

Interest Rates & Inflation

BONDS (Debt)

Interest Rates

Real vs. Nominal Rates:

Real Rates

- ① The percentage change in buying power.
- ② Rates that have been adjusted downward for **inflation**.
- ③ Base rate that does not take inflation into consideration.
- ④ percentage change in how much you can buy with your money

Nominal Rates

- ① The percentage change in number of dollars that you have.
- ② Rates that have **NOT** been adjusted downward for **inflation**.
- ③ Nominal Rates of interest include our desired real rate of return plus an adjustment for expected inflation

Example 1 →

# Money

P2

- ⊕ Money is as money does
- ⊕ It is better to define money by its functions:
  - Buy stuff
  - get paid a wage
  - save

① Money acts as a medium of exchange (without it, Barter)

② A standard of value, money is the unit in which the prices of goods & services are measured.  
Scale of value,  $20 \text{ units} = \text{sweater}$   
 $10 \text{ units} = \text{toy}$ , thus:  
 $2 \text{ toys} = 1 \text{ sweater}$

③ A store of value. A person can hold money & use it later.

## Example 1

(P3)

Jan 1, 2010 Milk cost = \$ 3.39

Jan 1, 2011 Milk cost = \$ 3.56

Milk INflation =  $\frac{3.56}{3.39} - 1 = 0.050147$

Jan 1 2010 Buy 100 cartons =  $3.39 * 100$   
= \$339.00

put in Bank & Earn 10%, n=1

APR (n=1) = Nominal = 10%

Jan 1 2011 FV =  $339 * (1 + .1) = 372.90$

~~★~~ we earn 10%, But can we

Q: Buy 10% more cartons of Milk?

can we buy 110 cartons of Milk?

A:  $\frac{\$372.90 \text{ \$ in bank}}{\$3.56 \text{ New Milk cost}} = 104.75$  } Number of cartons

change in buying power =  $\frac{104.75}{100}$

= 0.0475

~~★~~ we earned a Nominal Rate of 10% but our buying power (for Milk) went up by only 4.75%

# The Fisher Effect (Relationship between Real Rate, Nominal Rate, & Inflation)

$$1 + R = (1 + r) * (1 + h) \quad (P4)$$

R = Nominal Rate (Annual Rate with  $n=1$ )

h = Inflation Rate

r = Real Rate

## Example 2

$$R = 0.10, \quad h = 0.050147$$

$$1 + 0.10 = (1 + r) * (1 + 0.050147)$$

$$\frac{1.10}{1.050147} - 1 = r$$

$$0.047472 = r$$

(same as before)

$$\frac{104.75}{100} - 1 = 0.0475$$

### other formulas

$$1 + R = (1 + r) * (1 + h)$$

$$\frac{1 + R}{1 + h} = 1 + r$$

$$\frac{1 + R}{1 + h} - 1 = r$$

$$1 + R = (1 + r) * (1 + h)$$

$$\frac{1 + R}{1 + r} = 1 + h$$

$$\frac{1 + R}{1 + r} - 1 = h$$

$$1 + R = (1 + r) * (1 + h)$$

$$R = (1 + r) * (1 + h) - 1$$

$$1 + R = (1 + r)(1 + h)$$

Multiply out

$$1 + R = 1 + h + r + rh$$

$$R = 1 - 1 + h + r + rh$$

$$R = h + r + rh$$



Compensation for decrease in value in original amount invested

real

compensation for decrease in value of interest earned

$$R \approx h + r$$

# Example 1 (2nd time)

P6

Jan 1, 2010 Milk Cost = \$3.39

Jan 1, 2011 Milk Cost = \$3.56

$$\text{Milk Inflation} = \frac{\text{End}}{\text{Beg}} - 1 = \frac{3.56}{3.39} - 1 = 0.050147$$

Inflation went up, but what about the buying power of our money in the Bank?

$$\text{Annual Rate (n=1)} = \text{Nominal Rate} = 10\% = 0.10$$

$$\downarrow$$
$$\$3.39 * (1 + 0.1) = \$3.729$$

We earn 10%, but did our buying power increase by 10%?

$$\frac{\$3.729}{3.56} - 1 = 0.0474191$$

or

$$\frac{1 + 0.10}{1 + 0.050147} - 1 = \frac{1 + \text{Nominal}}{1 + \text{inflation}} - 1 = 0.047$$

0.0474191 = change in buying power

# The Fisher Effect

Relationship between  
Real, Nominal &  
Inflation

$$r = \frac{1 + R}{1 + h} - 1$$

R = Nominal Rate (Annual Rate  $n=1$ )

h = Inflation Rate

r = Real Rate = change in Buy Power

r = % change  
Buying  
power

$$= \frac{1 + R}{1 + h} - 1$$

Amount  
you would  
earn in  
1 year

what you could  
purchase give  
an inflation  
rate increase in  
Price of Goods/service

### Example 3:

If nominal Rate is 12%  
& Inflation is 6%,  
What is change in  
Buying power?

$$r = \frac{1.12}{1.06} - 1 = 0.0566$$

### Example 4:

If nominal Rate is 1%  
and Inflation is 1.5%,  
what is change in Buying  
power?

$$r = \frac{1.01}{1.015} - 1 = -0.004926$$

Example 5: in Excel workbook



# Bonds

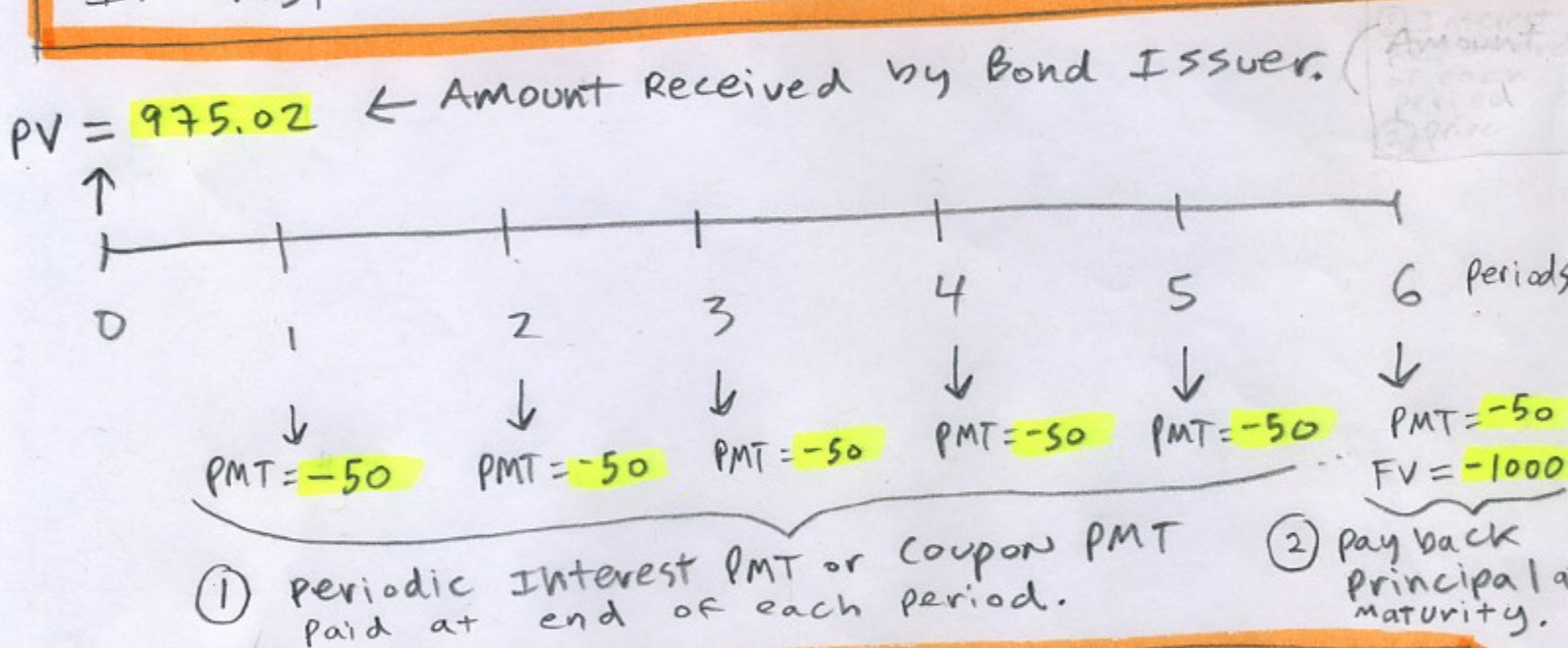
Loan contract = set of Future Cash Flows

