Borrow or Lend money?

These are flip sides of the coin.

① When you borrow money, someone else lends.

② when you lend money (deposits in bank are loans to bank), someone else borrows.

Interest = pay rent to use someone else's money.

① when you borrow money you must pay interest (interest expense, or cash going out)

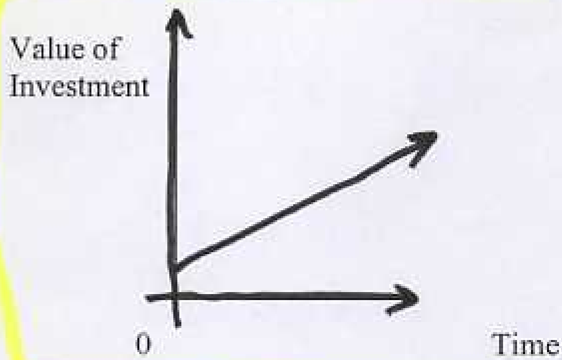
② when you lend money you receive interest (Interest Revenue, or cash received)

{The amount you borrow or lend} = Principal = {value of loan or investment}

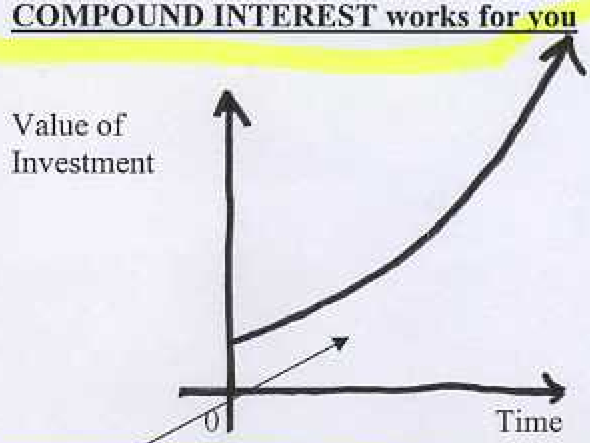
{If you borrow money} = loan

{If you lend money} = investment

SIMPLE INTEREST



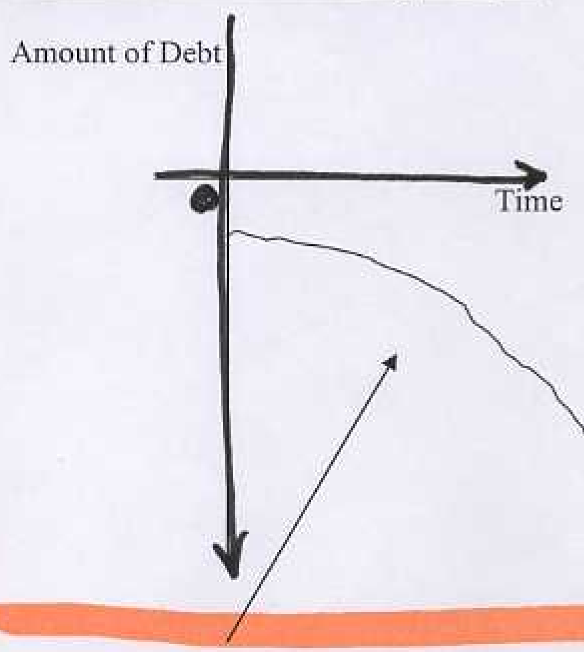
COMPOUND INTEREST works for you



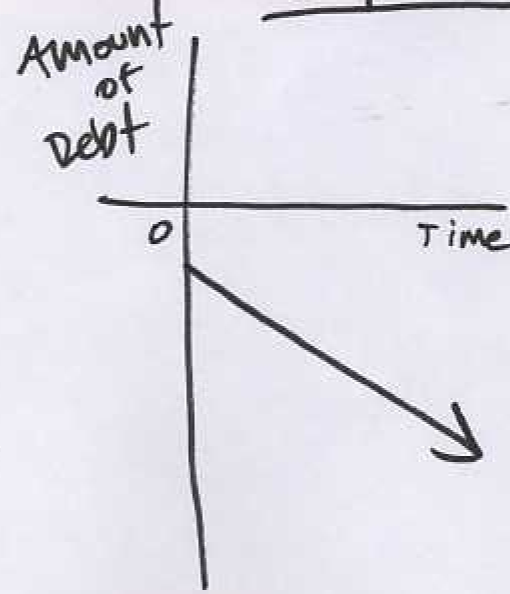
If you deposit $, As time goes by your investment grows

THE POWER OF COMPOUND INTEREST WORKING FOR YOU

COMPOUND INTEREST works against you



Simple Interest



If you borrow $ and pay it back all (principal and interest) at the end, as time goes by the amount you owe grows

THE POWER OF COMPOUND INTEREST WORKING AGAINST YOU

Fundamental Truth in Finance:

- A dollar received now is worth more than a dollar received later.
- This is true because of the ability of individuals to earn interest.

