}{n}\right)^{nx}
\]

\[
FV = 168.72
\]

\[
FV = PMT \times \left[\frac{(1 + \frac{i}{n})^{nx} - 1}{\frac{i}{n}}\right] = 50 \times \left[\frac{(1 + \frac{0.12}{1})^{1 \times 3} - 1}{\frac{0.12}{1}}\right]
\]

\[
= 50 \times \frac{1.4049 - 1}{0.12} = 50 \times 3.3744 = 168.72
\]

Excel

Remember: cash flow signs! PMT & PV are negative if you are investing.

\[
FV(rate, nper, pmt, pv, type) = FV\left(\frac{i}{n}, n \times x, PMT, -PV, \text{0 or 1}\right)
\]

\[
FV(0.12, 3, -50, 0) = 168.72
\]

Leave blank if ordinary
Example 12.5

Doe:  (Begin)

\[ \frac{r}{n} = 0.05 \]
\[ n \times x = 4 \]
\[ PMT = 500 \]
\[ FV = 2262.82 \]

<table>
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<th>1</th>
<th>2</th>
<th>3</th>
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<td></td>
<td>500</td>
<td>500</td>
<td>500</td>
<td>500</td>
<td></td>
</tr>
</tbody>
</table>

ordinary:  (End)

\[ \frac{r}{n} = 0.05 \]
\[ n \times x = 4 \]
\[ PMT = 500 \]
\[ FV = 2155.06 \]

<table>
<thead>
<tr>
<th>Time</th>
<th>0</th>
<th>1</th>
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<td></td>
<td>500</td>
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</tr>
</tbody>
</table>

Relationship:

Annuity Due = Annuity ordinary \times (1 + \frac{r}{n})

\$ 2262.82 = 2155.06 \times (1 + 0.05)

\checkmark check!!
Example 15:

Savings plan compounds interest 365 times a year, but you only put money in 12 times a year:

Monthly PMT = 250

\( X = 25 \)

\( N = 365 \) (Savings account n)

\( APR = \gamma = 0.08 \)

1. Solve for savings account EAR:

\[
EAR = \left(1 + \frac{0.08}{365}\right)^{365} - 1 = 0.083277571
\]

2. From EAR find APR for savings account:

\[
0.083277571 = \left(1 + \frac{\text{APR}}{12}\right)^{12} - 1
\]

Excel: =NOMINAL (.083277571, 12)

= 0.08025843577

3. Find period rate for monthly PMT:

\[
\frac{\gamma}{n} = \frac{0.08025843577}{12} = 0.006688203
\]

4. Solve for FV of monthly PMT:

\[
FV = PMT \times \left[ \frac{(1 + \frac{\gamma}{n})^n \times X - 1}{\frac{\gamma}{n}} \right] = 250 \times \left[ \frac{(1 + 0.006688)^{12 \times 25} - 1}{0.006688} \right]
\]

\( FV = 238,757.59 \)
What if we know how much we want in the future, but we don’t know how much to invest each period?

If this is true: \[ FV = PMT \times \left[ \frac{(1 + \frac{i}{n})^{nx} - 1}{\frac{i}{n}} \right] \]

Then

\[
\frac{FV}{\left[ \frac{(1 + \frac{i}{n})^{nx} - 1}{\frac{i}{n}} \right]} = PMT \times \left[ \frac{(1 + \frac{i}{n})^{nx} - 1}{\frac{i}{n}} \right]
\]

Formula for finding \( PMT \) for \( FV \) amount:
\[ PMT = \frac{FV}{\left[ \frac{(1 + \frac{i}{n})^{nx} - 1}{\frac{i}{n}} \right]} \]

Excel Function:
\[ =PMT(rate, nper, FV, type) \]

Where:
- rate is the interest rate per period
- nper is the total number of payment periods
- FV is the future value of the loan or investment
- type is 0 (ordinary annuity) or 1 (annuity due)
Example 16: How much do I have to invest at the end of each month to become a millionaire if I can earn 10% compounded monthly for the next 40 years?

