Discrete Probability Distributions

1. **Discrete**
   a. Gaps between numbers
   b. Counting
      Example: 1, 2, 3, or 1, 2, 3... or 1.1, 1.2...

2. **Continuous**
   a. Can assume any numerical value in an interval
   b. Depends on how small you want to go. Depends on measurement instrument.
      Example: \[ \mathbb{R} \] Lots of possible numbers

3. **Random Variable**
   A random variable is a numerical description of the outcome of an experiment.
   or
   A quantity resulting from an experiment that by chance, can assume different values.

   a. **Discrete Random Variable** "GAPs"
      Example: 1, 2, 3 or 1, 2, 3... or 1.1, 1.2...

   b. **Continuous Random Variable** "No GAPs"
      When timing, it depends on measurement instrument.
      1, 1.1, 1.01, or 1.001...
Discrete Random Variable "Gaps"

May assume either a finite number of values \((1, 2, 3)\) or an infinite sequence of values \((1, 2, 3, \ldots)\).

or

A variable that can only assume certain clearly separated values.

Examples:
- Roll die \(1, 2, 3, 4, 5, 6\)
- Scores for Dancer \(0, 1, 2, \ldots, 9.8, 9.9, 10\)
- Product Defective \(0 = \text{False} \ 1 = \text{True}\)
- Inspect to see if 100 Booms fly \(0, 1, 2, \ldots, 99, 100\)

Continuous Random Variable "No Gaps"

May assume any numerical value in an interval or collection of intervals.

Examples:
- Time to take timed Test \((\text{any time between } 0 \ \& \ 10 \ \text{min})\)
- Weight of cereal box
- Air pressure in air compressor
- Money (even though it seems discrete)
**Probability Distribution**

A listing of all the experimental outcomes of an experiment and the probabilities associated with each outcome.

Or

A description of how the probabilities are distributed over the values of the random variable.

Description/listing could be:

1. Table
2. Chart
3. Equation

**Discrete Probability Function**

A function, $f(x)$, that provides the probability that $X$ assumes a particular value for a discrete random variable. Examples coming...

**Required Conditions for a Discrete Probability Function**

\[ P(X) = f(x) \geq 0 \quad \text{any probability} \geq 0 \]

\[ \sum P(x) = \sum f(x) = 1 \quad \text{All of probability sum to 1} \]

*outcomes are mutually exclusive & collectively exhaustive.*
Probability Distribution

1. **Table**

<table>
<thead>
<tr>
<th>X # of Heads</th>
<th>P(X)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.125</td>
</tr>
<tr>
<td>1</td>
<td>0.375</td>
</tr>
<tr>
<td>2</td>
<td>0.375</td>
</tr>
<tr>
<td>3</td>
<td>0.125</td>
</tr>
</tbody>
</table>

   **Total**: 1

2. **Chart**

3. **Equation** *(later)*

   \[ F(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} \]

What is convenient about probability distributions is that you can then figure out any probability you want:

\[ P(x \geq 2) = P(2) + P(3) = 0.375 + 0.125 = 0.5 \]
How to Make a Discrete Probability Distr.

Consider random experiment: A coin tossed 3 times

- $X =$ random discrete variable = # of Heads
- $H =$ Heads
- $T =$ Tails

Sample space (list of all possible outcomes) = $SS$

- # of Trials = Toss = 3 times = 3
- Outcomes = 2

$2 \times 2 \times 2 = 8 =$ count of all outcomes = $SS$

### Table

<table>
<thead>
<tr>
<th>Count Outcomes</th>
<th>Coin Toss 1st</th>
<th>Coin Toss 2nd</th>
<th>Coin Toss 3rd</th>
<th># of Heads</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>H</td>
<td>H</td>
<td>T</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>H</td>
<td>T</td>
<td>H</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>H</td>
<td>T</td>
<td>H</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>T</td>
<td>H</td>
<td>H</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>T</td>
<td>H</td>
<td>T</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>T</td>
<td>T</td>
<td>H</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>0</td>
</tr>
</tbody>
</table>

Possible values of $x$:

- $0, 1, 2, 3$

Discrete Probability Distribution

<table>
<thead>
<tr>
<th># of Heads</th>
<th>$P(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\frac{1}{8} = 0.125$</td>
</tr>
<tr>
<td>1</td>
<td>$\frac{3}{8} = 0.375$</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{3}{8} = 0.375$</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{1}{8} = 0.125$</td>
</tr>
</tbody>
</table>