Because you can take the $1.00 today and put it in the bank

Today — 1 year from today

$1 received today — $1 received 1 year from today

The $1 received today is worth more than a dollar received 1 year from now because you can invest the dollar and earn interest.

If you invest the $1.00 in a bank account that earns 10% annual interest:

Today — (at 10% annual rate) — 1 year from today

$1.00 → grows to → $1.10

The terms we will use for this chapter are:

Present Value (Interest going backwards)

Future Value (Interest going forward)

The Present Value of the $1.00 I receive today is $1.00. But if I put it in the bank & earn 10% Annual Rate compounded once for the year, The Future Value of that $1.00 will be $1.10. Therefore I would rather receive the $1.00 today instead of waiting 1 whole year for $1.00.

"The future value of $1.00 is $1.10"
"The Present Value of $1.10 is $1.00"

9.1) Compound Interest

a. Find compound interest

Simple Interest:
Interest paid on only the principal.

Compound Interest:
1) Interest paid on principal and past interest
2) Compound interest is calculated on any interest previously paid to the account in addition to the original principal

Compound Amount:
Total amount that an investment earns at maturity (FV) MV = Maturity value or FV = Future value

Interest Earned:
Interest Earned = Compound Amount – Original Amount
**Works for both simple and compounded interest

Formula

Example:
Imagine you have deposited money in two different accounts:
1) $1000 in an account that pays 10% interest compounded annually
2) $1000 in an account that pays 10% simple interest
   a. If you invest the money in each account for 4 years, what are the compound amounts for each account?
   b. What is the difference between the two?
   c. Which type of interest payments earns more?

x = years = 4
n = number of compounding periods per year = 1
i = 10%
principal = present value = P = $1000.00

compound amount = maturity value = future value = FV = M

Find FV for 10% compounded Annually
Find FV for 10% simple interest

Find FV for 10% compounded Annually

After Year 1

I = PRT = 1000 * .1 * 1 = $100

FV at end of year 1 = 1000 + 100 = $1100

After Year 2

I = PRT = 1100 * .1 * 1 = $110

FV at end of year 2 = 1100 + 110 = $1210

After Year 3

I = PRT = 1210 * .1 * 1 = $121

FV at end of year 3 = 1210 + 121 = $1331

After Year 4

I = PRT = 1331 * .1 × 1 = $133.10

FV at end of year 4 = 1331 + 133.10 = $1464.10

Find FV for 10% simple Interest

I = PRT = 1000 * .1 * 4 = $400

FV at end of year 4 = 1000 + 400 = $1400

Future value = Maturity value = compound Amount = FV = M

FV w/ compound interest = 1464.10

− FV w/ simple Interest = 1400.00

= {Compound Interest earns this much more} ⇒ 64.10

Compound amount for $1000 in an account that pays 10% interest compounded annually:

After year 1:

I = PRT = 1000 x .10 x 1 = $100

Value at end of 1 year = 1000 + 100 = $1100

After year 2:

I = PRT = 1100 x .1 x 1 = $110

Value at end of 2 years = 1100 + 110 = $1210

After year 3:

I = PRT = 1210 x .1 x 1 = $121

Value at end of 3 years = 1210 + 121 = $1331

After year 4:

I = PRT = 1331 x .1 x 1 = $133.10

Value at end of 4 years = 1331 + 133.10 = $1464.10

Compound amount for compound interest = $1464.10

$1000 in an account that pays 10% simple interest:

After year 1:

I = PRT = 1000 x .10 x 1 = $100

Value at end of 1 year = 1000 + 100 = $1100

After year 2:

I = PRT = 1000 x .1 x 1 = $100

Value at end of 2 years = 1100 + 100 = $1200

After year 3:

I = PRT = 1000 x .1 x 1 = $100

Value at end of 3 years = 1200 + 100 = $1300

After year 4:

I = PRT = 1000 x .1 x 1 = $100

Value at end of 4 years = 1300 + 100 = $1400

Compound amount for simple interest = $1400

when we compare simple interest and compound interest our conclusions are

What is the difference between the two?

