# Chapter 4

## Introduction to Valuation: Time value of Money

Lump sum calculations: (1) Present value
(2) Future value

Present — 0 — 1 (years) — Future

$1.00

put $1 in Bank that pays 10% Interest Rate compounded 1 time a year. (cash Flow to you is −1 because it comes out of your pocket).

Bank pays 10% Interest

$1 + (1 * .1)$

$1 + .10$

$1.10

original Investment

Interest paid

$1.10

original Investment plus interest paid. (cash Flow to you is +1.10 because it comes into your pocket).

The Present value is $1.00

The Future value is $1.10

# Formula to Calculate Future Value

$$FV = PV\left(1+\frac{i}{n}\right)^{n*X}$$

text book:

$$\boxed{FV = C*(1+r)^{t}}$$

$FV =$ Future Value

$PV =$ Present Value

$i =$ Annual Interest Rate (also: APR)

$n =$ number of compounding periods per year

$X =$ # of years

$\frac{i}{n}$ = period Rate

$n*X$ = Total # periods

**Example 1:** If you put \$10,000 in bank at an annual rate of 6%, compounded monthly for 10 years how much will you have at maturity?

Future Value $= FV = ?$

Present Value $= PV =$ how much you deposit today $= \$10{,}000$

Annual Rate $i = 6\%$ or $0.06$

# compounding per year $n = 12$

years $= X \quad 10$

$$FV = 10{,}000\left(1+\frac{.06}{12}\right)^{10*12}$$

$$= 10{,}000\,(1+.005)^{120}$$

$$= 10{,}000\,(1.005)^{120}$$

$$= 10{,}000 * 1.8193967340322\,8$$

$$= \$18{,}193.97$$

# Excel FV Function

Future value = FV = FV

Present value = PV = PV

period rate = $\frac{i}{n}$ = rate

Total periods = $x*n$ = nper

= FV(rate, nper, , -PV)

Skip PMT argument

PV negative because FV understands cash flow. Putting in bank is a negative cash flow to you

$= FV\left(\frac{i}{n}, x*n, , -PV\right)$

= FV(.005, 120, , -10000)

= \$ 18,193.97

# Interest

: ① "Rent on money"

② When you use someone else's money, you must pay them to use the money: The payment given to use the money is called interest.

③ When you Borrow money, you pay interest

④ When you Lend money, you receive interest

Interest is in dollars

Interest Rate is in decimal or fractional or percentage terms (percentage of investment or loan).

## Simple Interest:

Interest Earned on the original investment only (or paid on original loan).

Example 2:

PV = investment = \$100 (cash out of pocket)
$i$ = Annual Interest Rate = .10 or 10%
$n$ = compounding periods per year = 1
$x$ = years = 4
FV = Future value of investment (Lump sum) = ?

simple interest = \$100.00 * .1 = \$10.00

| | Interest | Amount in Bank |
|---|---|---|
| year 0 | | \$100 |
| year 1 | \$10 | 10 + 100 = \$110 |
| year 2 | \$10 | 10 + 110 = \$120 |
| year 3 | \$10 | 10 + 120 = \$130 |
| year 4 | \$10 | 10 + 130 = \$140 |

## Compound Interest:

Interest earned on both original investment and interest reinvested from prior periods.

Example 3:

| | | |
|---|---|---|
| $PV =$ | \$100 | Investment |
| $i =$ | .10 or 10% | Annual Interest Rate |
| $n =$ | 1 | # compounding periods per year |
| $x =$ | 4 | years |
| $FV =$ | ? | Future value |

$FV_{year1} = 100 + 100 * .1 = 100 + 10 = 110$

$FV_{year2} = 110 + 110 * .1 = 110 + 11 = 121$

$FV_{year3} = 121 + 121 * .1 = 121 + 12.1 = 133.10$

$FV_{year4} = 133.10 + 133.10 * .1 = 133.1 + 13.31$

$= 146.41$

Compound Interest FV ⇒ \$146.41
Simple Interest FV ⇒ − 140.00

Interest on Interest = \$6.41

Interest earned on the reinvestment of previous interest payments

**Compounding:**

The process of accumulating interest in an investment over time to earn more interest

# Math:



$PV = \$100$

$i = .10$

$n = 1$

$x = 1$

$FV = ?$

original investment

interest

$FV = 100 + 100 * .1$

$FV = 100 + 10$

$FV = 110$

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Notice:

