Interest Rates

Real vs. Nominal Rates:

**Real Rates**
1. The percentage change in buying power.
2. Rates that have been adjusted downward for inflation.
3. Base rate that does not take inflation into consideration.
4. Percentage change in how much you can buy with your money.

**Nominal Rates**
1. The percentage change in number of dollars that you have.
2. Rates that have NOT been adjusted downward for inflation.
3. Nominal Rates of interest include our desired real rate of return plus an adjustment for expected inflation.

Example 1 →
Money

Money is as money does

It is better to define money by its function:
- Buy stuff
- Get paid a wage
- Save

1. Money acts as a medium of exchange (without it, barter)

2. A standard of value, money is the unit in which the prices of goods & services are measured. 

   Game of value, 2 units = sweater, 10 units = toy, thus: 2 toys = 1 sweater

3. A store of value. A person can hold money & use it later.
Example 1

Jan 1, 2000 Milk cost = $3.39
Jan 1, 2011 Milk cost = $3.56

Milk inflation = \[
\frac{3.56}{3.39} - 1 = 0.050147
\]

Jan 1 2010 Buy 100 cartons = 3.39 * 100 = $339.00

Put in Bank & Earn 10%, n=1

APR (n=1) = Nominal = 10%

Jan 1 2011 FV = 339 * (1 + 0.1) = 372.90

We earn 10%, but can we buy 10% more cartons of milk?

Q: Can we buy 110 cartons of milk?

A: $372.90 in bank

\[
\frac{3.56}{\text{New Milk cost}} = 104.75
\]

Change in buying power = \[
\frac{104.75}{100} = 0.0475
\]

We earned a nominal rate of 10% but our buying power (for milk) went up by only 4.75%.
The Fisher Effect

(\text{Relationship between Real Rate, Nominal Rate, \\
\& Inflation})

\[ 1 + R = (1 + r) \times (1 + h) \]

\( R = \text{Nominal Rate} \) (Annual Rate with \( n=1 \))
\( h = \text{Inflation Rate} \)
\( r = \text{Real Rate} \)

Example 2

\[ R = 0.10, \quad h = 0.050147 \]

\[ 1 + 0.10 = (1 + r) \times (1 + 0.050147) \]

\[ \frac{1.10}{1.050147} - 1 = r \]

\[ 0.047472 = r \]

\( \frac{104.75}{100} - 1 = 0.0475 \)

Other formulas

\[ \frac{1 + R}{1 + h} = 1 + r \]
\[ \frac{1 + R}{1 + h} - 1 = r \]

\[ \frac{1 + R}{1 + r} = 1 + h \]
\[ \frac{1 + R}{1 + r} - 1 = h \]

\[ R = (1 + r) \times (1 + h) - 1 \]
\[ 1 + R = (1 + r)(1 + h) \]

\[ 1 + R = 1 + h + r + rh \]

\[ R = 1 - 1 + h + r + rh \]

\[ R = h + r + rh \]

- Compensation for decrease in value in original amount invested
- Compensation for decrease in value of interest earned

\[ R = h + r \]
Example 1 (2nd time)

Jan 1, 2010 Milk Cost = $3.39
Jan 1, 2011 Milk Cost = $3.56

Milk Inflation = \( \frac{\text{End}}{\text{Beg}} - 1 \) = \( \frac{3.56}{3.39} - 1 \) = 0.050147

Inflation went up, but what about the buying power of our money in the Bank?

Annual Rate \((n=1)\) = Nominal Rate = \(10\% = 0.10\)

\($3.39 \times (1 + 0.10) = $3.729 \)

We earn 10\%, but did our buying power increase by 10\%?

