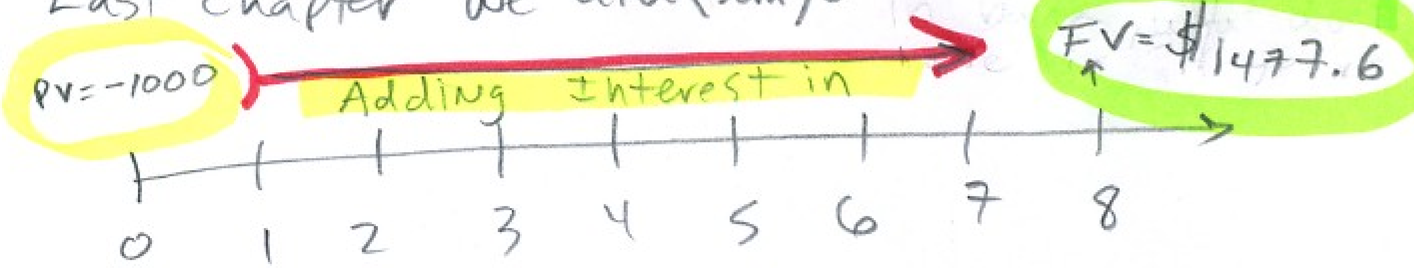
Chapter 5
Discounted Cash Flow Valuation
Interest Rates
Loans

Last chapter we did (One Lump Sum) • If I put $1000 in bank what do I have in 4 years? •

(1)



PV = $1000

i = 10% = .10

n = 2

$\frac{i}{n}$ = 10%/2 = 5% = .05

x = 4

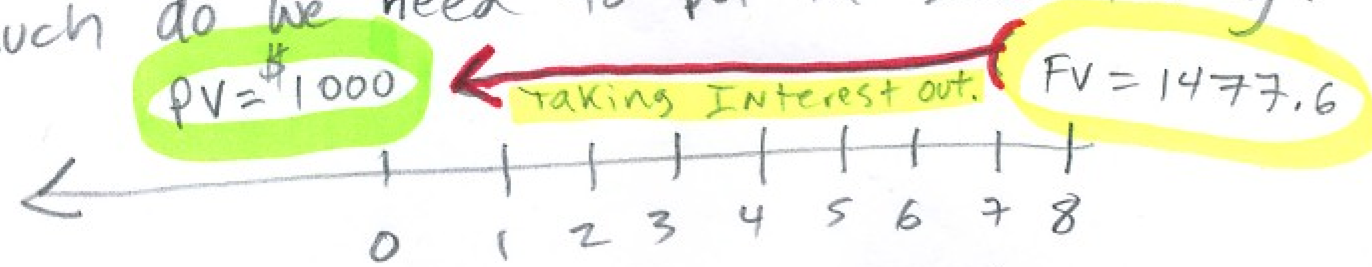
$n * x$ = 4 * 2 = 8

$$FV = PV * \left(1+\frac{i}{n}\right)^{n*x}$$

$$FV = 1000 * \left(1+\frac{.1}{2}\right)^{2*4}$$

$$FV = 1{,}477.46$$

(2) If we need $1,477.46 in 4 years how much do we need to put in bank today?



FV = 1,477.6

i = 10% = Discount Rate

n = 2

$\frac{i}{n}$ = 10%/2 = 5% = .05

x = 4

$n * x$ = 4 * 2 = 8

$$PV = \frac{FV}{\left(1+\frac{i}{n}\right)^{n*x}}$$

$$PV = \frac{1477.6}{\left(1+\frac{.1}{2}\right)^{2*4}}$$

$$PV = \$1000$$

Excel;

---

1) PV = $1000

Discount Rate = i = 10%

n = 2

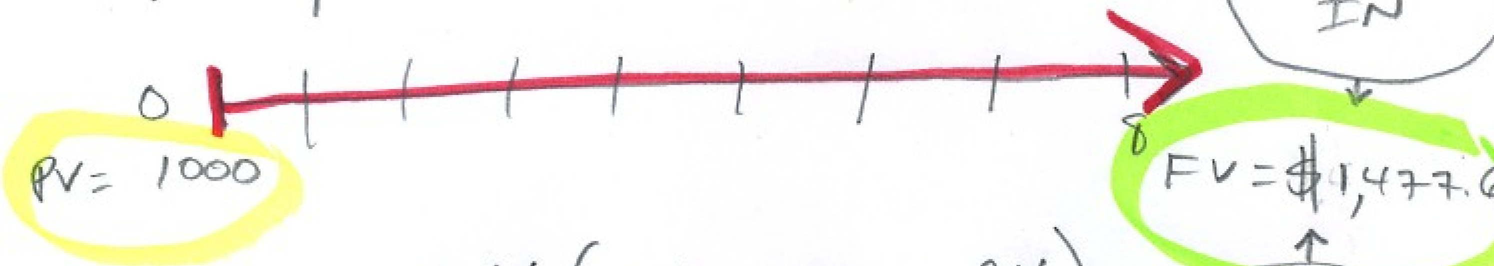
FV = ?

x = 4

$\frac{i}{n} = \frac{10\%}{2} = 5\%$

$n * x = 4 * 2 = 8$

Add all Interest IN



Solve for this

FV = FV (rate, nper, , PV)

FV ($\frac{i}{n}$, n*x , , PV)

FV (5%, 8, , -1000) = $1,477.6

---

2) i = 10%

Discount Rate = n = 2

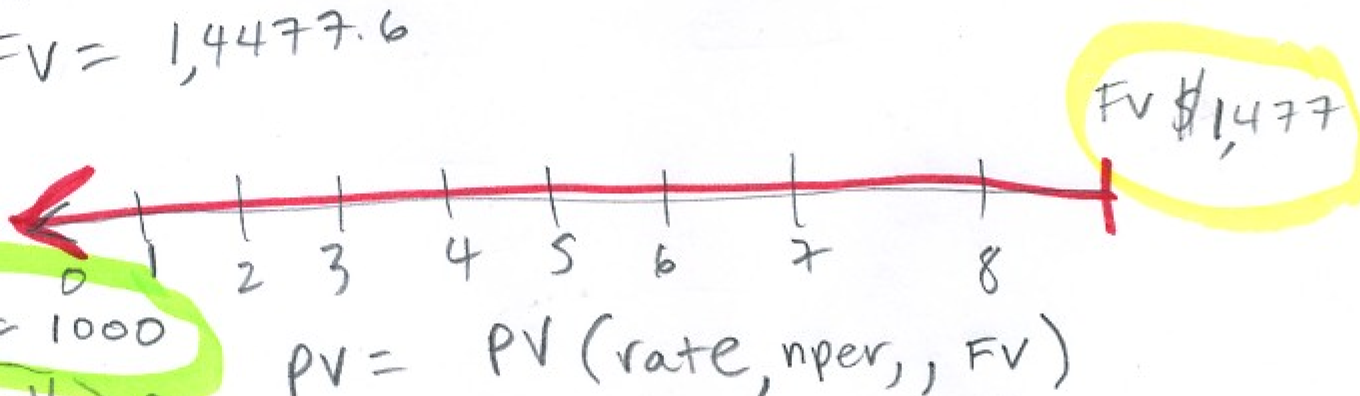
x = 4

FV = 1,4477.6

$\frac{i}{n} = \frac{10\%}{2} = 5\%$

$n * x = 4 * 2 = 8$

solve for this



Take all Interest out

PV = PV (rate, nper, , FV)

PV ($\frac{i}{n}$, n*x , , FV)

PV(.05, 8, 1447.6) = -$1000

In Excel Minus Because Excel knows cash flow

Chapter 4: what is value of one Lump sum cash amount?