100 in both places

$100 + 100 * .1$

Plus sign

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Notice:

$100 * 1 + 100 * .1$

If we put 1 here, it is still the same

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Notice that we can factor: (distributive property backwards)

$100 * 1 + 100 * .1$

$100 * (1 + .1) = 100 * (1 + .1) = 100 * 1.1$

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Conclusion: $100 + 100 * .1 = 100 * 1.1$

# Derive Easier Fomula for Compound Interest

## Lump sum Future value calculation:

| | | |
|---|---|---|
| Present value | $PV =$ | $100$ |
| Annual Rate | $i =$ | $.10$ |
| # com. periods per year | $n =$ | $1$ |
| year | $x =$ | $4$ |
| Future value | $FV =$ | ? |

$100(1+.1) = 100 * (1+.1)$

NO multiplication sign

multiplication sign

factor out

$100(1+.1)$

$FV_1 = 100 + 100 * .1 = 100 + 10 = 110 = 100(1+.1)$

$\boxed{\text{Thus}}$

$110 = 100(1+.1)$

$FV_2 = 110 + 110 * .1$

$\boxed{\text{Substitute}}$

$= 100(1+.1) + 100(1+.1) * .1$

$\boxed{\text{Factor out}}$

$= 100(1+.1) * 1 \; 100(1+.1) * .1$

$= 100(1+.1)(1+.1)$

$= 100(1+.1)^2 = 121$

$FV_3 = 121 + 121 * .1$

$\boxed{\text{Substitute}}$

$= 100(1+.1)^2 + 100(1+.1)^2 * .1$

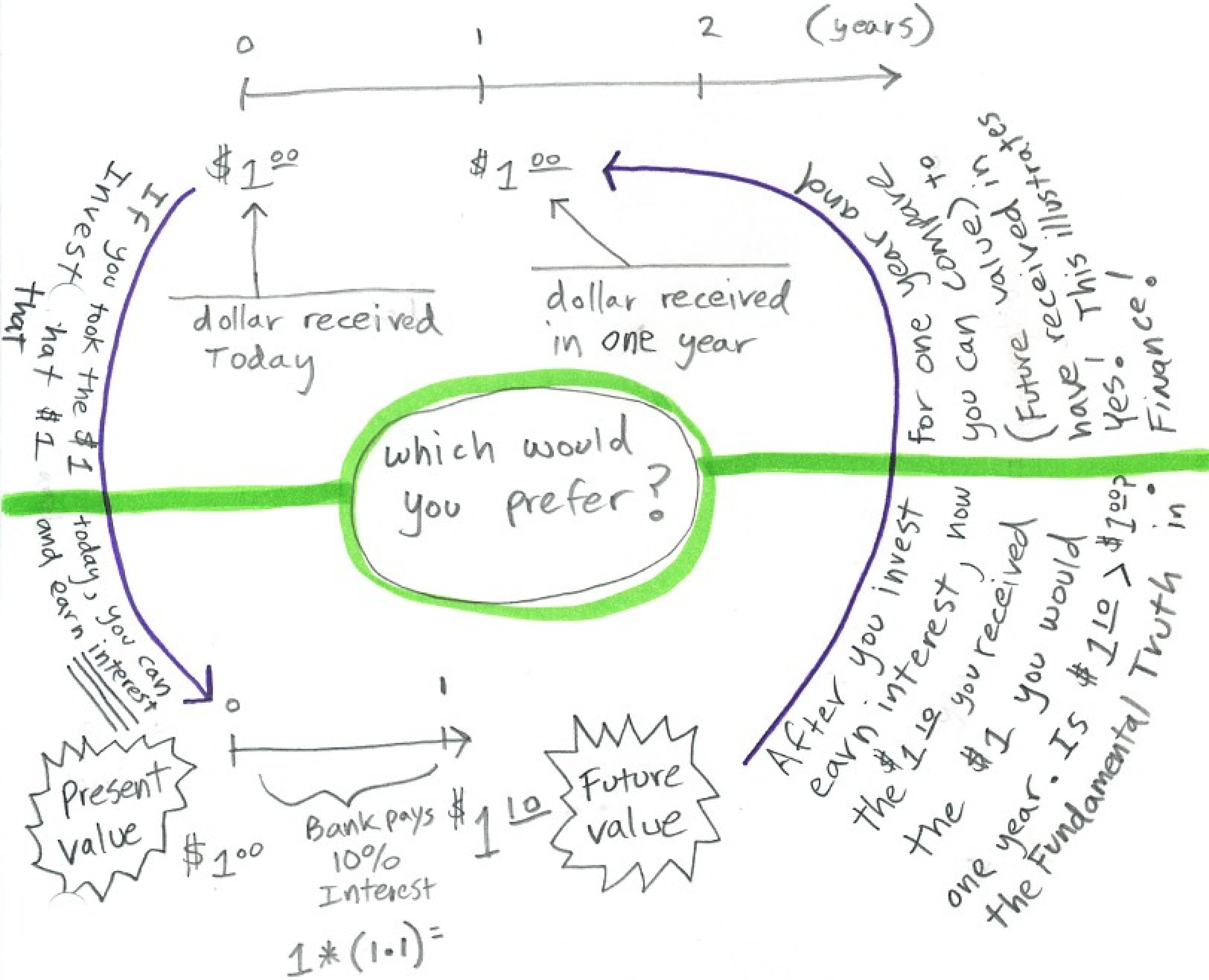
$= 100(1+.1)^2(1+.1) = 100(1+.1)^3 = 133.10$