\(\frac{3.729}{3.56} - 1 = 0.0474191\)

or

\(\frac{1 + 0.10}{1 + 0.050147} - 1 = \frac{1 + \text{Nominal}}{1 + \text{inflation}} - 1 = 0.047\)

0.0474191 = Change in buying power
The Fisher Effect

Relationship between Real, Nominal & Inflation

\[ r = \frac{1 + R}{1 + h} - 1 \]

\[ R = \text{Nominal Rate (Annual Rate } n = 1) \]
\[ h = \text{Inflation Rate} \]
\[ R = \text{Real Rate} = \text{Change in Buying Power} \]

\[ r = \text{\% Change Buying Power} = \frac{1 + R}{1 + h} - 1 \]

Amount you would earn in 1 year

What you could purchase give an inflation rate increase in price of goods/service
Example 3:

If nominal rate is 12% and inflation is 6%, what is change in buying power?

\[ r = \frac{1.12}{1.06} - 1 = 0.0566 \]

Example 4:

If nominal rate is 1% and inflation is 1.5%, what is change in buying power?

\[ r = \frac{1.01}{1.015} - 1 = -0.004926 \]
Bonds

Loan contract = set of future cash flows

**Borrower**

Bond Issuer = Bond Seller = Liability = Corporation or Government Debt

**How cash Flows might look:**

Interest only loan or coupon bond * coupon = interest

\[ PV = 975.02 \] Amount received by bond issuer.

\[ PMT = -50 \]

\[ FV = -1000 \]

1. Periodic interest PMT or coupon PMT paid at end of each period.
2. Pay back principal & maturity.

**Deep Discount Loan or Zero Coupon Bond**

\[ PV = 725.25 \] Amount received by bond issuer.

\[ FV = -1000 \]

1. No periodic interest paid.
2. All interest & principal paid back at maturity.
Lender

Bondholder = Bond Buyer = Asset = Bondholder is buying set of Future Cash Flows

How cash flows might look:

Interest only or Coupon Bond * coupon = Interest

PV = -975.02 ← Amount paid for Future set of cash flows

↑
0 1 2 3 4 5 6
PMT = 50 PMT = 50 PMT = 50 PMT = 50 PMT = 50...

1. Periodic interest PMT or coupon PMT received at end of each period.
2. Receive principal amount at maturity.

Deep Discount Loan or Zero Coupon Bond

PV = -725.25 ← Amount paid for Future set of cash flows.

↑
0 1 2 3 4 5 6
FV = 1000

1. No periodic interest paid. 2. All interest & principal paid back at maturity.
Bond Terms:

1. Bond = Loan contract issued by corporation or government that can be bought or sold in financial markets.

2. YTM = Yield To Market = Discount Rate = Market Rate = Rate for similar securities.

3. Usually Different

4. Face Value = Amount of loan to pay back at maturity = "par value" = principal.

5. Coupon Payment = Interest payment.

6. Years until Maturity = years until pay back face value & last coupon payment.

7. Maturity = specific date when face value & last coupon payment is paid.
Similar capital structure

Similar business activities

Similar credit ratings

The similarities could be:

- and financial risks
- considered (different than the one being
- investment vehicles issued by other
- securities are bonds or other

What are similar securities?
\[
\text{Bond Value} = \text{PMT} \times \left[ \frac{1 - \left(1 + \frac{\text{YTM}}{n}\right)^{-\text{nx}}}{\frac{\text{YTM}}{n}} \right] + \frac{\text{FV}}{(1 + \frac{\text{YTM}}{n})^{\text{nx}}} \\
= \text{PV} \left( \text{rate}, \text{nper}, \text{PMT}, \text{FV} \right) \\
\]

<table>
<thead>
<tr>
<th>Math</th>
<th>Bond Terms</th>
<th>Excel</th>
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<tbody>
<tr>
<td>FV</td>
<td>Face Value</td>
<td>FV</td>
</tr>
<tr>
<td>PV</td>
<td>Bond Value</td>
<td>PV</td>
</tr>
<tr>
<td>i</td>
<td>YTM</td>
<td>Rate</td>
</tr>
<tr>
<td>n</td>
<td># compound periods per year usually 2</td>
<td>n per</td>
</tr>
<tr>
<td>X</td>
<td>Years until Maturity</td>
<td></td>
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<tr>
<td>n*x</td>
<td>Total number of periods</td>
<td>n per</td>
</tr>
<tr>
<td>\frac{i}{n}</td>
<td>\frac{\text{YTM}}{n} = Discount Rate</td>
<td>Rate</td>
</tr>
<tr>
<td></td>
<td>= Market Rate</td>
<td></td>
</tr>
<tr>
<td>PMT</td>
<td>Periodic Interest ( \text{PMT} = \frac{\text{Coupon Rate} \times \text{Face Value}}{n} )</td>
<td>PMT</td>
</tr>
</tbody>
</table>
Zero coupon Bonds (Pure Discount Loan or "zeroes"):