Maturity value with compounded interest =	$1464.10
Maturity value with simple interest =	-$1400.00
Compounded interest earned this much more	= $64.10

Which type of interest payments earns more?

Account with compound interest always earns more!

Account with Compounded Interest:

Interest = Compound Amount – Original Amount

Compounded Interest = 1464.10 – 1000 = $464.10

Account with Simple Interest:

Interest Earned = Compound Amount – Original Amount

Simple Interest = 1400 – 1000 = $400

b. Decide on a period of compounding

Period Interest Rate: → Book calls this "Interest Rate per Compounding Period"

The interest rate earned for a stated period of time.

Hints for compounding interest:

- Compound Interest is often calculated more than one time a year
- Interest rates are usually given in annual terms
- This necessitates that you convert the annual interest rate to a period interest rate ← Interest is paid each ½ year

Example:

semi = half
semi annual = ½ year

If the 10% annual interest rate is compounded semiannually, what is the period interest rate?

i = Annual Interest Rate = 10%

n = {number of compounding periods per year} = 2

$$\{\text{Period Interest Rate}\} = \frac{i}{n} = \frac{.10}{2} = .05 \text{ or } 5\%$$

Example:

If the 10% annual interest rate is compounded quarterly, → ¼ what is the period interest rate?

i = Annual Interest Rate = 10%

n = {number of compounding periods per year} = 4

$$\{\text{Period Interest Rate}\} = \frac{i}{n} = \frac{.10}{4} = .025 \text{ or } 2.5\%$$

Example:

If the 10% annual interest rate is compounded daily, what is the period interest rate?

i = Annual Interest Rate = 10%

n = {number of compounding periods per year} = 365 (for none leap years)

$$\{\text{Period Interest Rate}\} = \frac{i}{n} = \frac{.10}{365} =$$

c. Use the formulas to find compound amount and compound interest

$$\frac{.10}{365} = .00027397260273926 \approx .0274\%$$

Most saving Accounts earn daily interest

Exponent Lesson: "carrot"

[redacted]

2^4 or $2\wedge4$

$= 2^4 = 2*2*2*2$

$= 4*2*2$

$= 8*2$

$= 16$

Remember:

Compound amount for $1000 in an account that pays 10% interest compounded annually:

After year 1:

I = PRT = 1000 x .10 x 1 = $100

Value at end of 1 year = 1000 + 100 = $1100

After year 2:

I = PRT = 1100 x .1 x 1 = $110

Value at end of 2 years = 1100 + 110 = $1210

After year 3:

I = PRT = 1210 x .1 x 1 = $121

Value at end of 3 years = 1210 + 121 = $1331

After year 4:

I = PRT = 1331 x .1 x 1 = $133.10

Value at end of 4 years = 1331 + 133.10 = $1464.10

Instead of ALL This, you could do this

Alternative method of calculating:

1000 x (1 + .1) x (1 + .1) x (1 + .1) x (1 + .1) = $1464.10

OR EVEN Better!!

$FV = M = 1000(1 + .1)^4 = \$1464.10$

FV = M = P x (1 + i) x (1 + i) x (1 + i) x (1 + i)...

$$FV = M = 1000 * \left(1 + \frac{.1}{1}\right)^{(4*1)}$$
$$= 1000 * \left(1 + \frac{.1}{1}\right)^4$$
$$= 1000 * (1 + .1)^4$$
$$= 1000 * (1.1)^4$$
$$= 1000 * 1.4641$$
$$= \$1464.10$$

OUR Offical Formula for this chapter

Formula from book:

$FV = M = P(1 + i)^n$

M = Maturity Value (Future Value)
P = Principal or amount deposited in account
i = interest rate per period
n = Total number of periods

(★) Note: we will [redacted] not use numbers from compound Interest Tables in text book. we will use formulas ↴

Formula that will work for all problems (no matter what the number of periods or period rate):

$FV = M = P(1 + i/n)^{nx}$

M = Maturity Value = Future Value
P = Principal amount invested
i = Annual interest rate
n = Number of periods in one year
x = Number of years

"Carrot" means exponent

FV = M = P * (1 + i/n) ^ (n * x)

All synonyms

FV = M = Future Value = Maturity Value = Compound Amount

P = Principal = Present value

i = Annual Rate

n = number of compounding Periods per Year

x = Years

x * n = total number of compounding periods

i/n = period interest Rate

in Excel we can use FV function

=FV(period interest Rate, total number of compounding periods, , -Present value)

★ Interest is paid each ½ year

Example:

If you deposit $2000 in and account that pays 10% annual interest compounded semiannually, what is the value of the account after 10 years? What is the total interest earned?