$FV_4 = 100(1+.1)^3 + 100(1+.1)^3 * .1$

$= 100(1+.1)^3(1+.1) = 100(1+.1)^4 = 146.41$

# Fundamental Truth in Finance

A dollar received today is worth more than a dollar received later. (This is true because of interest).



Present value — 0 — 1 year — Future value

$\$1^{00}$ — Bank pays 10% Interest — $\$1^{10}$

$1 * (1.1) = \$1^{10}$

$\$1^{00}$ Present value + $\$0.10$ Interest = $\$1^{10}$ Future value

$\$1^{00}$ → The Future value of $\$1^{00}$ is $\$1^{10}$ → $\$1^{10}$

$\$1^{00}$ ← The present value of $\$1^{10}$ is $\$1^{00}$ ← $\$1^{10}$

$\$1^{00}$ → This way you **add** interest → $\$1^{10}$

$\$1^{00}$ ← This way you **take out (subtract out)** interest ← $\$1^{10}$

$$FV = PV\left(1+\frac{i}{n}\right)^{x*n}$$

Divide Both Sides

$$\frac{FV}{\left(1+\frac{i}{n}\right)^{x*n}} = \frac{PV\left(1+\frac{i}{n}\right)^{x*n}}{\left(1+\frac{i}{n}\right)^{x*n}}$$

cancel

$$\frac{FV}{\left(1+\frac{i}{n}\right)^{x*n}} = \frac{PV\cancel{\left(1+\frac{i}{n}\right)^{x*n}}}{\cancel{\left(1+\frac{i}{n}\right)^{x*n}}}$$

PV on 1 side

$$\frac{FV}{\left(1+\frac{i}{n}\right)^{x*n}} = PV$$

Formula

$$PV = \frac{FV}{\left(1+\frac{i}{n}\right)^{x*n}}$$

$\boxed{\text{Derive Fomula for PV}}$

# Formula to calculate Present value

$$PV = \frac{FV}{\left(1+\frac{i}{n}\right)^{x*n}}$$

term used when making present value calculations

$FV$ = Future value
$PV$ = Present Value
$i$ = Annual Interest Rate = Discount Rate
$n$ = # of compounding periods per year
$x$ = # of years
$\frac{i}{n}$ = period Rate
$x*n$ = Total number of periods.

# Excel PV Function

Future Value = $FV$ = FV
Present value = $PV$ = PV
period rate = $\frac{i}{n}$ = rate
Total periods = $x*n$ = nper

PV Function will always give negative answer because this is the amount to invest

= PV ( rate, nper,, FV)

skip PMT by putting 2 commas

$= PV\left(\frac{i}{n}, x*n,, FV\right)$

Example 6: How much do we have to put in the bank today ($i = .1$, $n = 12$) to be a millionare in 40 years?