## Borrower

Bond Issuer = Bond seller = Liability  
= Corporation or Government Debt

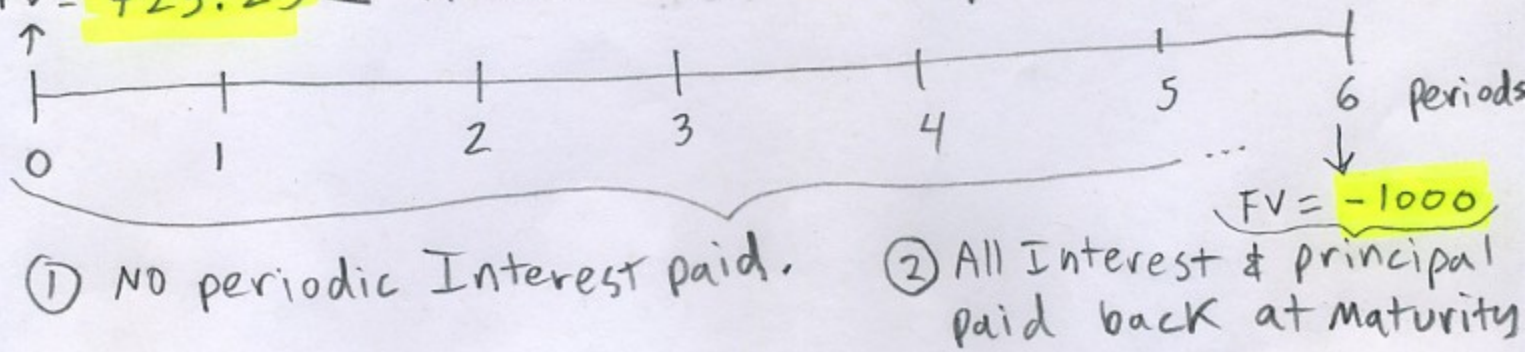
## How cash Flows might look

Interest ONLY Loan or Coupon Bond \* coupon = "interest"



## Deep Discount Loan or Zero Coupon Bond

PV = 725.25 ← Amount Received by Bond Issuer.



Lender

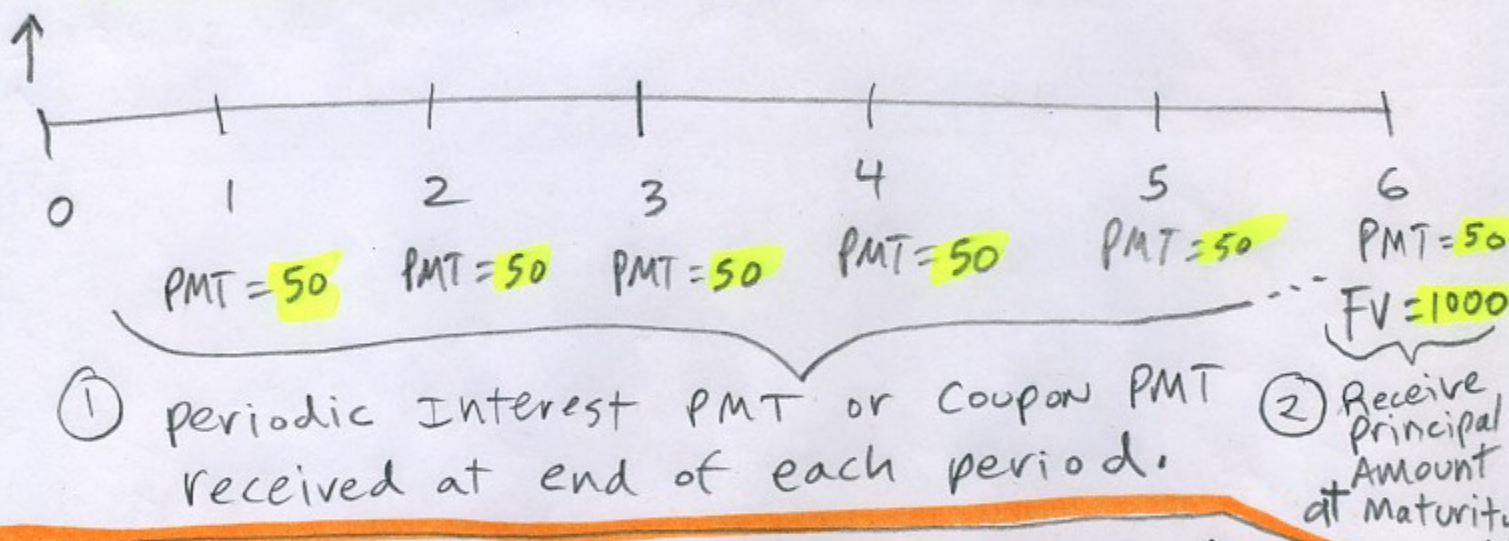
P 10

Bondholder = Bond Buyer = Asset  
= Bondholder is buying  
set of Future Cash Flows

How cash flows might look:

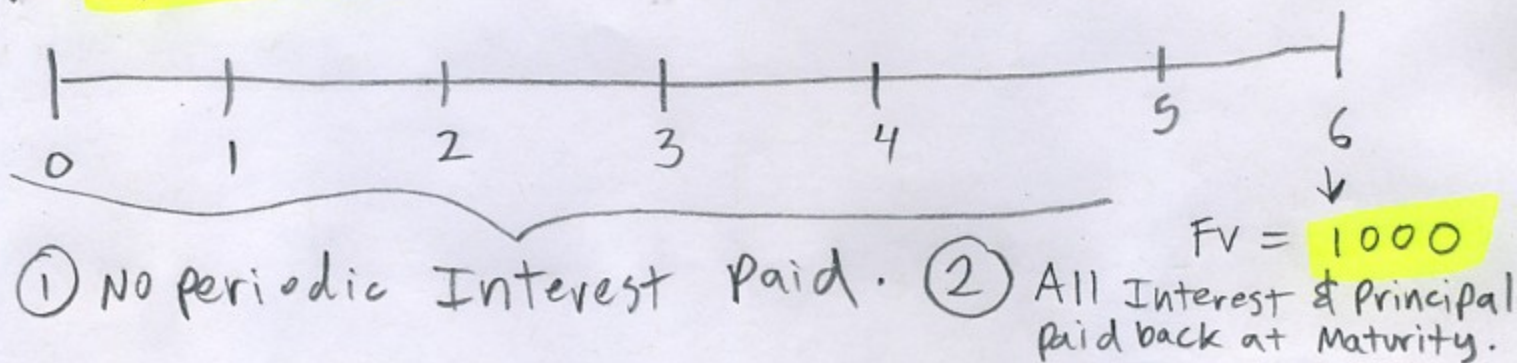
Interest only or Coupon Bond \* coupon = Interest

PV = -975.02 ← Amount Paid for Future set of Cash Flows



Deep Discount Loan or Zero Coupon Bond

PV = -725.25 ← Amount Paid for Future set (1) of cash Flows.



# Bond Terms:

P. 11

① Bond = Loan Contract issued by corporation or Government that can be bought or sold in Financial Markets.

② YTM = Yield To Market = Discount Rate = Market Rate = Rate for similar securities.

usually  
Different

③ Coupon Rate = contract interest rate for calculating Interest PMT.

④ Face value = Amount of loan to pay back at Maturity = "par value" = principal.

⑤ coupon payment = Interest payment.

⑥ Years until Maturity = years until pay back face value & last coupon payment.

⑦ Maturity = specific date when face value & last coupon payment is paid.

## What Are Similar Securities?

- Similar securities are bonds or other investment vehicles issue by other corporations (different than the one being considered) that have similar business and financial risks
- The similarities could be:
  - Similar credit ratings
  - Similar business activities
  - Similar capital structure

<sup>coupon</sup>  
Bond  
value

$$= PMT * \left[ \frac{1 - \left(1 + \frac{YTM}{n}\right)^{-nx}}{\frac{YTM}{n}} \right] + \frac{FV}{\left(1 + \frac{YTM}{n}\right)^{nx}}$$

Point of view of Bondholder

→ = PV (rate, nper, PMT, FV) \* →

= PV (  $\frac{YTM}{n} * n * X$ ,  $\frac{coupon * face}{n}$ , FV )

Math	Bond Terms	Excel
FV	face value	FV
PV	Bond Value	PV
i	YTM	rate
n	# compound periods per year usually 2	
X	years until maturity	
n * X	Total number of periods	nper
<del>i/n</del>	$\frac{YTM}{n}$ = Discount Rate = Market Rate	rate
<del>PMT</del>	Periodic Interest PMT = $\frac{coupon * face}{n}$	PMT

## Zero Coupon Bonds (Pure Discount Loan or "zeros"):

- ⊗ A Bond that makes no coupon payments, and thus is initially priced at a deep discount
- ⊗ From the Borrower's point of view: "Borrow an amount today, then pay back principal & all interest at the end of the loan period."
- ⊗ From the Lender's (Bondholder's) point of view: "Lend (pay) an amount today, and receive all principal & interest at the end of the loan period."

Zero Bond value

$$= \frac{FV}{\left(1 + \frac{YTM}{n}\right)^{nx}}$$
$$= PV(\text{rate}, nper, FV)$$
$$= PV\left(\frac{YTM}{n}, n * x, FV\right)$$

point of view of Bondholder

Note: Interest on Zeros is:

- ① Deductible for income tax for Bond Issuer
- ② Taxable for income tax for Bondholder

Why issue or buy zeros?

- ① Bond Issuer likes cash flow advantage of Deductible Non-cash Expense
- ② Bondholder likes predictability of future Bond Income.

① To solve for the  $\frac{YTM}{n} = \left\{ \begin{array}{l} \text{Discount Rate} \\ \text{per period} \end{array} \right\}$  p.13  
use:

Excel:  $=RATE(n * x, -PMT, PV, -FV, 0)$  Bond Issuer  
Point of  
view

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② To solve for the Effective Annual Yield  
use:

Excel:  $=EFFECT(YTM, n)$

Math:  $\left(1 + \frac{YTM}{n}\right)^n - 1$

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③ To solve for periodic Coupon PMT:  
use:

Excel:  $=PMT\left(\frac{YTM}{n}, n * x, -PV, FV, 0\right)$

↑  
Bondholders point of view

Q: For the Coupon Bond, How come Bond Issuer got only \$975.02 instead of \$1000?

A: ① Because Corporate & Government Bonds are bought & sold in the Financial Markets, AND Market Rates change often. (Because of New information).

② This means the interest rate written in the Bond contract is usually different from the interest Rate in the Financial Markets.



Bond Interest ONLY  
 Face = 1000  
 Coupon Rate = 10%  
 SEMI ANNUAL  
 Years to Maturity = 3



Bond Interest ONLY  
 Face = 1000  
 Coupon Rate = 10%  
 Semi Annual  
 Years to Maturity = 3

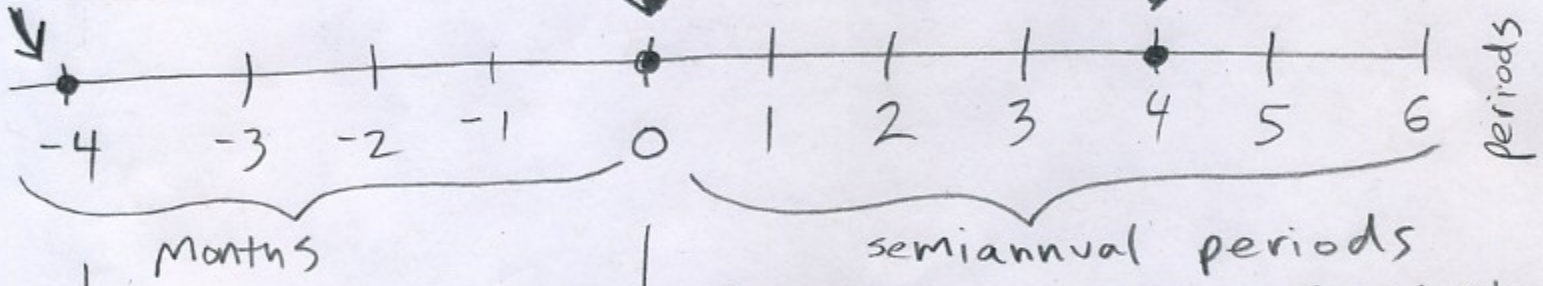
Bond Interest ONLY  
 Face = 1000  
 Coupon Rate = 10%  
 Semiannual  
 Years to Maturity = 1

P.15

Corporation & Investment Bank write up Bond Contract & say they will pay 10%,  $n=2$  coupon payments 4 months before they issue Bond.

But on the Day they sell the Bond contract the interest Rates have changed.  
 Coupon Rate = 10%  
 YTM = 11%

Later, if someone Buys the Bond from the original Bondholder, the Rates have changed again.