Formula: \[ \text{PMT} = \frac{\text{FV}}{(1 + \frac{r}{n})^{nt} - 1} \]

\[ x = 0.1 \]
\[ n = 12 \]
\[ \text{FV} = \$1,000,000 \]
\[ X = 40 \]

\[ \text{PMT} = \frac{1,000,000}{(1 + \frac{0.1}{12})^{12 \times 40} - 1} = \$158.13 \]

\[ \text{PMT} (\frac{0.1}{12}, 12 \times 40, 1,000,000, 0) = \$158.13 \]

2 commas because we are not using PV.

If I want to be a millionaire given a 10% annual rate compounded monthly for 40 years, I have to deposit 158.13 each month.

Timeline:

\[ 0 \quad 1 \quad 2 \quad 3 \quad \text{(time)} \ldots \]

\[ \text{PMT} \quad -158.13 \quad -158.13 \quad -158.13 \quad \ldots \quad -158.13 \]

\[ \text{FV} = 1,000,000 \]

\[ 12 \times 40 = 480 \]

\[ -158.13 \]
Example 17: \[ FV = 180,000 \]
\[ \ln = 0.08 \]
\[ n = 12 \]
\[ x = 18 \]
\[ PMT = -374.93 \]

Formula: \[ PMT = \frac{\frac{0.08}{12} \times 18 \times 12}{1 + \frac{0.08}{12}} - 1 \]

If I need $180,000 in 18 years to send my daughter to college, I should save $374.93 each month, assuming 8.6% APR compounded monthly.

Timeline:

\[ PMT \]
\[ -374.93 \]
\[ -374.93 \]
\[ -374.93 \]
\[ -374.93 \]
\[ FV = 180,000 \]

Notice that:

\[ PMT = \frac{180,000}{(1 + \frac{0.08}{12})^{18 \times 12} - 1} \]

is the same as:

\[ PMT = 180,000 \times \frac{0.08}{12} \]

In finance, you may see it either way because \[ \frac{1}{(1/2)} = 1 \times \frac{2}{1} = 2 \]
we have been calculating the future value of the PMT! That is, we put a certain PMT in each period, and then we want to know what that will be worth in the future. BUT, what if we wanted to withdraw a certain amount each period IN THE FUTURE, and we needed to determine how much to invest today (present value)?

For example:

\[ +50 \quad +50 \quad +50 \]

\[ 0 \quad 1 \quad 2 \quad 3 \]

\[ \downarrow \]

\[ PV \]

what if we needed $50 at the end of each period, and we wanted to know how much to invest today?  \[ \rightarrow \text{next page} \]
Present Value of Annuity
(Discounting Future Cash Flows)

\[ P \text{V}_{\text{Annuity}} = PMT \times \left[ \frac{1 - \left(1 + \frac{i}{n}\right)^{-nX}}{\frac{i}{n}} \right] \]

Minus Sign

See page 11 for variables defined

Excel

= PV(rate, nper, pmt, FV, type)

Period Rate total # of periods payment If there is some left over at final time period

If due = 1
End = ordinary leave blank

Sign of cash flow matters for Excel functions
Example 18:

If you want to withdraw $50 at the end of each year, for the next 3 years, how much do you have to deposit in the bank today if the APR = 0.12 compounded yearly?

\[ \text{PMT} = 50 \quad n = 1 \quad \dot{i} = 0.12 \quad x = 3 \quad n \times x = 3 \]

\[ \frac{\dot{i}}{n} = \frac{0.12}{1} = 0.12 \]

\[ PV = ? \]

\[ PV = \text{PMT} \left[ \frac{1 - (1 + \frac{i}{n})^{-nx}}{\frac{i}{n}} \right] = 50 \left[ \frac{1 - (1 + \frac{0.12}{1})^{-3 \times 1}}{\frac{0.12}{1}} \right] = -120.09 \]