$E = 1.000$

$P(0 \text{ Heads in } 3 \text{ tosses}) = 0.125$

$P(1 \text{ Head in } 3 \text{ tosses}) = 0.375$

$P(2 \text{ Heads in } 3 \text{ tosses}) = 0.375$

$P(3 \text{ Heads in } 3 \text{ tosses}) = 0.125$

$P(x)$ graph

![Graph of $P(x)$]
Steps for creating Probability Distributions

1. Define Random Variable
2. Build Frequency Distribution
3. Calculate Relative Frequency \( f(x) = \frac{fx}{\sum fx} \)
4. Check Requirement 1 \( f(x) \geq 0 \)
5. Check Requirement 2 \( \sum f(x) = 1 \)
6. Create column chart to visually portray Distribution (Discrete = Columns Do Not Touch)
7. Make predictions
Frequency Tables & Relative Frequency (Chapter 2) give us Probabilities based on the Relative Frequency method.

Example: over last 20 days the number of operating rooms used at TG Hospital were:
- on 3 days only 1 was used
- on 5 days 2 were used
- on 8 days 3 were used
- on 4 days all (4) were used

Let: Discrete Random variable = \( X \) = # of operating rooms used for 1 day

<table>
<thead>
<tr>
<th>X</th>
<th>Frequency</th>
<th>( P(X) )</th>
<th>( f(x) ) &gt; 0?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>( \frac{3}{20} = .15 )</td>
<td>( \checkmark )</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>( \frac{5}{20} = .25 )</td>
<td>( \checkmark )</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>( \frac{8}{20} = .40 )</td>
<td>( \checkmark )</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>( \frac{4}{20} = .20 )</td>
<td>( \checkmark )</td>
</tr>
<tr>
<td>( \Sigma = 20 )</td>
<td>( \Sigma = 1 )</td>
<td>( \Sigma f(x) = 1 )</td>
<td>( \checkmark )</td>
</tr>
</tbody>
</table>

\[ P(3 \text{ rooms used in 2 day}) = f(3) = .4 \]
\[ P(X > 2) = f(2) + f(3) + f(4) = .85 \]
\[ P(X = 1) = f(1) = .15 \]
\[ P(X = 0) = 0 \]
Discrete Uniform Probability Function

\[ f(x) = \frac{1}{n} \]

\( n = \# \) of values the random variable can assume

Example:

Roll Die

\( S = \{1, 2, 3, 4, 5, 6\} \)

Probability = \( f(x) = \frac{1}{6} \)

\( X = 1, 2, 3, 4, 5, 6 = \text{top face} \)

<table>
<thead>
<tr>
<th>Outcome</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ( \frac{1}{6} ) = 0.167</td>
<td></td>
</tr>
<tr>
<td>2 ( \frac{1}{6} ) = 0.167</td>
<td></td>
</tr>
<tr>
<td>3 ( \frac{1}{6} ) = 0.167</td>
<td></td>
</tr>
<tr>
<td>4 ( \frac{1}{6} ) = 0.167</td>
<td></td>
</tr>
<tr>
<td>5 ( \frac{1}{6} ) = 0.167</td>
<td></td>
</tr>
<tr>
<td>6 ( \frac{1}{6} ) = 0.167</td>
<td></td>
</tr>
<tr>
<td>( \Sigma = 1 )</td>
<td></td>
</tr>
</tbody>
</table>

chart

\( f(x) \)

\( x \)

Probability

\( \# \) of Face on Die
Mean & Standard deviation for Discrete Random Variable or Discrete Probability Distribution

\[ E(x) = \mu = \sum x f(x) \]

\[ \sigma = \sqrt{\sum (x-\mu)^2 f(x)} \]

\[ M = \text{mean} \]
\[ \sigma = \text{standard Deviation} \]
\[ x = \text{random variable} \]
\[ f(x) = p(x) = \text{probability of taking on a particular value of } x \]
\[ \sum = \text{add} \]

Greek Letters are used

Example:

<table>
<thead>
<tr>
<th>Rooms used</th>
<th>f(x) = p(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.15</td>
</tr>
<tr>
<td>2</td>
<td>.25</td>
</tr>
<tr>
<td>3</td>
<td>.40</td>
</tr>
<tr>
<td>4</td>
<td>.20</td>
</tr>
</tbody>
</table>

\[ p(2) = .25 \]
\[ p(x > 2) = .6 \]