Chapter 5: what is value of multiple cash Amounts (cash Flows)

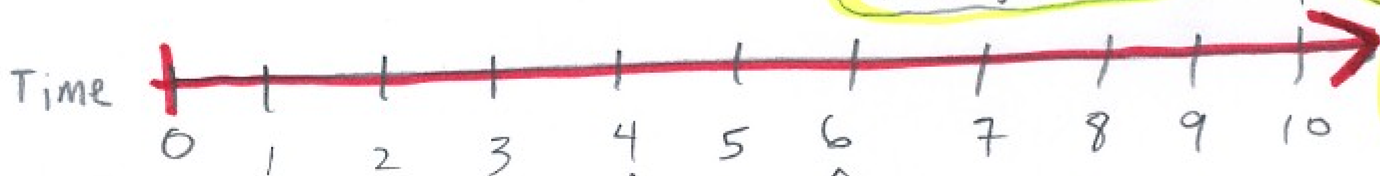
③ Example: Invest in bank: (savings plan)

$i = 10\%$
$n = 2$
$\frac{i}{n} = 10\%/2 = 5\%$
$x = 5$
$n * x = 2*5 = 10$

Time 0 = \$1000 invest
Time 4 = \$2000 invest
Time 6 = \$6000 invest
Time 10 = what is Bank Account worth?

Adding Interst → FV = \$11,602.12

solve for this



PV \$1000
$FV = PV * (1 + \frac{i}{n})^{nx}$
$FV = 1000(1 + .05)^{(10-0)}$

10 periods Left

PV \$2000
$FV = PV * (1 + \frac{i}{n})^{nx}$
$FV = 2000(1 + .05)^{(10-4)}$

only 6 periods Left

PV \$6000
$FV = PV * (1 + \frac{i}{n})^{nx}$
$FV = 6000 * (1 + .05)^{(10-6)}$

only 4 periods left

\$7,293.04
\$2,680.19
\$1,628.89

FV total = \$11,602.12

Adding all the interest

④ Example: we want to withdraw these cash Flows in future. How much do we have to put in Bank Today?

$i = 10\%$

$n = 2$

$\frac{i}{n} = 10\%/2 = 5\%$

$x = 5$

$n * x = 5 * 2 = 10$

withdraw:

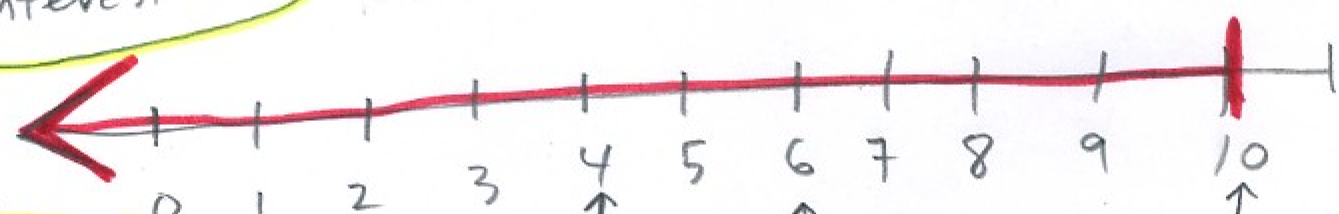
Time 4 = \$1000 withdraw

Time 6 = \$2000 withdraw

Time 10 = \$6000

PV = Time 0 = How much to put in Bank?

taking all Interest out



FV \$1000 | FV \$2000 | FV \$6000

PV = 5998.6

Solve for this

$$PV = \frac{FV}{(1+\frac{i}{n})^{nx}} \qquad PV = \frac{FV}{(1+\frac{i}{n})^{nx}} \qquad PV = \frac{FV}{(1+\frac{i}{n})^{nx}}$$

$$PV = \frac{1000}{(1+.05)^4} \qquad PV = \frac{2000}{(1+.05)^6} \qquad PV = \frac{6000}{(1+.05)^{10}}$$

\$ 822.70

\$ 1,492.43

\$ 3,683.48

PV = 5998.6 ← Taking all Interest out ←

(5) <u>Example:</u> <u>Price of Machine = $100,000</u>

what if you plan to buy a machine that you have estimated will yield these cash Flows:

Time 0 = Price
Time 1 = 50,000
Time 2 = 50,000
Time 3 = 40,000
Time 4 = 30,000

Assumed Discount Rate = i = 15%
n = 1
x = 4

Future positive cash Flows

Q: should we buy machine?

Time: 0, 1, 2, 3, 4

0	1	2	3	4	
	50000	50000	40000	30000	FV

By taking future cash Flows & discounting them back, we can compare future income to price

=PV(.15,1,50000) → -43,478.26

=PV(.15,2,50000) → -37,807.18

=PV(.15,3,40000) → -26,300.65

=PV(.15,4,30000) → -17,152.60

-124,738.69

A: If you are willing to pay out -124,738.69 for the cash flows, is -100,000, worth it? A: Yes!!!

Interest Rates

1 Period Rate

$$\boxed{\text{period interest rate} = \frac{i}{n}}$$

n = {# of compounding periods per year}

i = {Annual Interest Rate, other Names: Stated Rate, Quoted Rate, APR (Annual Percentage Rate)}

Example 6:

i = 12% or .12

n = 12

$$\left\{\begin{matrix}\text{Period}\\ \text{Rate}\end{matrix}\right\} = \frac{i}{n} = \frac{.12}{12} = .01 \text{ or } 1\%$$

2 APR (Annual Percentage Rate)

(also known as: Stated Rate, Quote Rate, Annual Interest Rate, Nominal Rate.)

Examples:

10% compounded monthly

12% compounded semiannually

5.55% compounded daily

APR = period rate * # of compounding periods per year $= \frac{i}{n} * n = i$

→ Truth-in-lending laws in USA require lenders to prominently display the APR on loan documents

☒ Ironic & untruthful because EAR > APR

Example 7:

If the monthly rate is .5%, what is APR?