Day they write up contract  
 Coupon Rate = 10%  
 YTM = 10%

"sold at Discount"  
 Day they issue Bond contract  
 Coupon Rate = 10%  
 YTM = 11%

"sold at Premium"  
 Day the Bond is bought by new person  
 Coupon Rate = 10%  
 YTM = 9%

Why do rates change? In Financial Markets, each day New Information about the economy & business cause people to change their rates (Risk).

# Example 6: Coupon Bond for Issuer

Bond Issuer = Corporation

Face = \$1000 (corp. usually issues many Bonds)

Coupon Rate = 10% Determines Interest Paid

$n = 2$

$$\left\{ \frac{\text{Coupon Rate}}{2} \right\} = \frac{10\%}{2} = 5\%$$

YTM = 11%

$$\frac{\text{YTM}}{n} = \frac{11\%}{2} = 5.5\%$$

Market Rate = Discount Rate for similar securities

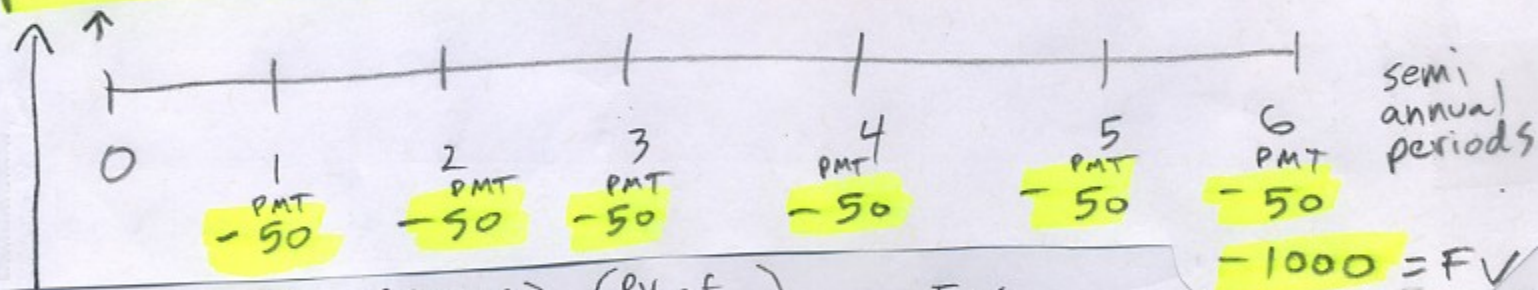
Years to Maturity = 3

Total periods =  $3 * 2 = 6$

Coupon PMT =  $5\% * 1000 = \$50$

## Coupon Bond

PV = SOLVE FOR THIS



$$PV = \left\{ \begin{array}{l} \text{cash} \\ \text{coming} \\ \text{into} \\ \text{corporation} \end{array} \right\} = \left\{ \begin{array}{l} \text{PV of} \\ \text{future} \\ \text{lump} \\ \text{sum} \end{array} \right\} + \left\{ \begin{array}{l} \text{PV of} \\ \text{Annuity} \\ \text{of} \\ \text{coupon} \\ \text{PMT} \end{array} \right\} = \frac{FV}{\left(1 + \frac{\text{YTM}}{n}\right)^{nx}} + \text{PMT} * \frac{1 - \left(1 + \frac{\text{YTM}}{n}\right)^{-nx}}{\frac{\text{YTM}}{n}}$$

$$PV = \frac{1000}{(1 + .055)^6} + 50 * \left[ \frac{1 - (1 + .055)^{-6}}{.055} \right]$$

$$PV = 725.245833 + 249.7765154$$

"sold at Discount"

$$PV = \$975.02$$

$$= PV(\text{rate}, \text{nper}, -\text{PMT}, -\text{FV}) = PV\left(\frac{\text{YTM}}{n}, n * x, -\text{PMT}, -\text{FV}\right)$$

$$= PV(.055, 6, -50, -1000) = \$975.02 \leftarrow \left\{ \begin{array}{l} \text{corp} \\ \text{receives} \end{array} \right\}$$

# Example 7: Zero coupon for Issuer

Bond Issuer = Corporation

Face value = \$1000 (corp usually issues many B.)

Coupon Rate = 10% (Determines Interest Paid)

n = 2

$$\left\{ \frac{\text{coupon rate}}{2} \right\} = \frac{10\%}{2} = 5\%$$

Market Rate or Discount Rate for "Similar security"

YTM = 11%

$\frac{YTM}{n} = \frac{11\%}{2} = 5.5\%$

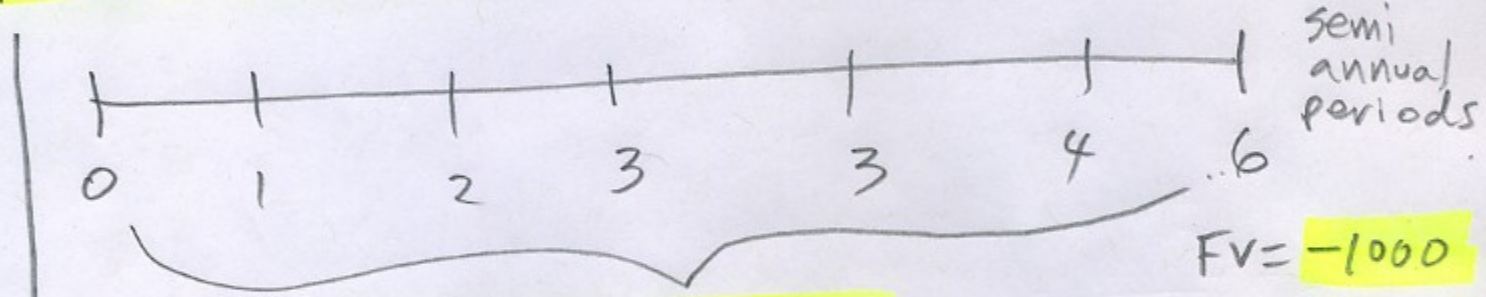
Years to Maturity = 3

Total periods = 3 \* 2 = 6

NO COUPON PMT !!

**zero coupon**

PV = SOLVE FOR THIS



$$PV = \left\{ \text{cash coming into corporation} \right\} = \left\{ \text{PV of Future Lump sum} \right\} = \frac{FV}{\left(1 + \frac{YTM}{n}\right)^{n \times x}}$$

$$PV = \frac{1000}{(1 + .055)^6} = \$725.25$$

Corp. receives \$

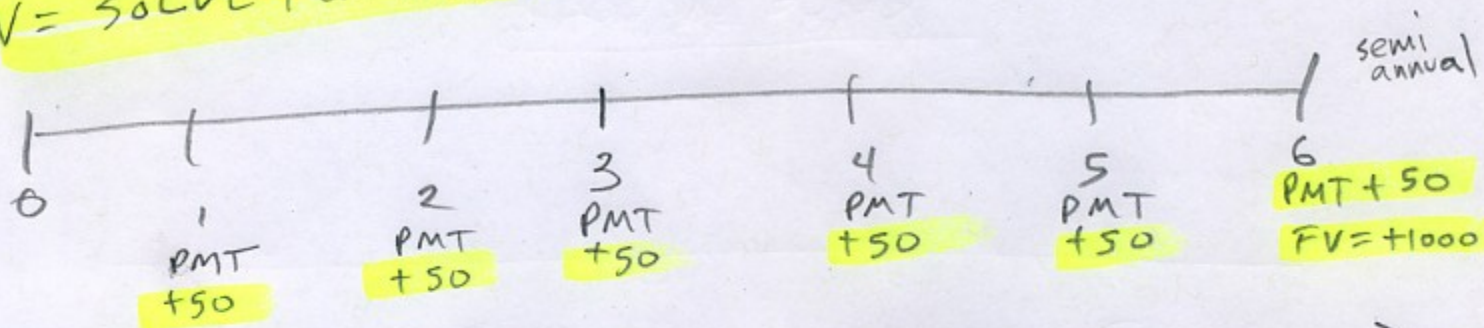
$$= PV(\text{rate}, \text{nper}, -FV) = PV\left(\frac{YTM}{n}, n \times x, -FV\right)$$