But what is a typical number of rooms used?
What is variation?

\[ E(x) = \mu = \text{Mean} = 1 \times .15 + 2 \times .25 + 3 \times .40 + 4 \times .2 \]
\[ = 2.65 \]

On any typical day, 2.65 rooms will be used. We want to get that exactly, but the number can be used for planning.
<table>
<thead>
<tr>
<th>$X$</th>
<th>$X - \mu$</th>
<th>$(X - \mu)^2$</th>
<th>$(X - \mu)^2 \cdot f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 - 2.65 = -1.65</td>
<td>2.7225</td>
<td>0.408345</td>
</tr>
<tr>
<td>2</td>
<td>2 - 2.65 = -0.65</td>
<td>0.4225</td>
<td>0.0105625</td>
</tr>
<tr>
<td>3</td>
<td>3 - 2.65 = 0.35</td>
<td>0.1225</td>
<td>0.0049</td>
</tr>
<tr>
<td>4</td>
<td>4 - 2.65 = 1.35</td>
<td>1.8225</td>
<td>0.3645</td>
</tr>
</tbody>
</table>

$\Sigma = -0.6$  

$\Sigma$ must equal to zero because we did not use all Raw Data.

$\Sigma = 19.275$  

$\sigma = \sqrt{19.275} = 0.96306801$

Used to compare our typical value 2.65 to other hospitals with 2.65 or close to 2.65 to tell which mean more fairly represents its data points.
Back to chapter 4:

**Probability Rules for Adding & Multiplying**

**Example:**

An insurance agent has appointments with 4 prospective clients tomorrow. From the past she knows that the probability of making a sale on any one appointment is 1 in 5 (\( \frac{1}{5} = 0.2 \)), what is likelihood that she will sell 3 policies in 4 tries?

Event = sell 3 policies in 4 tries

# of outcomes in 4 sales attempt = 2

sale = S (Assume 4 attempts are independent)

no sale = NS

Probability of success = \( P_S = \frac{1}{5} \) (same each time)

Probability of not success = \( P_{NS} = \frac{4}{5} = 0.8 \)

# of Trials (steps) = \( n = 4 \)

\( X \) = Discrete Random Variable = # sales = # successes = 3

1st: use multiplication rule (events are independent)

\[
P(NS, S, S, S) = P(NS) \times P(S) \times P(S) \times P(S) = 0.0064
\]

2nd use adding rule

Prob of Event

\[
P(\text{Event} = 3 \text{ sales in } 4 \text{ attempts}) = 0.0256
\]

Let's create whole distribution
**Build probability Distribution (Discrete)**

<table>
<thead>
<tr>
<th>Possible outcomes sample points</th>
<th>Attempted sales</th>
<th># of sales</th>
<th>Probability of occurrence</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>S S S S S</td>
<td>4</td>
<td>0.0016</td>
</tr>
<tr>
<td>2</td>
<td>S S S S NS</td>
<td>3</td>
<td>0.0064</td>
</tr>
<tr>
<td>3</td>
<td>S S NS S S</td>
<td>3</td>
<td>0.0064</td>
</tr>
<tr>
<td>4</td>
<td>S NS S S</td>
<td>3</td>
<td>0.0064</td>
</tr>
<tr>
<td>5</td>
<td>NS S S S</td>
<td>3</td>
<td>0.0064</td>
</tr>
<tr>
<td>6</td>
<td>S S NS NS</td>
<td>2</td>
<td>0.0125</td>
</tr>
<tr>
<td>7</td>
<td>S NS NS S</td>
<td>2</td>
<td>0.0125</td>
</tr>
<tr>
<td>8</td>
<td>NS NS S S</td>
<td>2</td>
<td>0.0125</td>
</tr>
<tr>
<td>9</td>
<td>S NS S NS</td>
<td>2</td>
<td>0.0125</td>
</tr>
<tr>
<td>10</td>
<td>NS S S NS</td>
<td>2</td>
<td>0.0125</td>
</tr>
<tr>
<td>11</td>
<td>NS NS S NS</td>
<td>1</td>
<td>0.0125</td>
</tr>
<tr>
<td>12</td>
<td>S NS NS NS</td>
<td>1</td>
<td>0.0125</td>
</tr>
<tr>
<td>13</td>
<td>NS S NS NS</td>
<td>1</td>
<td>0.0125</td>
</tr>
<tr>
<td>14</td>
<td>NS NS S NS</td>
<td>1</td>
<td>0.0125</td>
</tr>
<tr>
<td>15</td>
<td>NS NS NS S</td>
<td>0</td>
<td>0.04096</td>
</tr>
<tr>
<td>16</td>
<td>NS NS NS NS</td>
<td>0</td>
<td>0.4096</td>
</tr>
</tbody>
</table>