$FV = 1{,}000{,}000$
$i = .1$
$n = 12$
$X = 40$

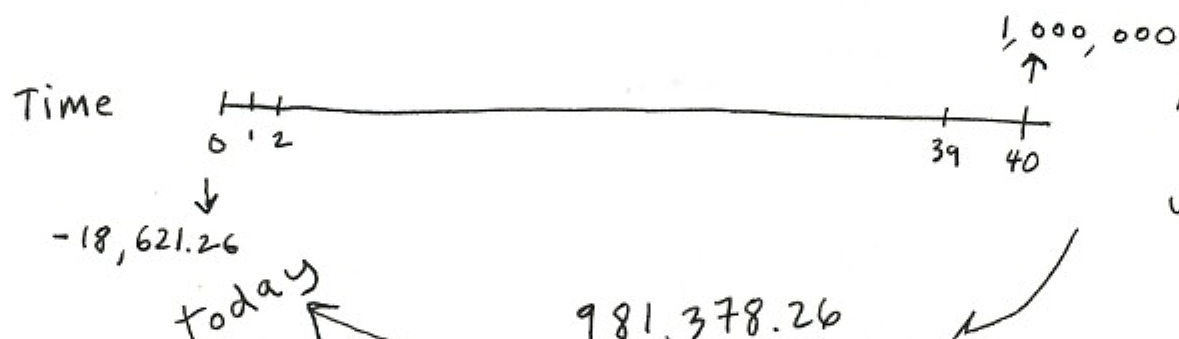
$$PV = \frac{1{,}000{,}000}{\left(1+\frac{.1}{12}\right)^{12*40}} = 18\,621.7$$

$$\underbrace{=PV\left(\tfrac{.1}{12}, 12*40,, 1000000\right) = -18{,}621.74}$$

Notice that Excel knows that the money going into the investment has a negative cash flow

Answer: If we want to be a millionare in 40 years & we could earn 10% compounded monthly, we would need to invest \$18,621.74 today.

$$\begin{array}{r} 1000\,000 \\ -\quad 18\,621.74 \\ \hline = 981{,}378.26 \end{array}$$ Interest earned




you know what you want in FV

981,378.26
Interest going backwards
"taking out all interest to see what to deposit

Example 7:

How much would we have to invest today, if we wanted to have \$150,000 for our daughter's college tuition in 18 years and we could earn an annual interest rate (discount rate) of 6.95% compounded daily (365 times a year)?

$PV = ?$

$FV = \$150,000$

$x = 18$

$i = .0695 \Rightarrow 6.95\%$

$n = 365$

$$PV = \frac{FV}{\left(1+\frac{i}{n}\right)^{n*x}}$$

$$PV = \frac{150\,000}{\left(1+\frac{.0695}{365}\right)^{(365*18)}} = \frac{150\,000}{(1.0001904109589041)^{6570}}$$

$$= \frac{150\,000}{3.4934190008 92} = \$42,937.88$$

Excel

$$= PV\left(\frac{.0695}{365}, 365*18, , 150\,000\right) = -42,937.88$$

**Answer:** If we want to have \$150,000 in 18 years to pay for our daughter's college tuition & we can earn 6.95% compounded daily, we would need to invest \$42,937.88 today.

**Notice:** Excel shows this as negative because this is a cash flow out of your wallet & into your bank

## Example 8:

If you want to buy a $350,000 C & C Router Machine to improve manufacturing efficiency and you have $200,000 today that you can invest at an annual rate of 8.5% compounded monthly, how long do you have to wait until your investment will grow to $350,000. (Assume machine will cost 350000 in future).

$PV = 200{,}000$

$i = .085$

$n = 12$

$x = ?$

$x * n = ?$

$FV = 350{,}000$

solve for Number of Periods

**Excel**

=NPER(rate,,-PV,FV)

$= NPER(\frac{i}{n},,-PV,FV)$

$= NPER(\frac{.085}{12},,-200000,350000)$

= 79.2841 = total periods

$\frac{79.2841}{12} = x = \text{years}$

6.607 = years

**MATH**

$$FV = PV * \left(1+\frac{i}{n}\right)^{nx}$$

$$350{,}000 = 200{,}000 * \left(1+\frac{.085}{12}\right)^{n*x}$$

$$\frac{35}{20} = (1.00708\overline{3})^{n*x}$$

$$\frac{LN\frac{35}{20}}{LN\ 1.00708\overline{3}} = n*x$$

$79.2841 = n*x$ = total periods

$$79.2841 = 12 * x$$

$\frac{79.2841}{12} = x$ = years

$6.607 = x$ = years

Answer: If you have $200,000 to invest today at 8.5% compounded monthly and you need $350,000 to buy the machine, you would have to wait 6.607 years.