We could make this Present Value calculation the long way or we can use the Financial Formula Handout.
Example 19: How much do you have to have in bank when you retire if you want to withdraw $3000 each month for the next 35 years if you can earn 6% compounded monthly?

\[ PV = ? \]
\[ PMT = +3000 \]
\[ n = 12 \]
\[ i = 0.06 \]
\[ \frac{i}{n} = \frac{0.06}{12} = 0.005 \]

\[ n \times x = 35 \times 12 = 420 \]

Timeline:

<table>
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<th>3000</th>
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\[ PV = \frac{3000 \left[ 1 - \left(1 + \frac{i}{n}\right)^{-nx} \right]}{\frac{i}{n}} \]

\[ PV = 3000 \left[ 1 - \left(1 + \frac{0.06}{12}\right)^{-420} \right] \]

\[ PV = 5,261,140.68 \]

Excel:

\[ = PV\left(\frac{i}{n}, n \times x, PMT\right) \]

\[ = PV\left(\frac{0.06}{12}, 12 \times 35, 3000\right) \]

\[ = -5,261,140.68 \]

---

You will need $5,261,140.68 when you retire if you want to withdraw $3000 per month from an account that yields 6% compounded monthly.
Example 20:

But if you need $526,140.68 when you retire, how much do you need to deposit each month if you are 28 years old now & plan to retire at age 70 ($i = 0.10$, $n = 12$)?

We already know how to do this:

\[
PMT = \frac{FV}{\left(1 + \frac{i}{n}\right)^{nx} - 1} \cdot \frac{i}{n}
\]

\[
PMT = ?
\]

\[
FV = 526,140.68
\]

\[
i = 0.10 \text{ or } 10\%
\]

\[
n = 12
\]

\[
x = (70-28) = 42
\]

\[
n = 42 \times 12 = 504
\]

\[
PMT = \frac{526,140.68}{\left(1 + \frac{0.10}{12}\right)^{12 \times 42} - 1} \cdot \frac{0.10}{12} = 67.9391
\]

Excel:

\[
= FV\left(\frac{0.10}{12}, 12 \times 42\right, 526,140.68) = -67,939.1 \approx -67.94
\]

Total paid over life of investment = 67.94 * 504 = 34,241.76

Total taken out over life of investment = 3000 * 420 = 1,260,000

\[
\text{Interest} = 1,260,000 - 34,241.76 = 1,225,758.24
\]

Answer: If we invest 67.94 for 504 months at an APR of 10% compounded 12 times a year, we will be able to then withdraw $3,000 for 420 months for a total net gain of $1,225,758.24
Example 21: If a new machine will yield a net cash flow of $10,000 per month for the next 5 years and your discount rate is 15% compounded monthly, how much should you pay for the machine?

\[
PV = 10,000 \left[ \frac{1 - (1.0125)^{-60}}{0.0125} \right] = 420,345.92
\]

Excel:

\[
= PV(\frac{0.15}{12}, n \times x, PMT), 0 \text{ or } 1) = PV(\frac{0.15}{12}, 12 \times 5, 10000 )
\]

We should pay $420,345.92 or less for the machine. ($420,345.92 is the max price we should pay.)
How do we solve for PMT?

$$PV = PMT \left[ \frac{1 - (1 + \frac{i}{n})^{-xn}}{\frac{i}{n}} \right]$$

Example 22: Your home mortgage loan is $300,000 with an annual rate of 6.5% compounded monthly for the next 30 years, what is your monthly PMT?

$$\text{PMT} = \frac{300,000}{1 - (1 + \frac{0.065}{12})^{-360}} = \$1896.20$$

Excel: $=PMT\left(\frac{0.065}{12}, 12*30, 300000\right)$
Example 23: If your retirement account shows $312,000 on the day you retire and you plan to live to be 100 (you are 70) how much can you withdraw each month if you can invest in a 38-year bond that yields 5% compounded monthly?