Σ = 1

<table>
<thead>
<tr>
<th># of sales</th>
<th>( P(x) )</th>
<th>( P(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( P(0) = 0.4096 \times 1 = 0.4096 )</td>
<td>0.4096</td>
</tr>
<tr>
<td>1</td>
<td>( P(1) = 0.1024 \times 4 = 0.4096 )</td>
<td>0.4096</td>
</tr>
<tr>
<td>2</td>
<td>( P(2) = 0.0256 \times 6 = 0.1536 )</td>
<td>0.1536</td>
</tr>
<tr>
<td>3</td>
<td>( P(3) = 0.0064 \times 4 = 0.0256 )</td>
<td>0.0256</td>
</tr>
<tr>
<td>4</td>
<td>( P(4) = 0.0016 \times 1 = 0.0016 )</td>
<td>0.0016</td>
</tr>
</tbody>
</table>

Σ = 1

---

**Discrete Probability Distributions with \( n = 4 \), \( p_i = 0.2 \)**

But there must be an easier way!!
Binomial Probability Distribution (Discrete P.D.)

List of all outcomes for a Binomial Experiment (common multi-step experiment that has many useful applications) and the probabilities associated with each experimental outcome (sample point).

Requirements for Binomial Experiment

1. The experiment consists of a sequence of \( n \) identical Trials. (Random variable counts the \# of successes in a Fixed \# of Trials. Fixed \# of Trials = \( n \)).

2. 2 outcomes are possible on each Trial, one is defined as a "success" and the other is "Not success" or "Failure." S or F.

3. Probability of success denoted as \( p \) or the greek letter \( \pi \) ("pi"), remains the same on each Trial. \( 1 - p \) does not change.

4. The Trials are independent (one does not affect next).

Examples:

1. Make 4 sales calls. \( n = 4 \), \( S = \text{sale} \), \( p = .2 \), \( \text{yes} \)
2. Test with 15 T/F questions. \( n = 15 \), \( S = \text{Correct} \), \( p = .5 \), \( \text{yes} \)
3. Flip coin 3 times. \( n = 3 \), \( S = \text{H} \), \( p = .5 \), \( \text{yes} \)
4. Drive across Bridge 7 times during Rush Hour Traffic. \( n = 7 \), \( S = \text{Traffic} \), \( p = .15 \), \( \text{yes} \)
5. Air Flight from Oak. to Seattle. \( n = 6 \), \( S = \text{late} \), \( p = .1 \), \( \text{yes} \)

6 Flights per day,
For our sales agent problem we have 16 total possible sample points but we needed only 4 of them:

<table>
<thead>
<tr>
<th>2</th>
<th>5, 5, 5, 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>5, 5, 5, 5</td>
</tr>
<tr>
<td>4</td>
<td>5, 5, 5, 5</td>
</tr>
<tr>
<td>5</td>
<td>5, 5, 5, 5</td>
</tr>
</tbody>
</table>

4 total

How can we calculate this $4$?

"Sample points"

**Number of Experimental outcomes that provide exactly $X$ successes in $n$ Trials**

\[
\left\{ \text{# experimental outcomes that have } X \text{ successes in } n \text{ Trials} \right\} = \frac{n!}{X!(n-X)!}
\]

\[
X = \text{# successes of Random Discrete Variable}
\]

*we use $X$ instead of $n$, because $X = \text{successes in } n \text{ trials}*

\[
n = \# \text{Fixed Trials}
\]

*we use $n$ instead of $N$ because $n = \# \text{ of Fixed Trials}*

\[
n = 4 \quad X = 3
\]

\[
\left\{ \text{# of experimental outcomes that have } X \text{ successes in } n \text{ trials} \right\} = \frac{4!}{3!(4-3)!} = \frac{4!}{1!*3!} = \frac{4}{1} = 4
\]