- A bond that makes no coupon payments, and thus is initially priced at a deep discount.

- From the Borrower’s point of view: "Borrow an amount today, then pay back principal & all interest at the end of the loan period."

- From the Lender’s (Bondholder’s) point of view: "Lend (pay) an amount today, and receive all principal & interest at the end of the loan period."

Zero Bond value

\[
    \text{Zero Bond value} = \frac{FV}{(1 + \frac{YTM}{n})^{nx}}
\]

Point of view of Bondholder

\[
    = PV(\text{rate}, \text{nper}, FV) = PV(\frac{YTM}{n}, nx, FV)
\]

Note: Interest on zeroes is:

- Deductible for income tax for Bond Issuer
- Taxable for income tax for Bondholder

Why issue or buy zeroes?

- Bond Issuer likes cash flow advantage of Deductible Non-cash Expense
- Bondholder likes predictability of future Bond Income
1. To solve for the YTM \( \frac{YTM}{n} \) = \{Discount Rate per period\}, use:

\[ = RATE(n \times x, -PMT, PV, -FV, 0) \]

Excel:

Bond Issuer's Point of View

2. To solve for the Effective Annual Yield, use:

\[ = EFFECT(YTM, n) \]

Excel:

Math:

\[ \left(1 + \frac{YTM}{n}\right)^n - 1 \]

3. To solve for periodic coupon PMT, use:

\[ = PMT\left(\frac{YTM}{n}, n \times x, -PV, FV, 0\right) \]

Excel:

Bondholders Point of View
Q: For the coupon bond, how come bond issuer got only $975.02 instead of $1000?

A: (1) Because corporate & government bonds are bought & sold in the financial markets, and market rates change often. (Because of new information).

(2) This means the interest rate written in the bond contract is usually different from the interest rate in the financial markets.
Corporate & Investment Bank write up Bond contract & say they will pay 10%, n=2 coupon payments, 4 months before they issue Bond.

But on the day they sell the bond contract the interest rates have changed. Coupon Rate = 10% YTM = 11%

Later, if someone buys the bond from the original bondholder, the rates have changed again.

Why do rates change? In financial markets each day new information about the economy & business cause people to change their rates (risk).
Example 6: Coupon Bond for Issuer

Bond Issuer = Corporation
Face = $1000 (corp. usually issues many bonds)

Coupon Rate = 10% Determines Interest Paid

\[ \text{Coupon Rate} = \frac{10\%}{2} = 5\% \]

\[ YTM = 11\% \]
\[ \frac{YTM}{n} = \frac{11\%}{2} = 5.5\% \]

Years to Maturity = 3
Total periods = 3*2 = 6
Coupon PMT = 5% * 1000 = $50

\[ \text{PV} = \text{SOLVE FOR THIS} \]

\[ \text{PV} = \frac{1000}{(1+.055)^6} + 50 \times \frac{1-(1+.055)^{-6}}{.055} \]

"sold at Discount"

\[ \text{PV} = 725.245833 + 249.07765154 \]
\[ \text{PV} = 974.32 \]

\[ \text{PV} (\text{rate, nper, PMT-FV}) = \text{PV} (\frac{YTM}{n}, n \times x, -\text{PMT}, -\text{FV}) \]
\[ = \text{PV} (.055, 6, -50, -1000) = 975.02 \]
{Corp Receives $}
Example 7: Zero coupon for Issuer