Given:
- PV = $312,000
- r = 0.05
- n = 12

We need to find PMT using the formula for the present value of an annuity:

\[ PMT = \frac{PV \cdot \frac{r}{n}}{1 - \left(1 + \frac{r}{n}\right)^{-n \cdot x}} \]

Substituting the given values:

\[ PMT = \frac{312000 \cdot \frac{0.05}{12}}{1 - \left(1 + \frac{0.05}{12}\right)^{-12 \cdot 30}} \]

\[ PMT = \$1,674.88 \]

Excel: negative because on the day that you retire you put it back in the bank.

\[ \text{PMT}(\frac{0.05}{12}, 12 \cdot 30, -312000) = \$1,674.88 \]

If I can get 5% compounded monthly on my retirement fund I can withdraw $1,674.88 per month for 30 years.
Example 24:

If you have a home loan for $230,000 and the monthly PMT is $3,250 for 10 years, what is the APR & EAR?

\[ PV = 230,000 \]
\[ PMT = 3,250 \]
\[ n = 12 \times 10 = 120 \]
\[ \frac{APR}{n} = \text{period rate} = \text{RATE}(12 \times 10, -3,250, 230,000, 0) \]
\[ = 0.009686459 \]

\[ APR = \frac{APR}{n} \times n = 0.009686459 \times 12 = 0.116237506 \]

\[ EAR = \left(1 + \frac{APR}{n}\right)^n - 1 = \left(1 + \frac{0.116237506}{12}\right)^{12} - 1 = 0.12399428 \]

**Excel**: \[ \text{EAR} = \text{EFFECT}(0.116237506, 12 \times 10) = 0.12399428 \]

**Answer**: The APR on the loan is 11.62% and the EAR is 12.32%.

**Timeline**:

\[ PV = 230,000 \]

\[ PMT = -3,250 \]

\[ 0 \quad 1 \quad 2 \quad 3 \quad (Time) \ldots \quad 120 \]

\[ PMT = -3,250 \quad -3,250 \quad -3,250 \quad -3,250 \]
Example 25: How long until you pay off your credit card balance of $2,000 with an APR = 18% compounded monthly & a minimum balance paid each month of $41? 

\[ PV = 2000 \]
\[ i = 0.18 \]
\[ n = 12 \]
\[ \frac{i}{n} = 0.015 \]
\[ x = ? \]
\[ xn = ? \]
\[ PMT = 41 \]

\[ PV = PMT \left[ \frac{1 - (1 + \frac{i}{n})^{-nx}}{\frac{i}{n}} \right] \]

\[ 2000 = 41 \left[ \frac{1 - (1.015)^{-nx}}{0.015} \right] \]

\[ \left[ \frac{2000}{41} \right] \times 0.015 = 1 - (1.015)^{-nx} \]

\[ \left[ \frac{2000}{41} \right] \times 0.015 - 1 = -(1.015)^{-nx} \]

\[ -0.268292683 = -(1.015)^{-nx} \]

\[ 0.268292683 = (1.015)^{-nx} \]

\[ 0.268292683 = \frac{1}{(1.015)^{nx}} \]

\[ (1.015)^{nx} = \frac{1}{0.268292683} \]

\[ nx = \ln \left( \frac{1}{0.268292683} \right) = 88.36799 \]

It will take 88 1/3 months to pay off the $2000 credit card bill, or 7 1/3 years.
perpetuity (consol)

- Annuity where cash flow continues forever
- Preferred stock are considered perpetuities
  \[ \text{contract to get defined dividend } \frac{1}{2} \frac{D}{E} \]

\[
PV = PMT \left[ \frac{1 - (1 + \frac{i}{n})^{-nx}}{\frac{i}{n}} \right] = PMT \left[ \frac{1}{(1 + \frac{i}{n})^{nx}} \right]
\]