$\frac{i}{n}$ = .5% or .005

n = 12

$i = \frac{i}{n} * n$ = .005 * 12 = .06 or 6% compounded Monthly

Note: If you are given an APR of 6% compounded Monthly you cannot find out what the semiannual rate might be!! We could figure out what the monthly rate is but not a semiannual or daily or quarterly or any other period rate.

Anytime we have # of compounding periods per year that is greater than 1, we must figure out what the real annual rate is. This is called the Effective Annual Rate:

3 EAR (Effective Annual Rate)

(more relevant rate for financial decisions.)

credit card

EAR = Annual Interest Rate expressed as if it were compounded once per year

$$EAR = \left(1 + \frac{i}{n}\right)^n - 1$$ (look familiar?)

Excel: =EFFECT(Nominal_Rate, Npery)
=EFFECT(i, n) * must be integer

Example 8:

Typical Credit Card Rate is 18%

If the APR is 18% compounded monthly, is 18% APR the same as the EAR (Effective Annual Rate)?

$i = .18$
$n = 12$

Answer: **No!**

$$EAR = \left(1 + \frac{.18}{12}\right)^{12} - 1 = (1.015)^{12} - 1 = 1.1956187 - 1$$

$$EAR = .19561817 \Rightarrow 19.56\%$$

Notes:
1. Never divide $\frac{EAR}{n}$ to get period rate! only $\frac{APR}{n}$ is used to find period rate ($\frac{i}{n}$).
2. Does APR = EAR? Answer: only when $n = 1$
3. EAR > APR when $n > 1$

Example 9:

which APR yields more interest: 11% compounded quarterly or 10.75% compounded?

$$EAR_1 = \left(1 + \frac{.11}{4}\right)^4 - 1 = .114621259$$
$$EAR_2 = \left(1 + \frac{.1075}{365}\right)^{365} - 1 = .113473238$$

Answer: 11% compounded quarterly earns more interest

Example 10:

MoneyTreeLoaning will give you \$200 today, if you pay them \$250 in 25 days. What is APR? What is EAR?

period length in days = 25

n = # of periods per year = $365/25 = 14.6$

$$\frac{i}{n} = \text{period rate} = \frac{\text{part}}{\text{whole}} = \frac{\text{Interest Paid}}{\text{Original Loan}} = \frac{250-200}{200} = \frac{50}{200} = \frac{5}{20}$$

$$\frac{i}{n} = \frac{1}{4} = .25 \Rightarrow 25\%$$

$$i = APR = \frac{i}{n} * n = .25 * 14.6 = 3.65 \Rightarrow 365\%$$

$$EAR = \left(1 + \frac{3.65}{14.6}\right)^{14.6} - 1 = (1.25)^{14.6} - 1 = 25.99478 - 1$$

$$= 24.99478 \Rightarrow 2{,}499.48\%$$

why is formula true

Example 11:

If EAR is 14.5% and $n = 2$, what is the APR?

$$APR = ((EAR+1)^{\wedge(1/n)} - 1) * n$$

$$APR = ((.145+1)^{\wedge(1/2)} - 1) * 2$$

$$APR = .140093456 \text{ or } 14.01\%$$

why?

$$EAR = \left(1 + \frac{APR}{n}\right)^n - 1$$

$$EAR + 1 = \left(1 + \frac{APR}{n}\right)^n$$

$$(EAR+1)^{1/n} = 1 + \frac{APR}{n}$$

$$(EAR+1)^{1/n} - 1 = \frac{APR}{n}$$

$$((EAR+1)^{1/n} - 1) * n = APR = i$$

Note

Note:

For EFFECT and NOMINAL

✪ npery must be an integer

Excel:

= NOMINAL (Effect_Rate, npery)

= NOMINAL (EAR, n)

What is an Annuity?

① pay an equal amount of cash each period

Example: Mortgage Loan Payment.

OR

② Receive an equal amount of cash each period

Example: Receive Retirement check each Month.

Define Annuity:

① All cash flow payments are equal in amount

② The time between each payment is equal (Total time fixed)

Types of Annuities:

① Ordinary Annuity

payments are made at the END of each Period.

Example: Monthly Mortgage Loan Payment.

② Annuity Due:

payments are made at the BEGINNING of each period.

Example: Lease payments.

Relationship between Annuity Due & Ordinary Annuity:

$$\text{Annuity Due Value} = \text{Ordinary Annuity} * \left(1 + \frac{i}{n}\right)$$

Note: All annuities are ordinary unless otherwise stated (for this class).

Future Value of Annuity
(Savings Plan)

Math

$$FV_{Annuity} = PMT * \left[\frac{\left(1+\frac{i}{n}\right)^{nx} - 1}{\frac{i}{n}} \right]$$

Formula for Annuity **ordinary** (Not Due)

Excel	Math	Meaning
FV =	FV =	Future value of all cash flows
	i =	Annual Interest Rate = APR
	n =	# of compounding periods per year
	x =	# of years
rate =	$\frac{i}{n}$ =	period Rate = $\frac{APR}{n}$
nper =	$n*x$ =	Total # of periods
PMT =	PMT =	equal payments at equal time intervals.
PV =	=	cash Flow at time zero (0)
Type =	=	Annuity Due = Beg = 1 ; ordinary = End = 0 or Leave Blank.

Excel

= FV(rate, nper, PMT, PV, type)

- rate: period Rate
- nper: total periods
- PMT: Payment
- PV: If there is an amount at time zero put here.
- type: 1 for Begin or Due; Leave Blank if ordinary or End.

* Sign of cash Flow Matters to Excel Functions.

(Examples 13, 14 in Excel workbook)

Example 12: If you deposit \$50 at the end of each year for the next 3 years and you earn 12% compounded yearly, what is the future value? (P.12)

Annual Interest Rate $= APR = i = .12$ or 12%

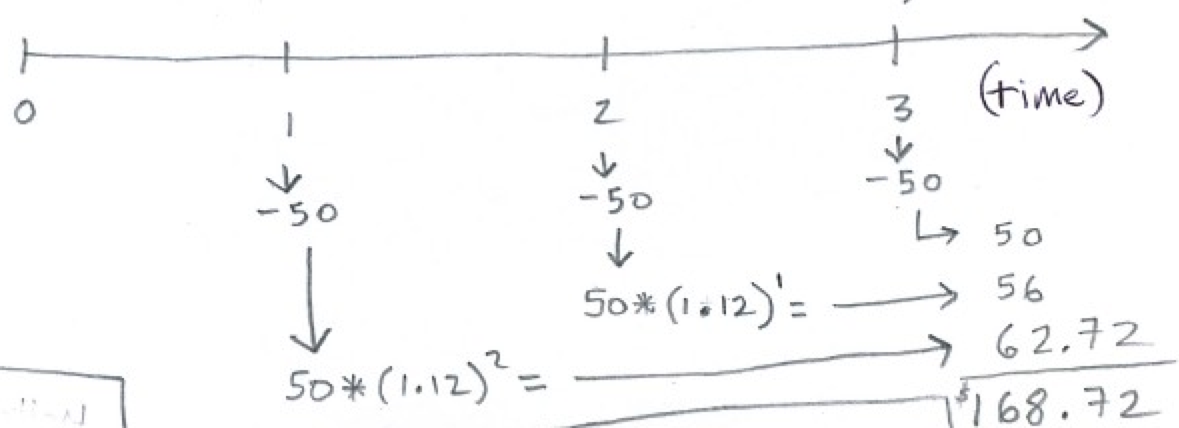
$n =$ # compounding periods per year $= n = 1$

