Geometric Series

skip if you dont want to see proof

a_1, $a_1 * r$, $a_1 * r^2$

$a_1 = 44.64$

$r = .892857143 = \frac{39.86}{44.64} = \frac{35.59}{39.86}$ ✓

$44.64, 44.64 * .892857143, 44.64 * .892857143^2$

$44.64, 39.86, 35.59$

Sum of Geometric series

→

Pages 34.0, 34.1, 34.2, 34.3 Are proofs that are Not required

Sum of Geometric series

34.1

Let S_n = sum of n terms of geometric series
r = common ratio

(A) $$S_n = a_1 + a_1 r + a r^2 \dots + a_1 r^{n-2} + a_1 r^{n-1}$$

multiply each side by r

(B) $$r S_n = a_1 r + a_1 r^2 + a r^3 \dots + a_1 r^{n-1} + a_1 r^n$$

minus (B) from (A)

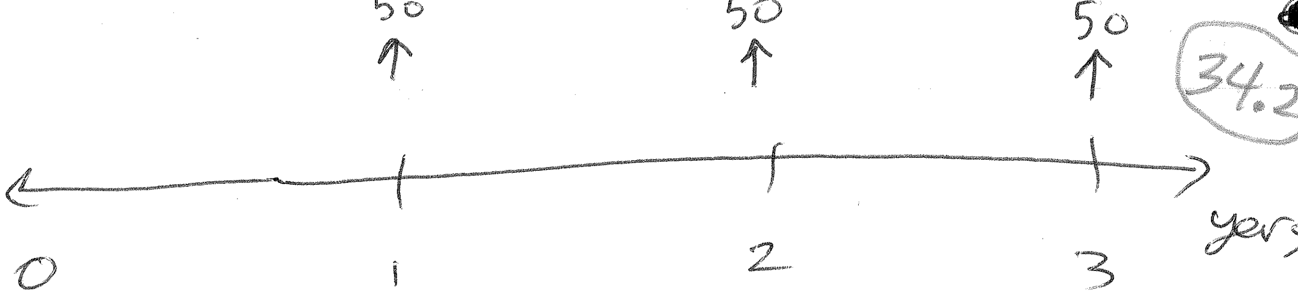
$$S_n - r S_n = a_1 - a_1 r + a_1 r - a_1 r^2 + a r^2 \dots + - a_1 r^n$$

$$S_n - r S_n = a_1 - a_1 r^n$$

$$S_n (1 - r) = a_1 (1 - r^n)$$

$$S_n = \frac{a_1 (1 - r^n)}{1 - r}$$

34.2



$PV_1 = 50(1+.12)^{-1}$

$PV_1 = 44.64$

$PV_2 = 50(1+.12)^{-2}$

$PV_2 = 39.86$

$PV_3 = 50(1+.12)^{-3}$

$PV_3 = 35.59$

$PV_1 = 44.64$

$PV_2 = 39.86$

$PV_3 = 35.59$

$PV_{total} = 120.09$

$PV_1 = 50(1+.12)^{-1} = PV_1 = PMT\left(1+\frac{i}{n}\right)^{-}$

$PV_2 = 50(1+.12)^{-2} = PV_2 = PMT\left(1+\frac{i}{n}\right)^{-2}$

$PV_3 = 50(1+.12)^{-3} = PV_3 = PMT\left(1+\frac{i}{n}\right)^{-3}$

$\vdots$

~~First term = $50(1+.12)^{-1}$~~

$PMT\left(1+\frac{i}{n}\right)^{-nx}$

Sum of all would be:

$PMT\left(1+\frac{i}{n}\right)^{-1} + PMT\left(1+\frac{i}{n}\right)^{-2} \dots + PMT\left(1+\frac{i}{n}\right)^{-nx}$

First term $= PMT\left(1+\frac{i}{n}\right)^{-1}$

Common ratio $= \left(1+\frac{i}{n}\right)^{-1}$

$$S_n = \frac{a_1 (1 - r^n)}{1 - r}$$

34.3

$n = n * x$ = total periods

a_1 = first term = $PMT\left(1+\frac{i}{n}\right)^{-1}$

r = common ratio = $\left(1+\frac{i}{n}\right)^{-1}$

S_n = sum of present value of all future cash flows = PV

$$PV = \frac{\left[PMT\left(1+\frac{i}{n}\right)^{-1}\right]\left(1-\left[\left(1+\frac{i}{n}\right)^{-1}\right]^{nx}\right)}{1-\left(1+\frac{i}{n}\right)^{-1}}$$

$$PV = \frac{\left[PMT\left(1+\frac{i}{n}\right)^{-1}\right] * \left(1-\left[\left(1+\frac{i}{n}\right)^{-1}\right]^{nx}\right)}{1-\left(1+\frac{i}{n}\right)^{-1}} * \frac{1+\frac{i}{n}}{1+\frac{i}{n}}$$

$$PV = \frac{\left[PMT * \left(1+\frac{i}{n}\right)^{-1}\right]\left(1+\frac{i}{n}\right) * \left(1-\left[1+\frac{i}{n}\right]^{-nx}\right)}{\left(1+\frac{i}{n}\right) - \left(1+\frac{i}{n}\right)^{-1} * \left(1+\frac{i}{n}\right)}$$

$$PV = \frac{PMT * \left(1-\left(1+\frac{i}{n}\right)^{-nx}\right)}{1+\frac{i}{n}-1}$$

$$PV = \frac{PMT\left(1-\left(1+\frac{i}{n}\right)^{-xn}\right)}{\frac{i}{n}}$$