Step 1: List variables

½ year means n=2

$P = 2000$

$i = 10\%$

→ $n = 2$

$x = 10$

Step 2: use formula to find value of account at maturity

Remember order of operations!

$FV = P * (1 + \frac{i}{n})^{\wedge(n*x)}$

$= 2000 * (1 + \frac{.10}{2})^{\wedge(2*10)}$

$= 2000 * (1 + \frac{.10}{2})^{\wedge 20}$

$= 2000 * (1 + .05)^{\wedge 20}$

$= 2000 * (1.05)^{\wedge 20}$

$= 2000 * 2.6532977051442$

$= 5306.5954 \approx \$5306.60$

[redacted]

**the number from the formula is more accurate than the number from the Compound Interest Table

Step 4: How much interest was earned?

$5306.60 - 2000 = 3306.60$ Interest was earned!

Difference between Compound Interest and Simple interest:

- With compounded Interest relatively small differences in interest rates can added up over many compounding periods

How to distinguish the difference between Simple Interest Problems and Compounded Interest Problems:

Simple Interest Problems:

"Simple Interest" "Simple Interest Notes" "Discount Rate"

Compounded Interest Problems:

"Compounded Annually" "6% per Quarter" "Compounded Daily"

} Hint for WORD problems

★ Interest is paid each 1/2 year

Example:
If you deposit $2000 in and account that pays 10% annual interest compounded semiannually, what is the value of the account after 10 years? What is the total interest earned?

Step 1: List variables

1/2 year means n=2

$P = 2000$
$i = 10\%$
→ $n = 2$
$x = 10$

Step 2: use formula to find value of account at maturity

Remember order of operations!

$FV = P * (1 + \frac{i}{n})^{\wedge (n * x)}$
$= 2000 * (1 + \frac{.10}{2})^{\wedge (2*10)}$
$= 2000 * (1 + \frac{.10}{2})^{\wedge 20}$
$= 2000 * (1 + .05)^{\wedge 20}$
$= 2000 * (1.05)^{\wedge 20}$
$= 2000 * 2.6532977051442$
$= 5306.59|54 \approx \$5306.60$

[redacted]

**the number from the formula is more accurate than the number from the Compound Interest Table

Step 4: How much interest was earned?

5306.60 − 2000 = 3306.60 Interest was earned!

Difference between Compound Interest and Simple interest:

- With compounded Interest relatively small differences in interest rates can added up over many compounding periods

How to distinguish the difference between Simple Interest Problems and Compounded Interest Problems:

Simple Interest Problems:
"Simple Interest" "Simple Interest Notes" "Discount Rate"

Compounded Interest Problems:
"Compounded Annually" "6% per Quarter" "Compounded Daily"

Hint for WORD problems

9.2) Savings Accounts: Daily Interest

a. Define the Passbook account or Savings account and Time deposit accounts

Most banks pay interest, compounded daily.

Passbook accounts or Savings accounts:

- "Regular savings accounts"
- Money can be deposited or withdrawn at any time
- The Truth in Savings Act of 1991 mandates that interest on savings accounts be paid based on exact number of days
- Most of these accounts are safe because they are insured by the government FDIC Insured (Federal Deposit Insurance Corporation).
- Historically earn from annual 2.5% to 6% interest rate
- Book assumes a 3 ½% annual interest rate compounded daily

Time Deposit accounts:

- You earn a higher interest rate when you promise to leave your money for a longer period of time.
- You are required to leave the money in the account for a specified period of time.
- The time deposit account will typically earn more than a savings account.

CD or Certificate of Deposit account:

- A minimum amount of money must be left in for a minimum amount of time.
- CD accounts typically earn a higher rate of return than the time deposit account.

Remember:

Formula for finding the compound amount after making an investment:

$FV = M = P(1 + i/n)^{nx}$

M = Maturity Value = Future Value
P = Principal amount invested
i = Annual interest rate
n = Number of periods in one year
x = Number of years

Finding the interest earned:

Interest = compound amount - principal

(No page 12)