*earlier formula: $\frac{N!}{n!(N-n)!}$

$N = \text{count of all objects = pop size}$

$n = \text{size of subset = sample size}$
Binomial Probability Function

\[ P(x) = f(x) = \frac{n!}{x!(n-x)!} \cdot p^x \cdot (1-p)^{(n-x)} \]

\[ P(x) = f(x) = \text{Probability of } x \text{ successes in } n \text{ Trials} \]
\[ n = \# \text{ of } \text{Fixed Trials} \]
\[ p = \text{Probability of success on any 1 Trial} \]
\[ 1-p = \text{Probability of failure on any 1 Trial} \]

Excel Binomial Probability Function

\[ = \text{BINOMDIST}(x, n, p, 0) \]
\[ x = \text{Discrete Random Variable count } \# \text{ successes} \]
\[ n = \# \text{ of Fixed Trials} \]
\[ p = \text{Probability of success} \]
\[ 0 \text{ for exactly } x \]
\[ 1 \text{ for less than or equal to } x \]

Example:

For our sales agent problem, what is probability of making exactly 3 sales in 4 attempts, \( p = 0.2 \)?

\[ n = 4 \]
\[ x = 3 \]
\[ p = 0.2 \]

\[ f(3) = P(3) = \frac{4!}{3!(4-3)!} \cdot 0.2^3 \cdot 0.8^{(4-3)} = \]
\[ = 4 \cdot 0.008 \cdot 0.8 = 0.0256 \]

Binomial?

1. Fixed # Trials? yes.
2. 2 outcomes on each Trial? yes.
3. Probability same each Trial? yes.
Example:

\[
\text{Find } P(X \leq 2) = F(x \leq 2)
\]

\[
\begin{align*}
&= P(0 \text{ or } 1 \text{ or } 2) = F(0) + F(1) + F(2) = \\
&= \frac{4!}{0!(4-0)!} \cdot .2^0 \cdot .8^4 + \frac{4!}{1!(4-1)!} \cdot .2^1 \cdot .8^{4-1} + \frac{4!}{2!(4-2)!} \cdot .2^2 \cdot .8^{4-2} \\
&= .4096 + .4096 + .1536 \\
&= .9728 = P(X \leq 2) = F(X \leq 2)
\end{align*}
\]

\[
= \text{BINOMDIST}(2, 4, .2, 1) = .9728
\]

Example:

\[
\text{Find } F(X \geq 3) = P(X \geq 3)
\]

\[
\begin{align*}
&= P(X = 3 \text{ or } X = 4) = P(3) + P(4) = \\
&= \frac{4!}{3!(4-3)!} \cdot .2^3 \cdot .8^{4-3} + \frac{4!}{4!(4-4)!} \cdot .2^4 \cdot .8^{4-4} \\
&= .0256 + .0016 \\
&= .0272 = P(X \geq 3) = F(X \geq 3)
\end{align*}
\]

\[
= 1 - \text{BINOMDIST}(3-1, 4, .2, 1) = .0272
\]

\[
\text{Excel Function always does cumulative from Low End up And All area = 1}
\]
Expected Value & Standard Deviation for the Binomial Distribution

\[ E(x) = \mu = \text{Mean} = n \times p \]
\[ \sigma = \text{Standard Deviation} = \sqrt{n \times p \times (1-p)} \]

\( n \) = # of Fixed Trials
\( p \) = probability of success

Example:

For our sales agent problem, what is the mean number of sales she will make in 4 attempts, and what is the standard deviation?

\( n = 4 \)
\( p = 0.2 \)

\[ E(x) = \text{mean} = 4 \times 0.2 = 0.8 \]

"For every 4 calls she can expect to sell 0.8 policies. If she has 40 calls planned, she can expect to sell

\[ \frac{40}{4} \times 0.8 = 8 \text{ policies} \]

Standard Deviation = \( \sqrt{0.8 \times (1-0.2)} = 0.8 \)

measures dispersion & could be used to compare to other sales people."
Binomial Experiment Examples:

1. A flight from Oakland to Seattle occurs 6 times per day. The probability that any one flight is late is 0.1. What is the probability that exactly 2 planes are late? What is the probability that less than 2 planes are late? Is this a binomial experiment? Mean? SD?

Binomial Experiment?

