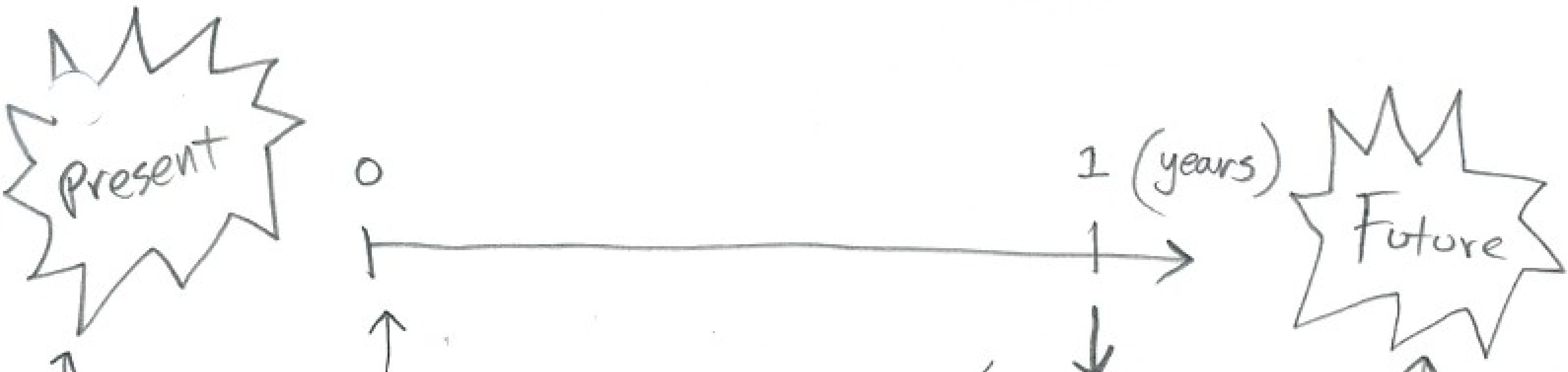
Chapter 4

Introduction to Valuation: Time Value of Money

Lump sum calculations: 1. Present value
2. Future value



$$\$1^{\underline{00}}$$

put \$1 in Bank that pays 10% Interest Rate compounded 1 time a year. (cash Flow to you is −1 because it comes out of your pocket).

Bank pays 10% Interest

$1 + (1 * .1)$

$1 + .10$

$\$1^{\underline{10}}$

original Investment

Interest paid

$$\$1^{\underline{.10}}$$

original Investment plus interest paid. (cash Flow to you is +1.10 because it comes into your pocket).

The Present value is $\$1^{\underline{00}}$

The Future value is $\$1^{\underline{10}}$

Formula to Calculate Future Value

$$FV = PV\left(1+\frac{i}{n}\right)^{n*x}$$

text book:

$$\boxed{FV = C*(1+r)^t}$$

$\frac{i}{n}$ = period Rate

$n*x$ = Total # periods

FV = Future Value

PV = Present Value

i = Annual Interest Rate (also: APR)

n = number of compounding periods per year

x = # of years

Example 1: If you put \$10,000 in bank at an annual rate of 6%, compounded monthly for 10 years how much will you have at maturity?

Future Value = FV = ?

Present Value = PV = how much you deposit today = \$10,000

Annual Rate i = 6% or 0.06

compounding per year n = 12

years x = 10

$$FV = 10{,}000\left(1+\frac{.06}{12}\right)^{10*12}$$

$$= 10{,}000\,(1+.005)^{120}$$

$$= 10{,}000\,(1.005)^{120}$$

$$= 10{,}000 * 1.8193967340328$$

$$= \$18{,}193.97$$

Excel FV Function

Future value $= FV = FV$

Present value $= PV = PV$

period rate $= \frac{i}{n} =$ rate

Total Periods $= x*n =$ nper

$= FV(\text{rate}, \text{nper}, , -PV)$

skip PMT argument

PV negative because FV understands cash flow. Putting in bank is a negative cash flow to you

$= FV\left(\frac{i}{n}, x*n, , -PV\right)$

$= FV(.005, 120, , -10000)$

$= \$18,193.97$

Interest : ① "Rent on money"

② when you use someone else's money, you must pay them to use the money: The payment given to use the money is called interest.