years $= x = 3$

Equal payments made at equal time intervals $= PMT = \$50$

Future value $= FV = ?$

$\boxed{\text{we could use our old formula } FV = PV * \left(1+\frac{i}{n}\right)^{nx}}$ → $FV = 168.72$



$$\boxed{FV = PMT * \left[\frac{\left(1+\frac{i}{n}\right)^{nx}-1}{\frac{i}{n}}\right]} = 50 * \left[\frac{\left(1+\frac{.12}{1}\right)^{1*3}-1}{\frac{.12}{1}}\right] = 50 * \left[\frac{1.4049-1}{.12}\right]$$

{ordinary FV factor (from tables)}

$$= 50 * \frac{.4049}{.12} = 50 * 3.3744 = \$168.72$$

Excel Remember: cash flow signs! PMT & PV are negative if you are investing.

$\boxed{= FV(\text{rate}, \text{nper}, -\text{pmt}, -\text{pv}, \text{type}) = FV\left(\frac{i}{n}, n*x, -PMT, -PV, 0 \text{ or } 1\right)}$

Leave Blank if ordinary

$= FV(.12, 3, -50, , 0) = \$168.72$

Example 12.5

Due: (Begin)

$\frac{i}{n} = .05$ P. 12.5

$n * x = 4$

$PMT = 500$

$FV = 2262.82$

Time	0	1	2	3	4
	500	500	500	500	

---

ordinary: (End)

$\frac{i}{n} = .05$

$n * x = 4$

$PMT = 500$

$FV = 2155.06$

Time	0	1	2	3	4
		500	500	500	500

Relationship:

$$\text{Annuity Due} = \text{Annuity ordinary} * \left(1 + \frac{i}{n}\right)$$

$$\$2262.82 = 2155.06 * (1 + .05)$$

✓ check!!

Example 15 :

- savings plan compounds interest 365 times a year, but you only put money in 12 times a year :

Monthly PMT = 250

x = 25

n = 365 (savings account n)

APR = i = .08

① solve for savings account EAR :

$$EAR = \left(1 + \frac{.08}{365}\right)^{365} - 1 = .08327757\text{-}1$$

(2) From EAR find APR for savings Account :

$$.083277571 = \left(1 + \frac{APR}{12}\right)^{12} - 1$$

Excel: =NOMINAL(.083277571,12)

= .0802584357 7

③ Find period rate for monthly PMT :

$$\frac{i}{n} = \frac{.0802584357 7}{12} = .006688203$$

④ Solve for FV of monthly PMT :

$$FV = PMT * \left[\frac{\left(1 + \frac{i}{n}\right)^{nx} - 1}{\frac{i}{n}}\right] = 250 * \left[\frac{(1 + .006688)^{12*25} - 1}{.006688}\right]$$

FV = \$238,757.59

What if we know how much we want in the future, but we don't know how much to invest each period?

If this is true: $FV = PMT * \left[\frac{\left(1+\frac{i}{n}\right)^{nx} - 1}{\frac{i}{n}}\right]$

Then

$$\frac{FV}{\left[\frac{\left(1+\frac{i}{n}\right)^{nx} - 1}{\frac{i}{n}}\right]} = \frac{PMT * \cancel{\left[\frac{\left(1+\frac{i}{n}\right)^{nx} - 1}{\frac{i}{n}}\right]}}{\cancel{\left[\frac{\left(1+\frac{i}{n}\right)^{nx} - 1}{\frac{i}{n}}\right]}}$$

$$\boxed{\frac{FV}{\left[\frac{\left(1+\frac{i}{n}\right)^{nx} - 1}{\frac{i}{n}}\right]} = PMT}$$

{Formula for find PMT for FV amount}

{Excel Function}

=PMT(rate, nper, , FV, type)

=PMT($\frac{i}{n}$, n*x, , FV, 0 for ordinary or 1 for Due)

If ordinary Leave Blank

Example 16: How much do I have to invest at the end of each month to become a millionaire if I can earn 10% compounded monthly for the next 40 years? (P.15)

Formula: $PMT = \left[\dfrac{FV}{\dfrac{(1+\frac{i}{n})^{nx}-1}{\frac{i}{n}}}\right]$

$PMT = ?$ $FV = \$1{,}000{,}000$

$i = .1$ $X = 40$

$n = 12$

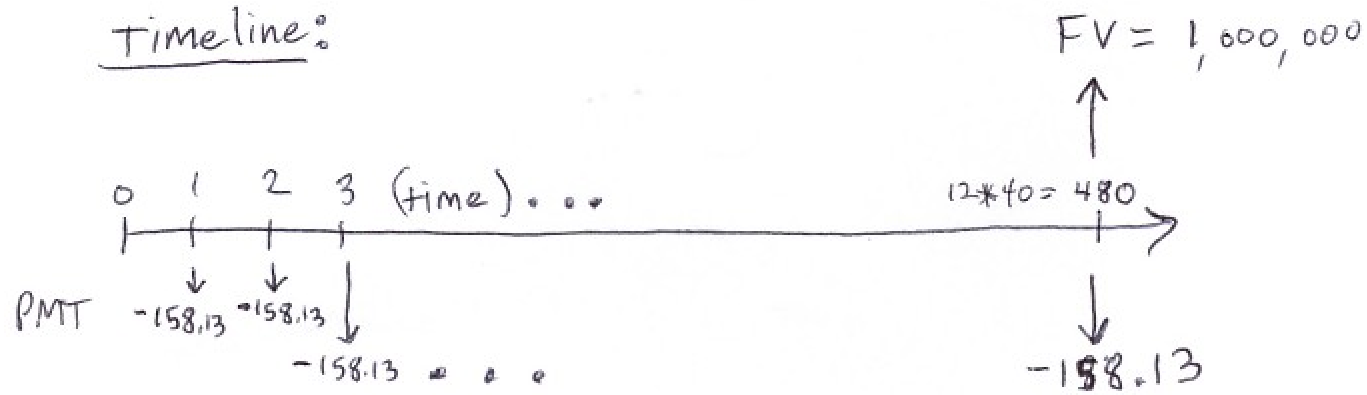
$$PMT = \frac{1{,}000{,}000}{\left[\dfrac{\left(1+\frac{.1}{12}\right)^{12*40}-1}{\frac{.1}{12}}\right]} = \$158.13$$

$$=PMT\left(\frac{.1}{12}, 12*40,, 1000000, 0\right) = \$-158.13$$

2 commas because we are not using PV.