1. Fixed # of trials (each count S/F)?
   - Yes ✓ n = 5
2. Each Trial Independent? Yes ✓ (more or less)
3. S/F each time? Yes late or Not late
4. Probability of success same each trial? Yes \( p = 0.1 \)

Variables

\( \pi = 0.1 = \text{success} = \text{late} \)
\( 1-\pi = 1-0.1 = 0.9 = \text{Not late} \)
\( n = 6 = \text{Fixed # of trials} \)

\[ P(\text{2 out of 6}) = \frac{6!}{(6-2)!2!} \times (0.1)^2 \times (0.9)^4 \]
\[ P(2) = \frac{6 \times 0.1 \times 0.9^4}{2} = 0.098415 \]
Probability of exactly 2 flights late is 0.098415

Excel:
\[ = \text{BINOMDIST}(x,n,\pi,0) \]
\[ = \text{BINOMDIST}(2,6,0.1,1) \]

\[ P(x < 2) = P(0) + P(1) \]
\[ P(x < 2) = 0.3543 + 0.3543 = 0.7086 \]
\[ P(x < 2) = 0.8857 \]
The probability that less than 2 flights will be late is 0.8857

Excel:
\[ = \text{BINOMDIST}(x,n,\pi,1) \]
\[ = \text{BINOMDIST}(2,6,0.1,1) \]

\[ M = np \]
\[ M = 6 \times 0.1 \]
\[ M = 0.6 \]
The mean amount late per day is 0.6 flights

\[ \sigma = \sqrt{np(1-\pi)} \]
\[ \sigma = \sqrt{6 \times 0.1 \times 0.9} \]
\[ \sigma = \sqrt{0.54} = 0.7348 \]
The Standard Deviation is 0.7348
Binomial Experiment Example 2:

The probability of sitting in traffic on the West Seattle Bridge during rush hour is 0.15. During your next 7 rush hour bridge crossings, what is the probability that you will sit in traffic 3 times? 5 or more times? Mean? SD?

Binomial?

Fixed # of trials? yes \( n = 7 \)
Independent? yes

S/F? Stuck in traffic/Not stuck in traffic

\( \tau \) same each time? yes \( \tau = 0.15 \)

\[ p(3) = \binom{7}{3} \cdot 0.15^3 \cdot (1-0.15)^{7-3} = \frac{7!}{3! \cdot 4!} \cdot 0.15^3 \cdot 0.85^4 = \frac{35 \cdot 0.03375 \cdot 0.5220625}{3.2} = 0.1662 \]

\[ p(x \geq 5) = p(4) + p(5) + p(6) + p(7) = 0.001522 + 0.000678 + 0.000017 = 0.002217 \]

\[ \mu = 0.15 \cdot 7 = 1.05 \]

\[ \sigma = \sqrt{0.15 \cdot 7 \cdot (1-0.15)} = 0.944727181 \]
Binomial Probability Distribution

\[ n = 7 \]
\[ p = 0.15 \]

# of successes
# of stuck in traffic

<table>
<thead>
<tr>
<th>X</th>
<th>P(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.3206</td>
</tr>
<tr>
<td>1</td>
<td>0.3960</td>
</tr>
<tr>
<td>2</td>
<td>0.2097</td>
</tr>
<tr>
<td>3</td>
<td>0.0617</td>
</tr>
<tr>
<td>4</td>
<td>0.0109</td>
</tr>
<tr>
<td>5</td>
<td>0.0012</td>
</tr>
<tr>
<td>6</td>
<td>0.0001</td>
</tr>
<tr>
<td>7</td>
<td>0.0000</td>
</tr>
</tbody>
</table>
For Binomial Distribution:

1. As $p(\pi)$ approaches 0.5, the distribution becomes symmetrical.

2. As $n$ gets larger, the distribution becomes symmetrical.
Problem 2

An insurance representative has appointments with four prospective clients tomorrow. From past experience she knows that the probability of making a sale on any appointment is 1 in 5 or 0.20. Use the rules of probability to determine the likelihood that she will sell a policy to 3 of the 4 prospective clients.

\[ \pi = 0.2 \]

Let \( NS = \text{No sale} \)  \( P_{NS} = 1 - 0.2 = 0.8 \)

\( S = \text{Sale} \)  \( P_S = 0.2 \)