Skip PMT "sold at Discount" →  $= PV(.055, 6, -1000) = \$725.25$

## Example 8: Coupon Bond for Bondholder #1 P.18

⊛ Details same as example 6, but for Bondholder

PV = SOLVE FOR THIS



$$= PV(\text{rate}, \text{nper}, \text{PMT}, \text{FV}) = PV\left(\frac{\text{YTM}}{n}, n \times x, \text{PMT}, \text{FV}\right)$$

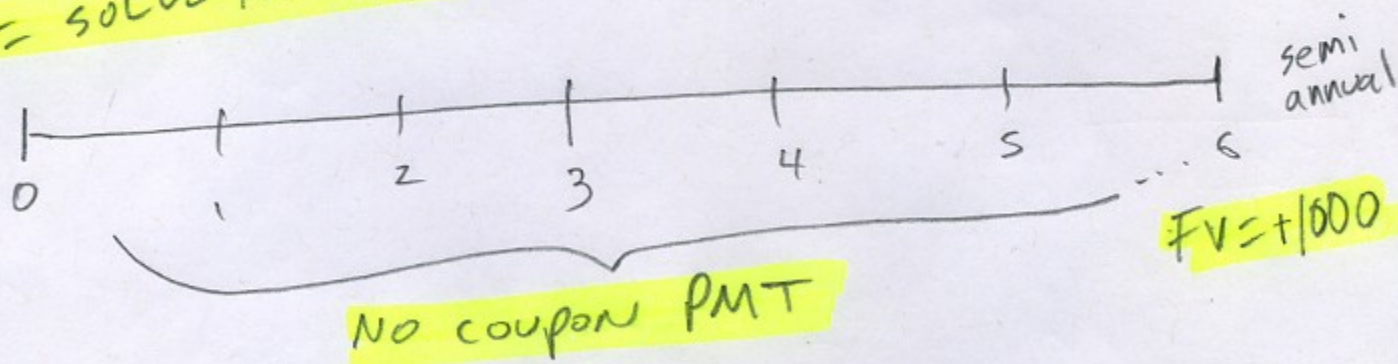
$$= PV(.055, 6, 50, 1000) = -\$975.02 \leftarrow \text{"Bought at Discount"}$$

Bondholder is willing to pay \$975.02 for set of Future Cash Flows!!!

## Example 9: Zero coupon Bond for Bondholder #1

⊛ Details same as example 7, but for Bondholder

PV = SOLVE FOR THIS



$$PV = PV\left(\frac{\text{YTM}}{n}, n \times x, \text{FV}\right) = PV(.055, 6, 1000) =$$

$$\rightarrow PV = -725.25$$

Bondholder is willing to pay \$725.25 now in order to receive all the interest & principal (\$1000) later.

"Bought at Discount"

### Example 10:

### Coupon Bond for Bondholder # 2

(P.19)

Bondholder # 1 sells Bond to Bondholder # 2 at the end of year 2, and the rate in the market (YTM, or Rate on "similar security") is 9%

Bondholder # 1 = sells Bond

Bondholder # 2 = Buys Bond in secondary Market

Face value = \$1000 = FV

Coupon Rate = 10% (Determines Interest Paid)