If I want to be a millionaire given a 10% annual rate compounded monthly for 40 years, I have to deposit 158.13 each month

Timeline:



Example 17: $FV = 180{,}000$

$i = .08$

$n = 12$

$x = 18$

$PMT = -374.93$

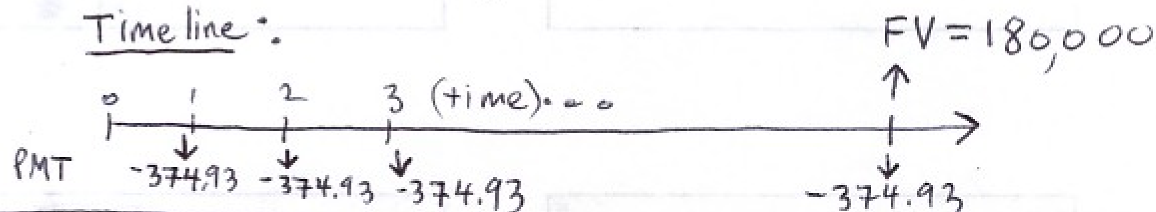
college fund for daughter

Formula = $PMT = \dfrac{FV}{\left[\dfrac{(1+\frac{i}{n})^{nx} - 1}{\frac{i}{n}}\right]}$

$$PMT = 180{,}000 * \frac{\frac{.08}{12}}{(1+\frac{.08}{12})^{18*12} - 1} = 374.93$$

If I need \$180,000 in 18 years to help send my daughter to college, I should save 374.93 each month, assuming 8% APR compounded monthly

Time line:



* Notice that:

$$PMT = \frac{180\,000}{\left[\dfrac{(1+\frac{.08}{12})^{\wedge 18*12} - 1}{\frac{.08}{12}}\right]}$$

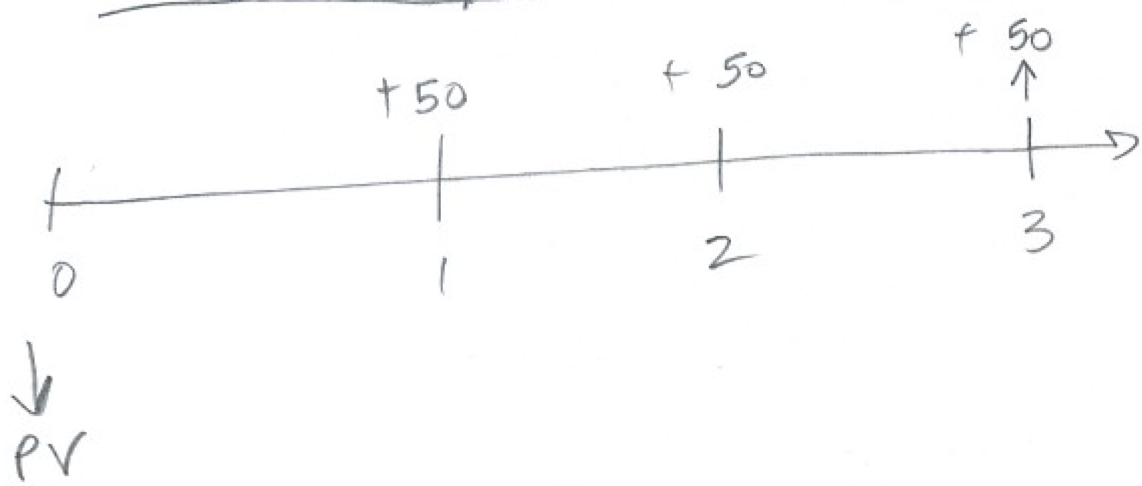
In finance you may see it either way because $\dfrac{1}{(\frac{1}{2})} = 1 * \dfrac{2}{1} = 2$

is the same as:

$$PMT \quad 180{,}000 * \left[\frac{\frac{.08}{12}}{(1+\frac{.08}{12})^{\wedge 18*12} - 1}\right]$$

we have been calculating the future value of the PMT! That is, we put a certain PMT in each period, and then we want to know what that will be worth in the future. BUT, what if we wanted to withdraw a certain amount each period IN THE FUTURE, and we needed to determine how much to invest today (present value)?

<u>For simple example:</u>



what if we needed \$50 at the end of each period, and we wanted to know how much to invest today?

→ next page

Present Value of Annuity
(Discounting Future cash Flows)

Math

MINUS SIGN

$$PV_{Annuity} = PMT * \left[\frac{1-\left(1+\frac{i}{n}\right)^{-nx}}{\frac{i}{n}} \right]$$

ordinary Annuity ONLY

see page 11 for variables defined

Excel

=PV(rate, nper, PMT, FV, type)

- rate: Period Rate
- nper: total # of periods
- PMT: payment
- FV: If there is some left over at final time period
- type: Beg = Due = 1; End = ordinary, leave Blank.

* Sign of cash flow matters for Excel functions

Example 18:

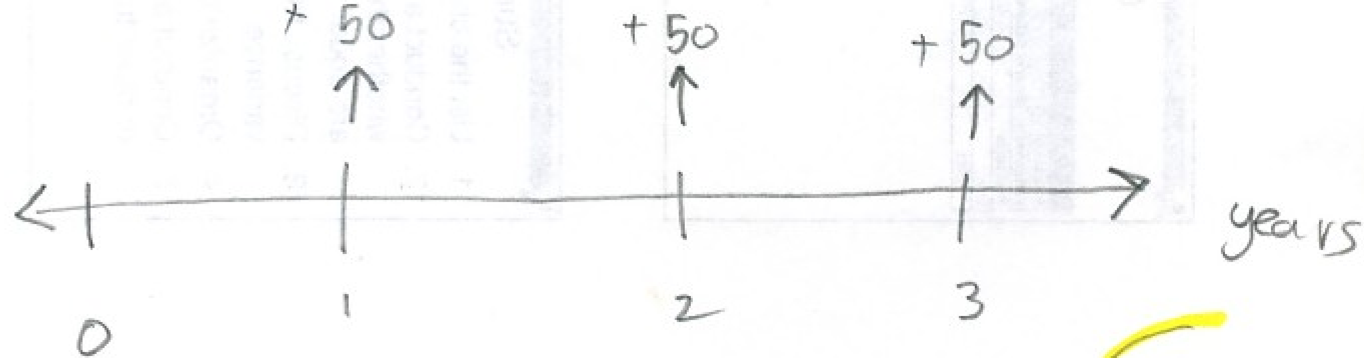
If you want to withdraw \$50 at the end of each year, for the next 3 years, how much do you have to deposit in the bank today if the APR = .12 compounded yearly?