Example 9:



If you want to buy a \$350,000 C & C Router to improve manufacturing efficiency and you can invest \$ 250,000 today for the next five years (compounding 2 times a year), what annual interest rate (APR) do you need to find so that you can afford the machine?

$PV = 250{,}000$

$n = 2$

$x = 5$

$FV = 350{,}000$

$\frac{i}{n}$ = period rate

$i$ { Annual Interest Rate, Annual Discount Rate, Rate of Return Rate, Internal Rate of Return } all synonyms

**Math**

$$FV = PV * \left(1 + \frac{i}{n}\right)^{nx}$$

$$350{,}000 = 250{,}000 * \left(1 + \frac{i}{2}\right)^{2*5}$$

$$\frac{35}{25} = \left(1 + \frac{i}{2}\right)^{10}$$

$$(1.4)^{\wedge\frac{1}{10}} = 1 + \frac{i}{2}$$

$$1.034219694 = 1 + \frac{i}{2}$$

$$1.034219694 - 1 = \frac{i}{2}$$

$$.034219694 = \frac{i}{2}$$

$$.034219694 * 2 = i$$

$$.068439388 = i$$

**Answer:** If you want to buy the \$350,000 machine & you have \$250,000 to invest today for the next 5 years (compounded semi annually), you would need an APR of 6.84%.

**Excel** Solve for Rate

= Rate(nper,,-PV, FV) = {Period Rate} = $\frac{i}{n}$

= Rate(n * x ,,-PV, FV)

= Rate (10 ,, 250,000, 350,000)

= .034219694 = $\frac{i}{n}$ = half year rate

$i$ = .034219694 * 2 = .068439388

Annual Rate = .068439388

# Time Value of Money

The formula for the "Time Value of Money" (TVM) forms the basis for many of the most common financial transactions we take for granted each day. TVM explains why a dollar today is worth more than a dollar in the future. TVM is why most of us don't keep all of our money under our mattress. We understand that inflation will slowly erode the value of the savings over time and it's a better idea to invest the money in some vehicle with the potential to generate interest like a stock or savings account.

While the evidence is not definitive, financial archeologists believe that Leonardo of Pisa, commonly known as "Fibonacci", was the first person to publish the formulas that have evolved into what we now know as TVM. It should be noted that much Fibonacci's work was built on top of prior work by Arabic and Indian mathematicians.

Fibonacci published his work in a book called *Liber Abaci* in 1202. Given the period in which he lived, its no surprise that Fibonacci's work focused on everyday mercantile issues like the TVM, interest rates and foreign exchange rates. Fibonacci published his work at a time when Italy was a collection of independent city states, each with their own currency trading with other merchants from Northern Africa, China, the Middle East and other parts of the Mediterranean.

*Liber Abaci* also included groundbreaking research into another area of mathematics that should be familiar to today's technical trader. The Fibonacci Series is a sequence of numbers where the each number is the sum of the prior two numbers in the series (1, 1, 2, 3, 5, 8...). While the series itself may not seem that exciting, the quotient of sequential numbers is always 1.618 or the inverse 0.618 which is more commonly referred to as the "Golden Mean". The Golden Mean is found throughout nature and human endeavors. In nature the number of female bees in a hive divided by male bees is always close to 1.618 and the Golden Mean can also be found in the spirals of a nautilus shell. The Golden Mean is also prevalent in modern and ancient architecture. Perhaps the most famous example is the Parthenon in Greece whose proportions adhere to the Golden Mean.

The Fibonacci Series also has a place in modern trading. Many investor use Fibonacci numbers in technical analysis to generate lines of support and resistance in stocks. According to Investopedia, the most popular Fibonacci studies are arcs, fans, retracements and time zones.

http://post.polls.yahoo.com/quiz/financeresults.php

Rule of 72 for estimating the Rate you will need to have your investment double.

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Example 10:

Rule of 72: $\frac{i}{n} \approx \frac{72}{n*X}$

if you want to double your investment in 5 years where the interest is compounded monthly.

$$\frac{i}{n} \approx \frac{72}{n*X} = \frac{72}{12*5} = \frac{72}{60} = 1.2$$

$$i \approx 1.2 * 12 = 14.4$$

But this is not .144

So to get final estimate:

$$\frac{14.4}{100} = .144 \approx i$$