$$\left\{ \frac{\text{Coupon Rate}}{2} \right\} = \frac{10\%}{2} = 5\%$$

YTM = 9% = Discount Rate = Market Rate

$$\frac{YTM}{2} = \frac{9\%}{2} = 4.5\%$$

Years to maturity = 1

total periods = 1 \* 2

Coupon Payment = PMT =  $1000 * .05 = \$50$

### Coupon Bond for Bondholder # 2

PV = solve For This

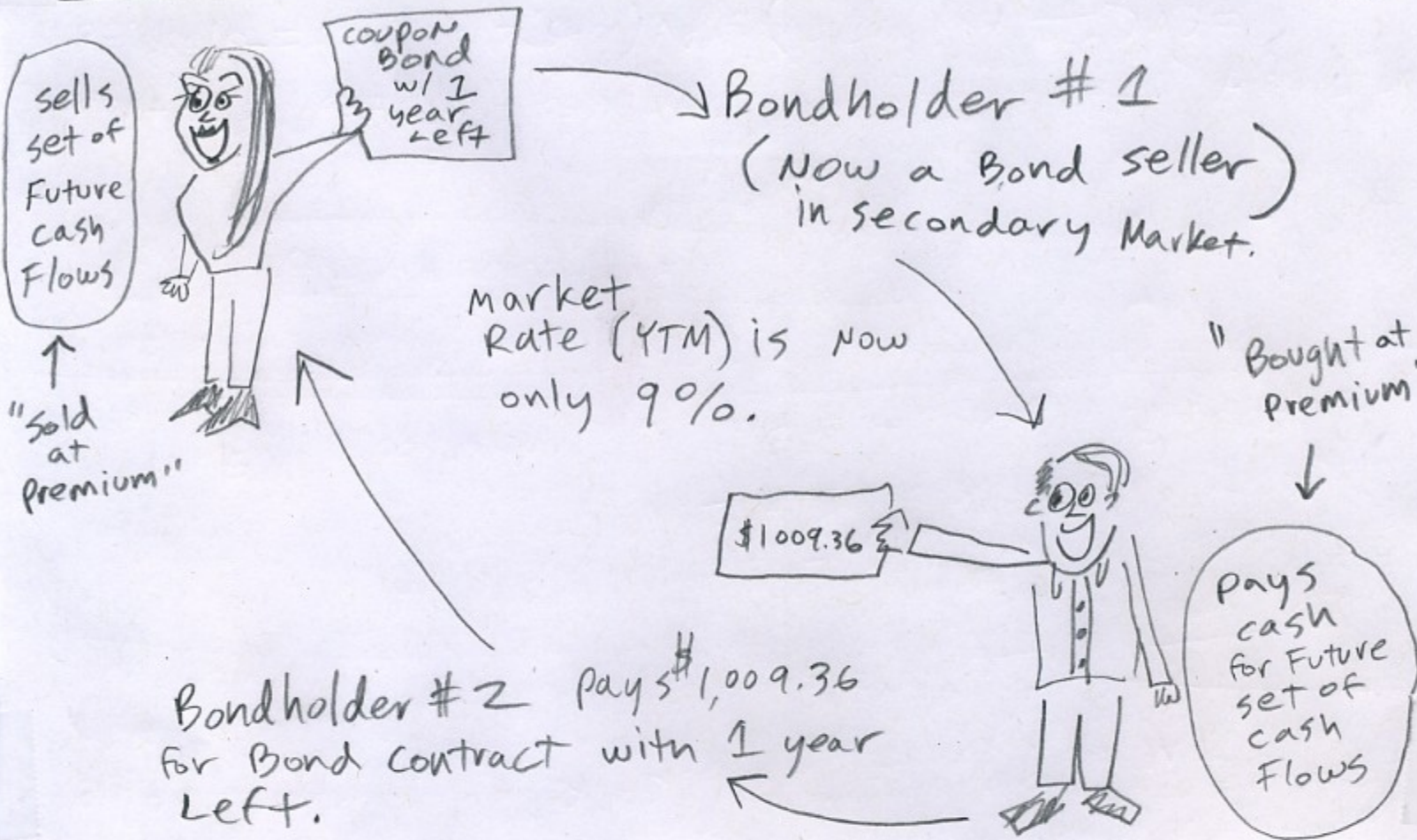


$$= PV\left(\frac{YTM}{n}, n * X, PMT, FV\right) = PV(.045, 2, 50, 1000) = -\$1009.36 \leftarrow$$

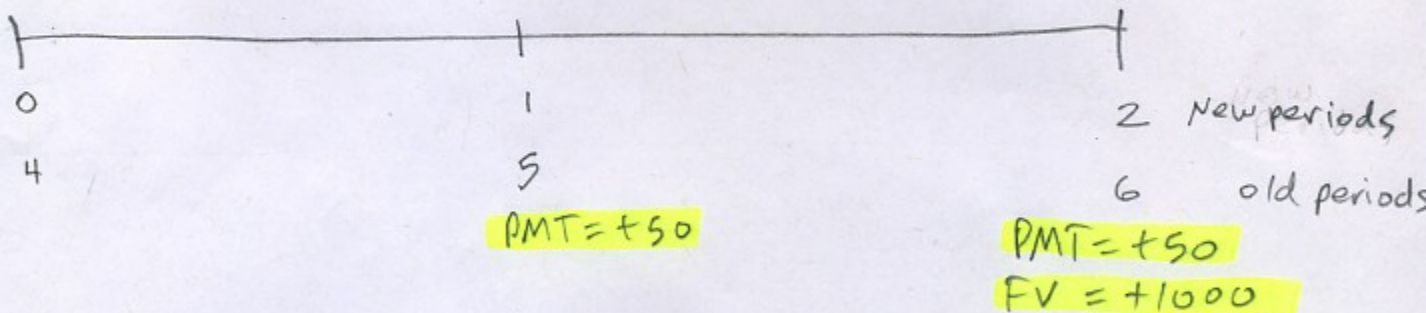
Bondholder # 2 pays Bondholder # 1 \$1009.36 now in order to get future cash flows from corporation "Bought at Premium"

# Example 10 continued:

P. 20



$$PV = 1,009.36$$



- ① Present value of all future cash Flows at YTM of 9% is Bond Value.
- ② Bond with a 10% coupon is priced to yield 9% at \$1,009.36.

## Example - II :

P, 21

But what if the buyer & the seller just had price = \$1,009.36 & not the YTM? Could we figure out YTM?

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Sell price for Bond = \$-1,009.36 = PV

Face = \$1000 = FV

Semi-annual Interest payment = \$50

Total number of periods left = 2

$$\frac{YTM}{2} = ?$$

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$$= RATE(nper, PMT, -PV, FV) = \frac{YTM}{2} =$$

$$= RATE(2, 50, -1009.36, 1000) \approx 0.045$$

$$\frac{YTM}{2} = 0.045 \Rightarrow YTM = 0.045 * 2 = 0.09 \approx 9\%$$

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$$\begin{aligned} \text{Effective Annual Yield} &= \left(1 + \frac{YTM}{n}\right)^n - 1 = 1.045^2 - 1 \\ &= 0.092025 \Rightarrow 9.2\% \end{aligned}$$

"1 Year Bond with a 10% coupon is priced to yield 9% at \$1,009.36."

"Bond is priced at a Premium."

"Bond's Effective Annual Yield is 9.2%."

## Example 12:

P.22

Sometimes you see Bond prices  
Quoted like this:

$$1.00936$$

or

$$100.936\%$$

or it is written like this:

" 1 year 10% coupon Bond is priced at 100.936% "  
" 100.936% " (assumed to be semiannual)

From statement we  
can get info. →

$$\text{Face} = \$1000 = FV$$

$$\text{semiannual coupon} = \$50 = PMT$$

$$\text{Price} = 1000 * 1.00936 = \$1009.36 = PV$$

$$\text{Years to maturity} = 1$$

$$\text{Total periods} = 2 \quad (\text{assumed } n=2)$$

Point of  
view of  
Bondholder

$$YTM = \text{RATE}(nper, PMT, -PV, FV) * n$$

$$YTM = \text{RATE}(2, 50, -1009.36, 1000) * 2 =$$

$$YTM = 0.0900035$$



# Three Principles in Bond Finance

Example 13:

## ① Rates are inversely related to Price

YTM ↑ then Bond Price ↓

Face value = 1000  
Coupon Rate = 6%  
YTM = 6%  
years = 30  
n = 2  
price = \$1,000  
sell at Par

FV = 1000  
Coupon Rate = 6%  
YTM = 7%  
years = 30  
n = 2  
price = \$875.28  
sell at Discount

FV = 1000  
Coupon Rate = 6%  
YTM = 5%  
years = 30  
n = 2  
price = \$1,154.54  
sell at Premium

## ② Bonds sell at Par, Discount or Premium

Example 13.5

Coupon Rate = 6%  
YTM = 6%  
face = price  
1000 = 1000  
C.R. = YTM  
6% = 6%  
PAR