PMT = 50 $\quad$ n = 1

i = .12 $\quad$ X = 3

PV = ?. $\quad$ n * X = 3

$\frac{i}{n} = \frac{.12}{1} = .12$



0 ↓ − PV

$\frac{50}{1.12}$ → 44.64

$\frac{50}{(1.12)^2}$ → 39.86

$\frac{50}{(1.12)^3}$ → 35.59

PV = 120.09

we could make this Present value calculation the long way

OR we can use the Financial Formula Handout

$$PV = PMT\left[\frac{1-\left(1+\frac{i}{n}\right)^{-nx}}{\frac{i}{n}}\right] = 50\left[\frac{1-\left(1+\frac{.12}{1}\right)^{-3*1}}{\frac{.12}{1}}\right] = 120.09$$

Excel: = PV(rate, nper, PMT, , type)

= PV($\frac{i}{n}$, n*X, PMT)

= PV(.12, 3, 50) = −120.09

Remember cash flow

Example 19: How much do you have to have in bank when you retire if you want to withdraw \$3000 each month for the next 35 years if you can earn 6% compounded monthly?

$PV = ?$

$PMT = +3000$

$x = 35$

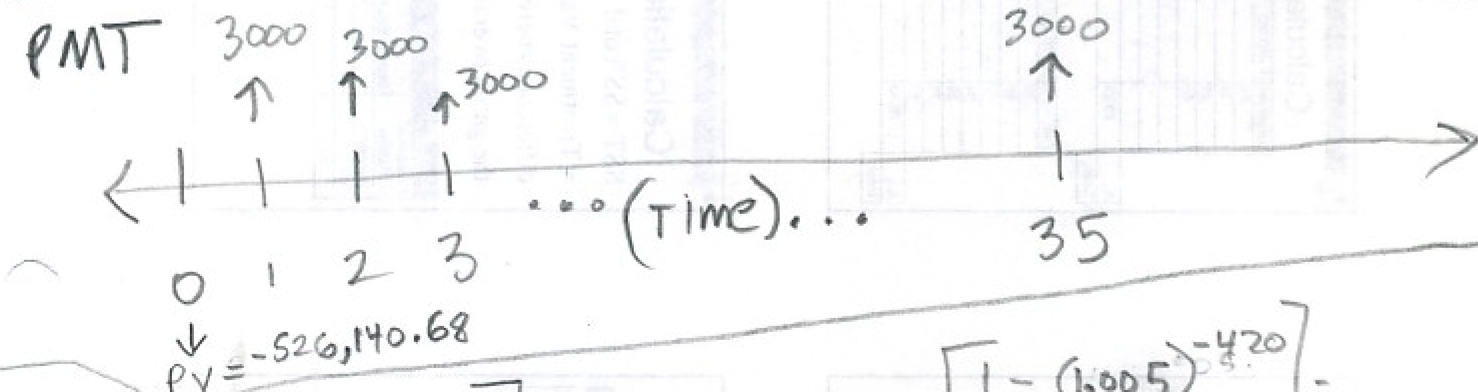
$n = 12$

$n * x = 35 * 12 = 420$

$i = .06$

$\frac{i}{n} = \frac{.06}{12} = .005$

Timeline:



Formula:

$$PV = PMT * \left[\frac{1-(1+\frac{i}{n})^{-nx}}{\frac{i}{n}}\right]$$

$$PV = 3000\left[\frac{1-(1.005)^{-420}}{.005}\right] =$$

$$PV = \$526\,140.68$$

Excel:

$= PV(\frac{i}{n}, n*x, PMT)$ $= PV(\frac{.06}{12}, 12*35, 3000) = \$-526,140.68$

You will need \$526,140.68 when you retire if you want to withdraw \$3000 per month from an account that yields 6% compounded monthly

Example 20:

But if you need \$ 526,140.68 when you retire, how much do you need to deposit each month if you are 28 years old now & plan to retire at age 70 $(i = .10, n = 12)$?

we already Know how to do this:

$$PMT = \frac{FV}{\left[\dfrac{\left(1+\frac{i}{n}\right)^{nx} - 1}{\frac{i}{n}}\right]}$$

PMT = ?
FV = 526,140.68
$i = .1$ or 10%
$n = 12$
$x = (70-28) = 42$
$n = 42 * 12 = 504$

$$PMT = \frac{526,140.68}{\left[\dfrac{\left(1+\frac{.1}{12}\right)^{12*42} - 1}{\frac{.1}{12}}\right]} = 67.9391$$

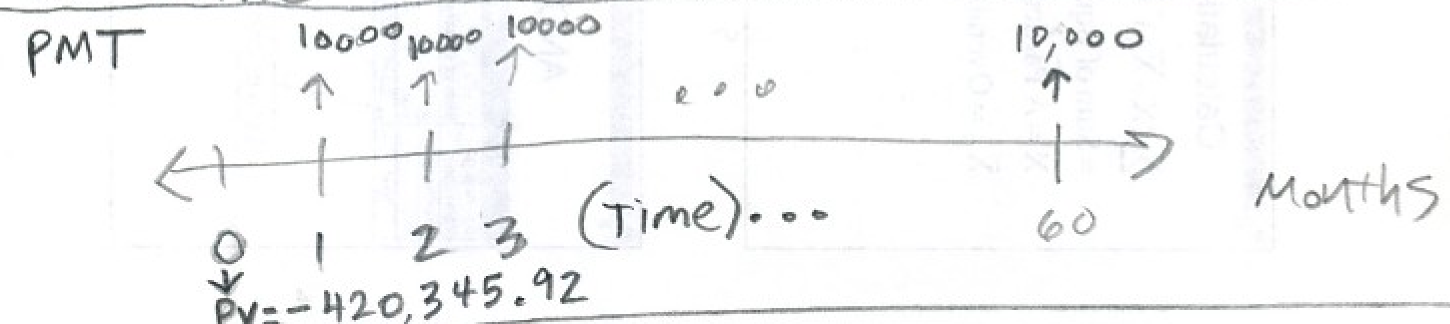
Excel:
$= FV(\frac{.1}{12}, 12*42, , 526,140.68) = -67.9391 \approx -67.94$

Total paid over life of investment = 67.94 * 504 = 34,241.76
total taken out over life of investment = 3000 * 420 = 1,260,000
−
Interest = 1225,758.

Answer: If we invest 67.94 for 504 months at an APR of 10% compounded 12 times a year, we will be able to then withdraw \$3,000 for 420 months for a total net gain of \$1,225,758.24

Example 21: If a new machine will yield a net cash flow of \$10,000 per Monthy for the next 5 years & your discount rate is 15% compounded monthly, how much should you pay for machine?