③ when you Borrow money, you **pay** interest

④ when you Lend money, you receive interest

Interest is in dollars

Interest Rate is in decimal or fractional or percentage terms (percentage of investment or loan).

Simple Interest :

Interest Earned on the original investment only (or paid on original loan).

Example 2:

PV = investment = \$100 (cash out of pocket)

i = Annual Interest Rate = .10 or 10%

n = compounding periods per year = 1

x = years = 4

FV = Future value of investment (Lump sum) = ?

simple interest = \$100.00 * .1 = \$10.00

	Interest	Amount in Bank
year 0		\$100
year 1	\$10	10 + 100 = \$110
year 2	\$10	10 + 110 = \$120
year 3	\$10	10 + 120 = \$130
year 4	\$10	10 + 130 = \$140

Compound Interest:

Interest earned on both original investment and interest reinvested from prior periods.

Example 3:

$PV = \$100$	Investment
$i = .10$ or 10%	Annual Interest Rate
$n = 1$	# compounding periods per year
$x = 4$	years
$FV = ?$	Future value

$FV_{year1} = 100 + 100 * .1 = 100 + 10 = 110$

$FV_{year 2} = 110 + 110 * .1 = 110 + 11 = 121$

$FV_{year 3} = 121 + 121 * .1 = 121 + 12.1 = 133.10$

$FV_{year 4} = 133.10 + 133.10 * .1 = 133.1 + 13.31$

$= 146.41$

Compound Interest $FV \Rightarrow \$146.41$
Simple Interest $FV \Rightarrow -140.00$

Interest on Interest $= \$6.41$

Interest earned on the reinvestment of previous interest payments

Compounding:

The process of accumulating interest in an investment over time to earn more interest

Math:

$PV = \$100$

$i = .10$

$n = 1$

$x = 1$

$FV = ?$

original investment

interest

$FV = 100 + 100 * .1$

$FV = 100 + 10$

$FV = 110$

Notice:

100 in both places

$100 + 100 * .1$

Plus sign

Notice:

$100 * 1 + 100 * .1$

If we put 1 here, it is still the same

Notice that we can factor: (distributive property) backwards

$100 * 1 + 100 * .1$

$100 * (1 + .1) = 100 * (1 + .1) = 100 * 1.1$

Conclusion: $100 + 100 * .1 = 100 * 1.1$

Derive Easier Fomula for Compound Interest

Lump sum Future value calculation:

Present value $PV = 100$

Annual Rate $i = .10$

com. periods per year $n = 1$

year $x = 4$

Future value $FV = ?$

factor out

$$FV_1 = 100 + 100 * .1 = 100 + 10 = 110 = 100(1+.1)$$

$100(1+.1)$ *

$100(1+.1) = 100*(1+.1)$

NO multiplication sign

multiplication sign

$\boxed{\text{Thus}}$

$$110 = 100(1+.1)$$

$$FV_2 = 110 + 110 * .1$$

$\boxed{\text{Substitute}}$

$$= 100(1+.1) + 100(1+.1) * .1$$

$\boxed{\text{Factor out}}$

$$= 100(1+.1) * 1 \quad 100(1+.1) * .1$$

$$= 100(1+.1)(1 + .1)$$

$$= 100(1+.1)^2 = 121$$

$$FV_3 = 121 + 121 * .1$$

$\boxed{\text{Substitute}}$

$$= 100(1+.1)^2 + 100(1+.1)^2 * .1$$

$$= 100(1+.1)^2(1+.1) = 100(1+.1)^3 = 133.10$$

$$FV_4 = 100(1+.1)^3 + 100(1+.1)^3 * .1$$

$$= 100(1+.1)^3(1+.1) = 100(1+.1)^4 = 146.41$$

Fundamental Truth in Finance

A dollar received today is worth more than a dollar received later. (This is true because of interest).

0 1 2 (years)

$\$1^{00}$ — dollar received Today

$\$1^{00}$ — dollar received in one year

which would you prefer?

If you took the $1 today, you can Invest that $1 and earn interest

0 1

Present value $\$1^{00}$

Bank pays 10% Interest

$1 * (1.1) =$ $\$1^{10}$

Future value

After you invest for one year and earn interest, now you can compare the $\$1^{10}$ you received to the $1 you would have (Future value) in one year. Is $\$1^{10} > \$1^{00}$? Yes! This illustrates the Fundamental Truth in Finance.