Bond Issuer = Corporation
Face Value = $1000 (corp usually issues many billions)
Coupon Rate = 10% (Determines Interest Paid)

\[ n = 2 \]
\[ \left\{ \frac{\text{coupon rate}}{2} \right\} = \frac{10\%}{2} = 5\% \]

Years to Maturity = 3
Total periods = 3 * 2 = 6
NO COUPON PMT!!!

\[ PV = \text{SOLVE FOR THIS} \]

\[ FV = -1000 \]

\[ PV = \left\{ \text{cash coming into corporation} \right\} = \left\{ \text{PV of Future Lump Sum} \right\} = \frac{FV}{(1 + \frac{YTM}{n})^n} \]

\[ PV = \frac{1000}{(1 + .055)^6} = \$725.25 \]

\[ = PV(\text{rate, nper, -FV}) = PV\left(\frac{YTM}{n}, n \times x, -FV\right) \]

 Skip PMT "sold at Discount"

\[ = PV(.055, 6, -1000) = \$725.25 \]
Example 8: Coupon Bond for Bondholder

Details same as example 6; but for Bondholder

\[ PV = \text{SOLVE FOR THIS} \]

\[
\begin{array}{ccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 \\
0 & \text{PMT} + 50 & \text{PMT} + 50 & \text{PMT} + 50 & \text{PMT} + 50 & \text{PMT} + 50 & \text{FV} = +1000 \\
\end{array}
\]

\[
= PV \left( \frac{\text{rate}, n, \text{PMT}, FV}{n} \right) = PV(\frac{0.05}{6}, 6, 50, 1000) = - \$ 975.02 \left( \text{Bought at Discount} \right)
\]

Bond holder is willing to pay \$ 975.02 for set of future cash flows !!

Example 9: Zero coupon Bond for Bondholder

Details same as example 7; but for Bondholder

\[ PV = \text{SOLVE FOR THIS} \]

\[
\begin{array}{ccccccc}
0 & 1 & 2 & 3 & 4 & 5 & \ldots & 6 \\
0 & \text{FV} = +1000 \\
\end{array}
\]

\[ PV = \text{PV} \left( \frac{\text{YTM}}{n}, n, X, FV \right) = \text{PV}(0.05, 6, 1000) = \]

\[ PV = -725.25 \]

Bondholder is willing to pay \$ 725.25 now in order to receive all the interest & principal ($1000) later.
Example 10:  

Bondholder #1 sells Bond to Bondholder #2 at the end of year 2, and the rate in the market (YTM, or Rate on "similar security") is 9%.

Bondholder #1 = Sells Bond
Bondholder #2 = Buys Bond in secondary market

Face value = $1000 = FV
Coupon rate = 10% (Determines Interest Paid)

\[
\text{YTM} = \frac{\text{Coupon Rate}}{2} = \frac{10\%}{2} = 5\%
\]

\[
\text{YTM} = \frac{9\%}{2} = 4.5\%
\]

Years To Maturity = 1
Total Periods = 1 * 2
Coupon Payment = \( PMT = 1000 \times 0.05 = $50 \)

---

Coupon Bond for Bondholder #2

\[
PV = \text{Solve For This}
\]

\[
0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6
\]

\[ PV = \text{Solve For This} \]

\[ PMT = +50 \]

\[ FV = +1000 \]

\[ PVT = +50 \]

\[ PMT = +50 \]

at Time 4

\[
PV = \left( \frac{\text{YTM}}{n}, n \times x, PMT, FV \right) = PV \left( 0.05, 2, 50, 1000 \right) = -$1009.36
\]

Bondholder #2 pays Bondholder #1 $1009.36 now in order to get future cash flows from corporation. "Bought at Premium"
Example 10 Continued:

Bondholder #1 (Now a Bond seller) in secondary market.

Market Rate (YTM) is now only 9%.

"Bought at Premium" for $1,009.36.

Bondholder #2 pays $1,009.36 for Bond contract with 1 year left.

PV = 1,009.36

1. Present value of all future cash flows at YTM of 9% is Bond Value.
2. Bond with a 10% coupon is priced to yield 9% at $1,009.36.
Example 11.8

But what if the buyer & the seller just had price = $1,009.36 & not the YTM? Could we figure out YTM?