TIMELINE:



$n = 12$ $\qquad n*x = 60$

$x = 5$ $\qquad i = .15$

$PMT = 10{,}000$ $\qquad \frac{i}{n} = \frac{.15}{12} = .0125$

$$PV = 10{,}000\left[\frac{1-(1.0125)^{-60}}{.0125}\right] = 420{,}345.92$$

Excel:

$= PV(\frac{i}{n}, n*x, PMT, , 0 \text{ or } 1)$ $\qquad =PV(\frac{.15}{12}, 12*5, 10000)$

positive because this is cash coming into business!

$= \$420{,}345.92$

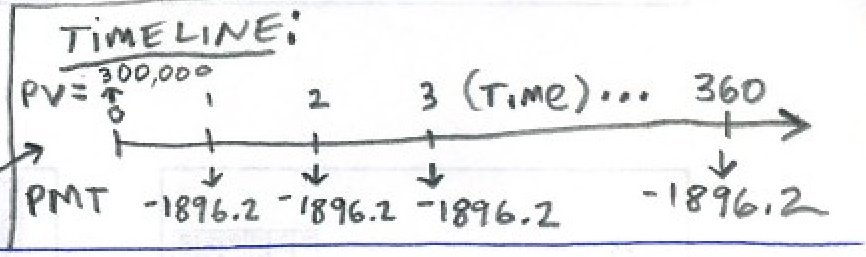
we should pay \$420,345.92 or less for the machine. (\$420,345.92 is the max price we should pay.)

How do we solve for PMT?:

$$PV = PMT\left[\frac{1-\left(1+\frac{i}{n}\right)^{-xn}}{\frac{i}{n}}\right]$$

$$\frac{PV}{\left[\frac{1-\left(1+\frac{i}{n}\right)^{-xn}}{\frac{i}{n}}\right]} = \frac{PMT * \cancel{\left[\frac{1-\left(1+\frac{i}{n}\right)^{-xn}}{\frac{i}{n}}\right]}}{\cancel{\left[\frac{1-\left(1+\frac{i}{n}\right)^{-xn}}{\frac{i}{n}}\right]}}$$

$$\frac{PV}{\left[\frac{1-\left(1+\frac{i}{n}\right)^{-xn}}{\frac{i}{n}}\right]} = PMT$$



Example 22: Your home mortgage loan is \$300,000 with an annual rate of 6.5% compounded monthly for the next 30 years, what is your monthly PMT?

$x = 30$ $\quad i = .065$

$n = 12$ $\quad \frac{i}{n} = \frac{.065}{12}$

$x * n = 360$

$PV = 300{,}000$

$$PMT = \frac{300{,}000}{\left[\frac{1-\left(1+\frac{.065}{12}\right)^{-360}}{\frac{.065}{12}}\right]} = \$1{,}896.20$$

The monthly PMT for the mortgage is 1896.20 @ an annual rate of 6.5% compounded monthly for 30 years.

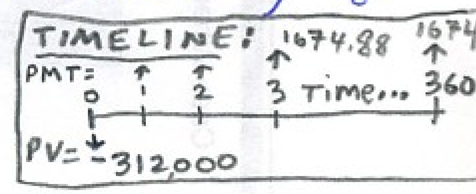
Excel: $= PMT\left(\frac{i}{n}, n*x, PV\right)$

positive cash flow - it comes into your pocket (from bank)

$= PMT\left(\frac{.065}{12}, 12*30, 300{,}000\right)$

$= -1{,}896.20$

Example 23: If your retirement account (p.24) shows \$312,000 on the day you retire & you plan to live to be 100 (you are 70) how much can you w/draw each month if you can invest in a 30 year bond fund that yields 5% compounded monthly?



$PV = \$312{,}000$ $\qquad x = 30$

$i = .05$ $\qquad PMT =$

$n = 12$ $\qquad n * x = 12 * 30 = 360$

$\frac{i}{n} = \frac{.05}{12}$

$$PMT = \frac{312000}{\left[\dfrac{1-\left(1+\frac{.05}{12}\right)^{-360}}{\frac{.05}{12}}\right]} = \$1{,}674.88$$

Excel:

negative because on the day that you retire you put it back in the bank.

$= PMT(\frac{i}{n}, n*x, -PV)$ $\qquad = PMT(\frac{.05}{12}, 12*30, -312000)$

$= 1{,}674.88$

If I can get 5% compounded monthly on my retirement fund I can w/draw \$1,674.88 per month for 30 years.

Example 24:

If you have a home loan for \$230,000 & the monthly PMT is \$3,250 for 10 years, what is the APR & EAR?

PV = 230,000

PMT = 3250

x = years = 10

n = 12

$\frac{i}{n}$ = period rate = RATE(12*10, -3250, 230000,, 0)

= .009686459

★ can't solve for this with formula directly because exponent > 5

$APR = i = \frac{i}{n} * n = .009686459 * 12 = .116237506$

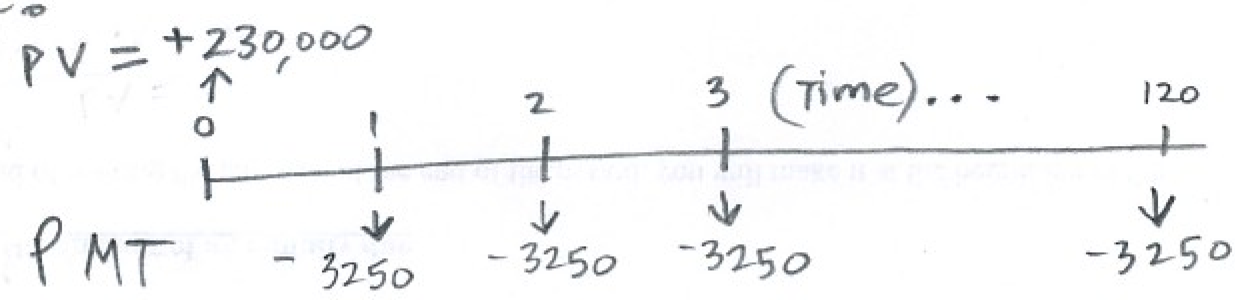
$$EAR = \left(1 + \frac{APR}{n}\right)^n - 1 = \left(1 + \frac{.116237506}{12}\right)^{12} - 1 = .123199428$$

OR

Excel: EAR = EFFECT(.116237506, 12*10) = .123199428

Answer: The APR on the loan is 11.62% and the EAR is 12.32%.

Timeline:



Example #25: How long until you pay off your credit card balance of \$2,000 with an APR = 18% compounded monthly & a minimum balance paid each month of \$41.00?