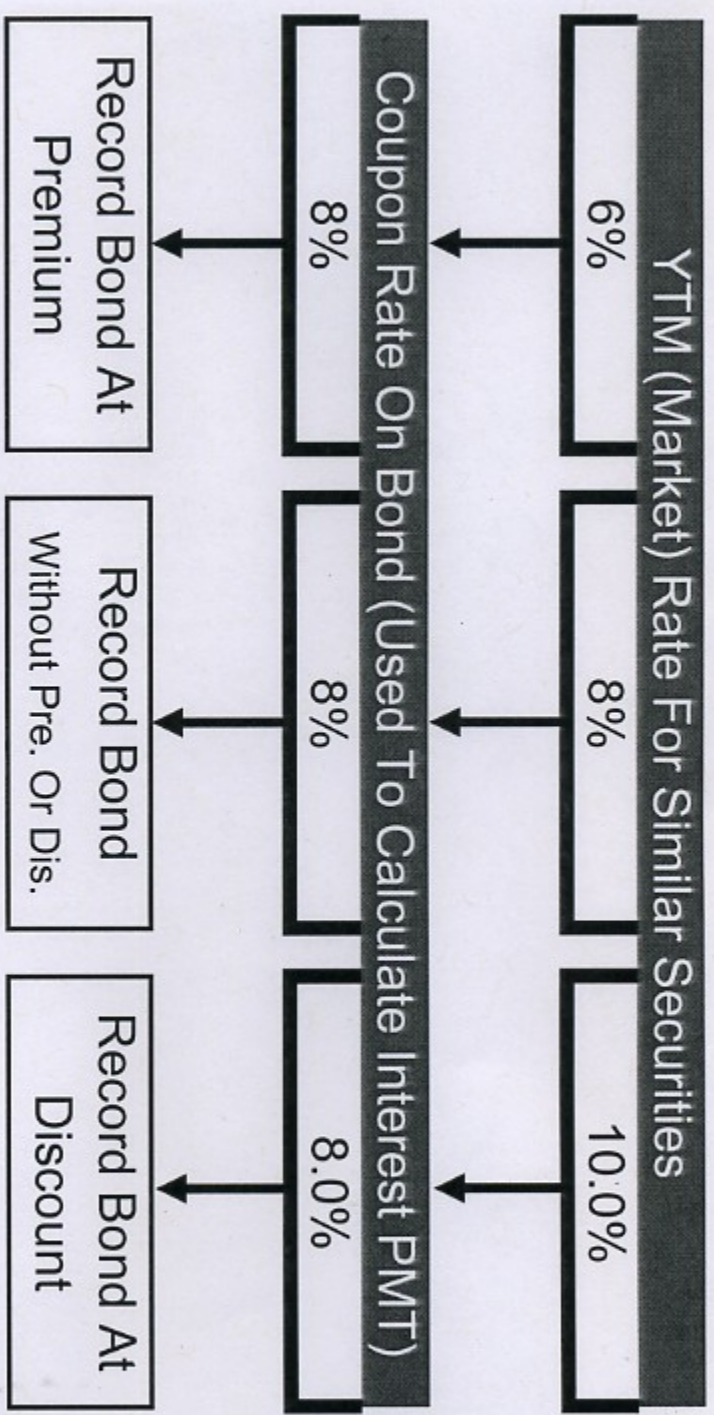
Coupon Rate = 6%  
YTM = 7%  
face > price  
1000 > 875.28  
C.R. < YTM  
6% < 7%  
Discount

Coupon Rate = 6%  
YTM = 5%  
face < par  
1000 < 1,154.54  
C.R. > YTM  
6% > 5%  
Premium

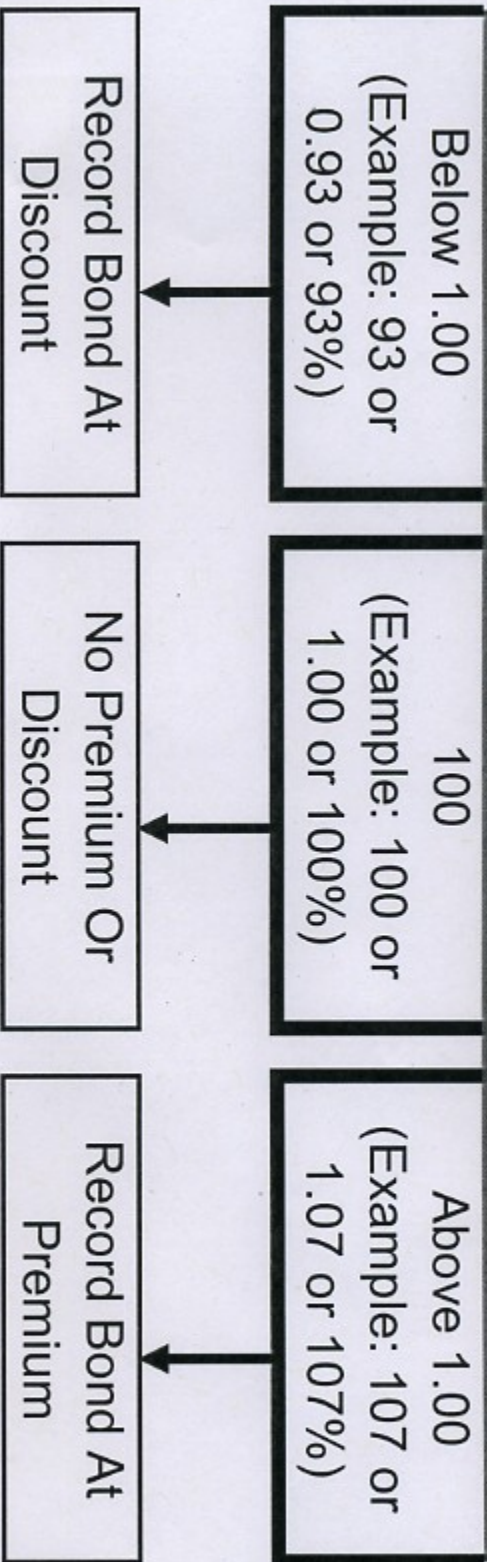
## ③ The more years to Maturity, the higher the Interest Rate Risk becomes.

Example 14 & 14.5

- ① The longer to maturity, the more the YTM affects the Bond price.
- ② For long maturity (years=30), a small increase in YTM can cause a large drop in Bond value.
- ③ Low coupon Rates have more risk than large coupon Rates because of pattern of cash flows