As \( x \) gets large (years are forever) \( (1 + \frac{i}{n})^x \)
approaches infinity, then, \( A \), \( (1 + \frac{i}{n})^x \) approaches infinity,
\( \frac{1}{\infty} \) approaches zero & \( \frac{1 - 0}{\frac{i}{n}} \)
becomes \( \frac{PMT}{\frac{i}{n}} \)

\[ PV = \frac{PMT}{\frac{i}{n}} \]
How much should you pay for preferred stock where it is assumed:

1. Quarterly dividend = $1.25
2. Quarterly discount rate = 4%

\[ PMT = \frac{1.25}{\frac{0.04}{n}} \]

\[ PV = \frac{1.25}{0.04} = 31.25 \]
Loans:

- **Interest only Loan**
  - pay fixed interest each period
  - pay back principal (loan amount) at end of term

  Example: Corporate Bonds

  Borrow 1,000,000 @ 10% compounded semi-annually for 5 years

  Interest = 1,000,000 * \( \frac{0.1}{2} \) = 50,000

  \[ 
  \begin{array}{c|c|c}
  \text{Period} & \text{Interest Paid} & \text{Principal Paid} \\
  \hline
  0 & 10,000 & \\
  1 & 5,000 & 2,500 \\
  2 & 2,500 & 2,500 \\
  3 & 1,250 & 2,500 \\
  4 & 0 & 2,250 \\
  \end{array}
  \]

  **Amortized Loans**: Each period payment is part principal & part interest

  - Medium-term business loans
    1. Each period Equal Principal is paid back
    2. Interest paid each period goes down
    3. Total payment each period goes down

  Example: Borrow 10,000 @ 10% compounded Annual for 4 years

  (Not in Excel workbook)
(B) Consumer loans (Amortized)

1. Each period PMT is equal
2. Each period amount of principal paid back goes up
3. Each period amount of interest paid goes down

\[
\text{Interest} = \{\text{Principal Balance}\} \times \{\text{Period Rate}\}
\]

\[
\{\text{paid on principal}\} = \text{PMT} - \text{Interest}
\]

Example 1: Mortgage = $325,000, \lambda = 7.75\% \text{, } n = 12, \text{ } x = 30

\[
PMT = \frac{PV}{\left[1-(1+\frac{\lambda}{n})^{-nx}\right]} = \frac{325000}{\left[1-(1+\frac{7.75\%}{12})^{-12 \times 30}\right]} = \#2328.34
\]

Example 2: \(PV = 10,000\) \(\lambda = .01\) \(n = 1\) \(x = 3\)

\[
PMT = \frac{10000}{1-(1+0.01)^{-3}} = 4021.15
\]

<table>
<thead>
<tr>
<th>Period</th>
<th>PMT</th>
<th>Interest</th>
<th>Principal Reduction</th>
<th>Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td>10000</td>
</tr>
<tr>
<td>1</td>
<td>4021.15</td>
<td>10000 * 0.01</td>
<td>4021.15 - 1000</td>
<td>6978.85</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>6978.85 * 0.01</td>
<td>4021.15 - 6978.85</td>
<td>3655.59</td>
</tr>
<tr>
<td>3</td>
<td>365.56</td>
<td></td>
<td>3655.59</td>
<td>0</td>
</tr>
</tbody>
</table>
pure Discount Loan

- Receive loan amount
- Pay back all interest & principal at end of loan term
- Use this method when cash flow for entity is limited until the end of term

Examples:
① US Treasury Bills (time < 1 year)
② Savings Bonds
③ Some Corp. Bonds

Examples not in Excel workbook:

Example 1:

\[ \text{FV} = 15,000 \]
\[ i = 0.0381 \]
\[ n = 1 \]
\[ x = 1 \]
\[ \text{PV} = \frac{15,000}{1.0381} = 14,494.89 \]

Example 2:

Corporate Bond \[ \text{FV} = 1,000,000 \]
\[ i = 0.07 \]
\[ x = 25 \]
\[ n = 1 \]
\[ \text{PV} = \frac{1,000,000}{(1.07)^{25}} = 184,249.18 \]