(p.26)

$PV = 2000$

$i = .18$

$n = 12$

$\frac{i}{n} = .015$

$x = ?$

$xn = ?$

$PMT = 41$

$$PV = PMT\left[\frac{1-\left(1+\frac{i}{n}\right)^{-nx}}{\frac{i}{n}}\right]$$

$$2000 = 41\left[\frac{1-(1.015)^{-nx}}{.015}\right]$$

$$\frac{2000}{41} = \frac{1-(1.015)^{-nx}}{.015}$$

$$\left[\frac{2000}{41}\right] * .015 = 1-(1.015)^{-nx}$$

$$\left[\frac{2000}{41}\right] * .015 - 1 = -(1.015)^{-nx}$$

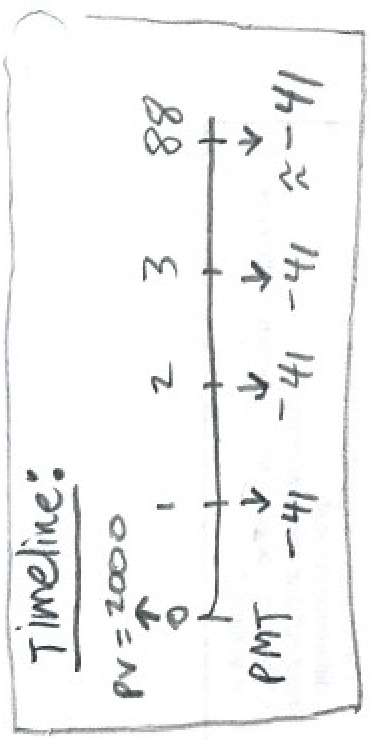
$$-.268292683 = -(1.015)^{-nx}$$

$$.268292683 = (1.015)^{-nx}$$

$$.268292683 = \frac{1}{(1.015)^{nx}}$$

$$(1.015)^{nx} = \frac{1}{.268292683}$$

$$nx = \frac{\ln\left(\frac{1}{.268292683}\right)}{\ln 1.015} = 88.36799$$



Excel: =NPER($\frac{.18}{12}$, -41, 2000), = 88.37 periods

years = 88.37 ÷ 12 = 7.364 years.

It will take $88\frac{1}{3}$ months to pay of the 2000 credit card bill - or $7\frac{1}{3}$ years!!!

Perpetuity (consol)

- Annuity where cash flow continues forever
- Preferred stock are considered perpetuities
  → (contract to get defined dividend $\frac{1}{2}$ D $\frac{1}{2}$ E)

$$PV = PMT\left[\frac{1-\left(1+\frac{i}{n}\right)^{-nx}}{\frac{i}{n}}\right] = PMT\left[\frac{1-\frac{1}{\left(1+\frac{i}{n}\right)^{nx}}}{\frac{i}{n}}\right]$$

becomes 0

gets large

As x gets large (years are forever) $\left(1+\frac{i}{n}\right)^{nx}$ approaches infinity, then, As $\left(1+\frac{i}{n}\right)^{nx}$ approaches ∞, $\frac{1}{\infty}$ approaches zero & $\frac{1-0}{\frac{i}{n}} * PMT$ becomes $\frac{PMT}{\frac{i}{n}}$

Example 26:

How much should you pay for preferred stock where it is assumed:

1. quarterly dividend = \$1.25
2. quarterly discount rate = 4%

$PMT = \$1.25$

$\frac{i}{n} = .04$

$$PV = \frac{1.25}{.04} = \$31.25$$

Loans:

(P. 29)

(1) Interest only Loan

- pay fixed interest each period
- pay back principal (loan Amount) at end of term
- Example : Corporate Bonds

Example 38:

	Interest Paid	Principal paid
1	50,000	
2		
3		
4		
5		
6		
7		
8		
9		
10		1,000,000

Borrow 1,000,000 @ 10% compounded semi annually for 5 years

$$\text{Interest} = 1{,}000{,}000 * \frac{.1}{2} = 50{,}000$$

(2) Amortized Loans : Each period payment is part Principal & part interest

(A) medium-term business loans

(1) Each period Equal Principal is paid back
(2) Interest paid each period goes down
(3) Total payment each period goes down

Example : Borrow 10,000 @ 10% compounded Annual for 4 years
(Not in Excel workbook)

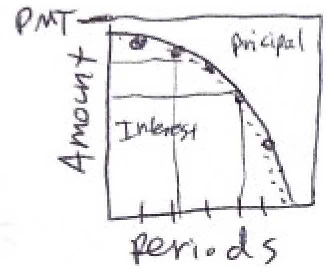
Amortazation Table

Period	Balance	Interest	Principal Paid	Total Payment
0	10,000			
1	7500	10000 * .1 1000	2500	3500
2	5000	750	2500	3250
3	2500	500	2500	3000
4	0	250	2500	2750

(B) Consumer loans (AMortized)

(4)

1. Each period PMT is equal
2. Each period amount of principal paid back goes up (p. 30)
3. Each period amount of interest paid goes down



$$\text{Interest} = \left\{\begin{matrix}\text{principal}\\ \text{Balance}\end{matrix}\right\} * \left\{\begin{matrix}\text{period}\\ \text{Rate}\end{matrix}\right\}$$

$$\left\{\begin{matrix}\text{paid on}\\ \text{Principal}\end{matrix}\right\} = PMT - \text{Interest}$$

Examples not in Excel workbook

Example 1 Mortgage = $325,000, $i = 7.75\%$ $n=12$, $x=30$

$$PMT = \frac{PV}{\left[\dfrac{1-(1+\frac{i}{n})^{-xn}}{\frac{i}{n}}\right]} = \frac{325000}{\left[\dfrac{1-(1+\frac{.0775}{12})^{-12*30}}{\frac{.0775}{12}}\right]} = \$2328.34$$

Example 2 PV = 10,000 $i = .1$ $n = 1$ $x = 3$

$$PMT = \frac{10000}{\dfrac{1-(1.1)^{-3}}{.1}} = 4021.15$$

period	PMT	Interest	Principal Reduction	Balance
0				10,000
1	4021.15	10000 * .1 1000	4021.15 − 1000 3021.15	10000 − 3021.15 6978.85
2	↓	6978.85 * .1 697.89	4021.15 − 697.89 3,323.26	3655.59
3	↓	365.56	3655.59	0

(3) Pure Discount Loan

p. 31

- Receive loan amount
- pay back all interest & principal at end of loan term
- use this method when cash flow for entity is limited until the end of term
- Examples: (1) US treasury Bills (time < 1y)
  (2) Savings Bonds
  (3) Some Corp. Bonds

Examples Not in Excel workbook:

Example 1:

I year Treasury Bill that promises to pay $15,000 in one year with $i = 3.81\%$ $n = 1$ how much will you lend the government?

$FV = 15000$
$i = .0381$
$n = 1$
$x = 1$

$$PV = \frac{15000}{1.0381} =$$

$14,449.48

Example 2:

Corporate Bond $FV = 1{,}000{,}000$
$i = .07$ $x = 25$ $n = 1$

$$PV = \frac{1{,}000{,}000}{(1.07)^{25}} = 184{,}249.18$$