Sell price for Bond = $1,009.36 = PV
Face = $1000 = FV
Semi-annual Interest payment = $50
Total number of periods left = 2

\[
\frac{YTM}{2} = ?
\]

\[
= RATE(nper, pmt, pv, fv) = \frac{YTM}{2} =
\]

\[
= RATE(2, 50, -1009.36, 1000) \approx 0.045
\]

\[
\frac{YTM}{2} = 0.045 \implies YTM = 0.045 \times 2 = 0.09 = 9%
\]

Effective Annual Yield = \((1 + \frac{YTM}{n})^n - 1\) = 1.045^2 - 1

= 0.092025 \implies 9.2% 

"1 Year Bond with a 10% coupon is priced to yield 9% at $1,009.36."

"Bond is priced at a Premium."

"Bond's Effective Annual Yield is 9.2%."
Example 12:

Sometimes you see bond prices quoted like this:

1.00936

or

100.936%

Or it is written like this:

"A year 10% coupon bond is priced at 100.936%" (assumed to be semiannual)

From this statement we can get info.

Face = $1000 = FV

Semiannual coupon = $50 = PMT

Price = 1000 * 1.00936 = $1009.36

Years to Maturity = 1

Total periods = 2 (assumed n = 2)

\[
\text{YTM} = \text{RATE}(\text{nper}, \text{PMT}, -\text{PV}, \text{FV}) \times n
\]

\[
\text{YTM} = \text{RATE}(2, 50, -1009.36, 1000) \times 2 = 0.0900035
\]
Three Principles in Bond Finance

1. Rates are inversely related to price
   - YTM ↑ then Bond Price ↓

   \[
   \begin{align*}
   \text{Face value} & = 1000 \\
   \text{Coupon Rate} & = 6\% \\
   \text{YTM} & = 6\% \\
   \text{Years} & = 30 \\
   n & = 2 \\
   \text{Price} & = \$1000 \\
   \text{Sell at Par}
   \end{align*}
   \]

   \[
   \begin{align*}
   \text{Face value} & = 1000 \\
   \text{Coupon Rate} & = 6\% \\
   \text{YTM} & = 7\% \\
   \text{Years} & = 30 \\
   n & = 2 \\
   \text{Price} & = \$875.28 \\
   \text{Sell at Discount}
   \end{align*}
   \]

   \[
   \begin{align*}
   \text{Face value} & = 1000 \\
   \text{Coupon Rate} & = 6\% \\
   \text{YTM} & = 5\% \\
   \text{Years} & = 30 \\
   n & = 2 \\
   \text{Price} & = \$1,154.54 \\
   \text{Sell at Premium}
   \end{align*}
   \]

2. Bonds sell at Par, Discount or Premium

   - Coupon Rate = 6\%
     - YTM = 6\%
     - Face = Price
     - 1000 = 1000
     - C.R. = YTM
     - 6\% = 6\%
     - PAR

   - Face > Price
     - 1000 > 875.28
     - C.R. < YTM
     - 6\% < 7\%
     - Discount

   - Face < Par
     - 1000 < 1,154.54
     - C.R. > YTM
     - 6\% > 5\%
     - Premium

3. The more years to Maturity, the higher the Interest Rate Risk becomes.

   - The longer to maturity, the more the YTM affects the Bond price.
   - For long maturity (years = 30), a small increase in YTM can cause a large drop in bond value.
   - Low coupon rates have more risk than large coupon rates because of pattern of cash flows.
Selling Price For Bond

Premium Record Bond At

Discount Record Bond At

No Premium Or Discount

1.07% (Example: 107 or Above 1.00)

1.00% (Example: 100 or 100)

0.93% (Example: 93 or Below 1.00)
rate, the greater the interest rate risk.

All things being equal, the lower the coupon maturity, the greater the interest rate risk.