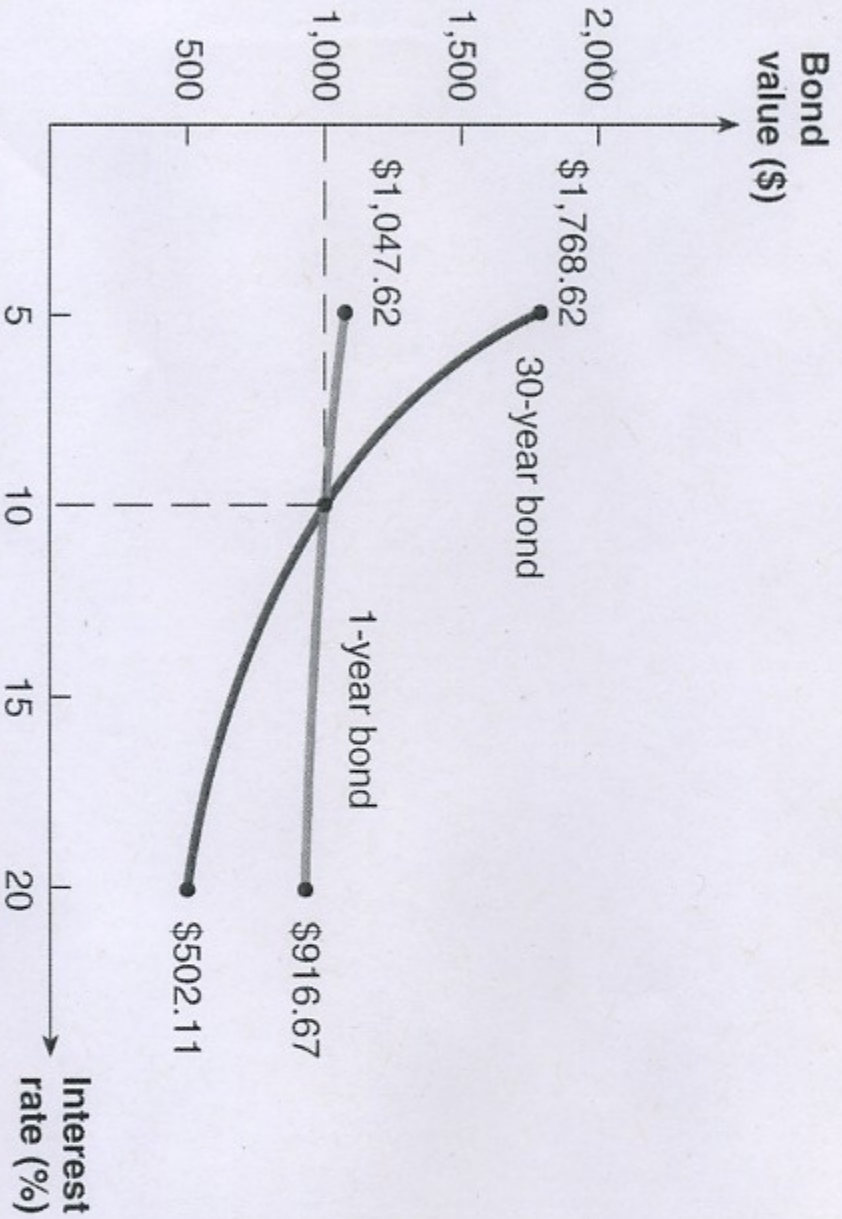
Selling Price For Bond



# Interest Rate Risk

- The risk that arises for bond owners from fluctuating interest rates
- How much interest rate risk a bond has depends on:
  - How sensitive its price is to interest rate change
    - The sensitivity depends on two things:
      - All things being equal, the longer the time to maturity, the greater the interest rate risk
      - All things being equal, the lower the coupon rate, the greater the interest rate risk

# Interest Rate Risk And Time To Maturity



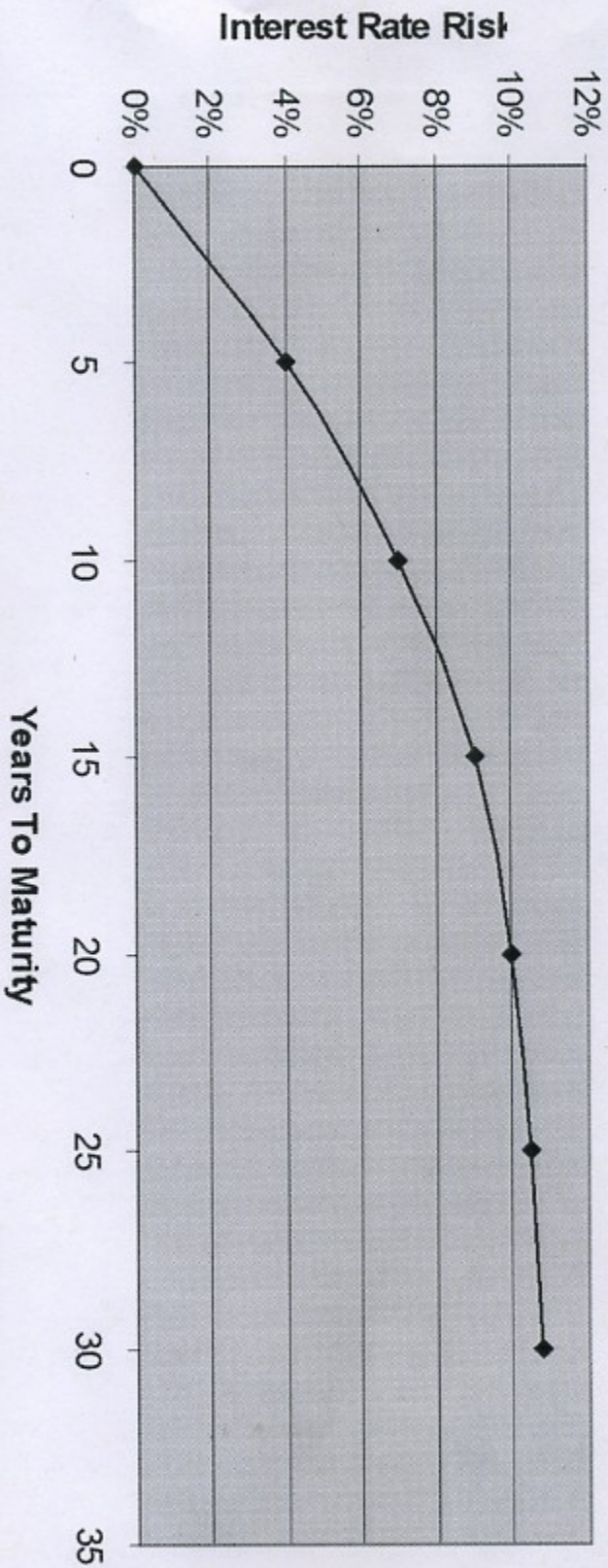
Value of a Bond with a 10 Percent Coupon Rate for Different Interest Rates and Maturities

Interest Rate	Time to Maturity	
	1 Year	30 Years
5%	\$1,047.62	\$1,768.62
10	1,000.00	1,000.00
15	956.52	671.70
20	916.67	502.11

### Interest Rate Risk To Loss Of Principal (current price)

- **Longer time to maturity**
  1. Small changes in market rate have substantial affect on bond value
  2. Face value is discounted over many periods and thus compounding magnifies small interest rate changes
- **Lower Coupon rate**
  1. Bond with lower coupon rate is proportionally more dependent on the face value
    - (Bond with larger coupon rate has a larger cash flow early in life, so value less sensitive to discount rate)

# Interest Rate Risk Increases At A Decreasing Rate



## Bonds

- ① Bring cash into Corp.
- ② Debt to Corp.
- ③ Bond holders are creditors
- ④ Interest must be paid (Fixed claim to cash flow)
- ⑤ Bond holders get paid interest & principal, but no more.
- ⑥ Generally no voting rights
- ⑦ Interest is tax deductible.
- ⑧ Bondholders have first claim in Bankruptcy.
- ⑨ Excess debt can lead to bankruptcy.

## Stocks

P. 30

- ① Bring cash into Corp.
- ② Equity to Corp.
- ③ stockholders are owners
- ④ Dividends are not a liability - they are paid only if Board of Directors Declare them (residual claim to cash flow)
- ⑤ If corporation is very successful, owners may get paid a lot of Dividends (No upper limit)
- ⑥ Generally, voting rights
- ⑦ Dividends are not tax deductible
- ⑧ Stockholders are in line behind creditors in Bankruptcy
- ⑨ An all equity firm can not go bankrupt.



Government Bonds (USA biggest borrower in world) (p. 31)

Federal Level

Treasury Bill

Years < 1

Treasury Note

1 < Years < 7

Treasury Bonds

other

Treasury Debt (Fed. Gov.)

- ① considered default free  
⊗ No default risk
- ② Taxed at Federal Level only (Not State).
- ③ Highly Liquid to Bondholder.

Municipal Bonds "Munis":

State & Local Level

⊗ Exempt from Federal Income Tax

Example: If your tax bracket is 25% for Federal Income tax, would you prefer 5% Corporate Bond or 3.9% muni Bond?

Example 15:

$$.039 > .05 * (1 - .25) = .05 * .75 = .0375$$

.039 Muni > .0375 corporate