All things being equal, the longer the time to:

- The sensitivity depends on two things:
  - Change

  How sensitive its price is to interest rate change:

  How much interest rate a bond has:

- Fluctuating interest rates:

- The risk that arises for bond owners from:

  Interest Rate Risk
Interest Rate Risk And Time To Maturity

Value of a Bond with a 10 Percent Coupon Rate for Different Interest Rates and Maturities

<table>
<thead>
<tr>
<th>Time to Maturity</th>
<th>Interest Rate</th>
<th>Bond Value ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 Years</td>
<td>5%</td>
<td>$1,768.62</td>
</tr>
<tr>
<td>1 Year</td>
<td>7%</td>
<td>$1,047.62</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>1,000.00</td>
</tr>
<tr>
<td></td>
<td>15%</td>
<td>671.70</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>502.11</td>
</tr>
</tbody>
</table>

Interest (rate (%))
(Bond with larger coupon rate has a larger value proportionally more dependent on the face value)

1. Bond with lower coupon rate

2. Interest rate changes

3. Smaller changes in market rate have substantial affect on bond value

4. Longer time to maturity

Interest Rate Risk To Loss Of Principal (Current price)
Decreasining Rate
Interest Rate Risk Increases At A
<table>
<thead>
<tr>
<th>Bonds</th>
<th>Stocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Bring cash into Corp.</td>
<td>1. Bring cash into Corp.</td>
</tr>
<tr>
<td>2. Debt to Corp.</td>
<td>2. Equity to Corp.</td>
</tr>
<tr>
<td>3. Bond holders are creditors</td>
<td>3. Stockholders are owners</td>
</tr>
<tr>
<td>4. Interest must be paid (Fixed claim to cash flow)</td>
<td>4. Dividends are not a liability - they are paid only if Board of</td>
</tr>
<tr>
<td></td>
<td>Directors Declare them (residual claim to cash flow)</td>
</tr>
<tr>
<td>5. Bondholders get paid interest &amp; principal, but no more.</td>
<td>5. If corporation is very successful, owners may get paid a lot of</td>
</tr>
<tr>
<td></td>
<td>Dividends (No upper limit)</td>
</tr>
<tr>
<td>7. Interest is tax deductible</td>
<td>7. Dividends are not tax deductible</td>
</tr>
<tr>
<td>8. Bondholders have first claim in Bankruptcy,</td>
<td>8. Stockholders are in line behind creditors in Bankruptcy</td>
</tr>
<tr>
<td>9. Excess debt can lead to bankruptcy</td>
<td>9. An all equity firm can not go bankrupt.</td>
</tr>
<tr>
<td><strong>Government Bonds</strong> (USA biggest borrower in world)</td>
<td></td>
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<tr>
<td>---------------------------------------------------</td>
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<tr>
<td><strong>Federal Level</strong></td>
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<tr>
<td><strong>Treasury Bill</strong></td>
<td></td>
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<tr>
<td>Years &lt; 1</td>
<td></td>
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<tr>
<td><strong>Treasury Note</strong></td>
<td></td>
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<tr>
<td>1 ≤ Years ≤ 7</td>
<td></td>
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<tr>
<td><strong>Treasury Bonds</strong></td>
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<tr>
<td>other</td>
<td></td>
</tr>
<tr>
<td><strong>Treasury Debt</strong> (Fed. Gov.)</td>
<td></td>
</tr>
<tr>
<td>1. considered default free</td>
<td></td>
</tr>
<tr>
<td><em>No default risk</em></td>
<td></td>
</tr>
<tr>
<td>2. Taxed at Federal Level only (Not State).</td>
<td></td>
</tr>
<tr>
<td>3. Highly Liquid to Bondholder.</td>
<td></td>
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</tbody>
</table>

**Municipal Bonds** "Munis":

- Exempt from Federal Income Tax

Example: If your tax bracket is 25% for Federal Income tax, would you prefer 5% corporate bond or 3.9% muni bond?

\[
0.039 > 0.05 \times (1 - 0.25) = 0.05 \times 0.75 = 0.0375 \]

0.039 Muni > 0.0375 corporate