<table>
<thead>
<tr>
<th>Location of NS</th>
<th>Order of Occurrence</th>
<th>Probability of Occurrence</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>NS, S, S, S</td>
<td>((0.8)(0.2)(0.2)(0.2) = 0.0064)</td>
</tr>
<tr>
<td>2</td>
<td>S, NS, S, S</td>
<td>((0.2)(0.8)(0.2)(0.2) = 0.0064)</td>
</tr>
<tr>
<td>3</td>
<td>S, S, NS, S</td>
<td>((0.2)(0.2)(0.8)(0.2) = 0.0064)</td>
</tr>
<tr>
<td>4</td>
<td>S, S, S, NS</td>
<td>((0.2)(0.2)(0.2)(0.2) = 0.0064)</td>
</tr>
</tbody>
</table>

\[ \sum = 0.0256 \]

\[ P(\text{of exactly 3 sales in 4 tries}) = 0.0256 \]

Problem 3

Now let's use formula [6-3] for the binomial distribution to compute the probability that the sales representative in Problem 2 will sell a policy to exactly 3 out of the 4 prospective clients.

\[ P(x) = \frac{n!}{x!(n-x)!} \pi^x (1-\pi)^{n-x} \]

\[ P(3) = \frac{4!}{3!(4-3)!} (0.2)^3 (1-0.2)^{4-3} = 4 (0.008)(0.8) = 0.0256 \]

Table \[ P(3) = 0.0256 \]
Problem 4

In Problems 2 and 3 the probability of 3 sales resulting from 4 appointments was computed using both the rules of addition and multiplication and the binomial formula. A more convenient way of arriving at the probabilities for 0, 1, 2, 3, or 4 sales out of 4 appointments is to refer to a binomial table. We use the binomial table to determine the probabilities for all possible outcomes.

<table>
<thead>
<tr>
<th>Number of Successes (x)</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.410</td>
</tr>
<tr>
<td>1</td>
<td>0.410</td>
</tr>
<tr>
<td>2</td>
<td>0.154</td>
</tr>
<tr>
<td>3</td>
<td>0.026</td>
</tr>
<tr>
<td>4</td>
<td>0.002</td>
</tr>
</tbody>
</table>

*Slight discrepancy due to rounding*
Problem 5

Use the information regarding the insurance representative, where \( n = 4 \) and \( \pi = 0.20 \), to compute the probability that the representative sells more than two policies. Also determine the mean and variance of the number of policyholders.

\[
P(x > 2) = 0.0272
\]

\[
\mu = n \pi = 0.8
\]

\[
\sigma = \sqrt{n \pi (1-\pi)} = \sqrt{0.8 \times 0.8} = 0.8
\]

Exercise 6.3

Check your answers against those in the ANSWER section.

Labor negotiators estimate that 30 percent of all major contract negotiations result in a strike. During the next year, 12 major contracts must be negotiated. Determine the following probabilities using Appendix A:

a. no major strikes  
   b. at least 5  
   c. between 2 and 4 (that is 2, 3, or 4).

\[
\pi \text{ of strike } = 0.3
\]

\[
1 - \pi = 0.7
\]

\[
n = 12
\]

\[
P(0) = 0.0048
\]

\[
P(x \geq 5) = 0.2763
\]

\[
P(2 \leq x \leq 4) = 0.6386
\]
Exercise 6.2

Check your answers against those in the ANSWER section.

It is known that 60 percent of all registered voters in the 42nd Congressional District are Republicans. Three registered voters are selected at random from the district. Compute the probability that exactly 2 of the 3 selected are Republicans, using:

a. The rules of probability
b. The binomial formula.

c. table

\[ n = 3 \]
\[ \pi = 0.6 \]
\[ X = 2 \]
\[ 1 - \pi = 0.4 \]

\[ \begin{align*}
\text{Binomial?} & \quad \text{Fixed trials?} \quad \text{Yes} \quad n = 3 \\
\text{Independent?} & \quad \text{Yes} \\
\text{S or F?} & \quad \text{R or Not} \quad \text{Yes} \\
\text{Try same each time?} & \quad \text{Yes} \quad \pi = 0.6 \\
\end{align*} \]

<table>
<thead>
<tr>
<th>Location</th>
<th>Occurrence</th>
<th>Probability of Occurrence</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>R, R, NR</td>
<td>((0.6)(0.6)(0.4) = 0.144)</td>
</tr>
<tr>
<td>1</td>
<td>NR, R, R</td>
<td>((0.4)(0.6)(0.6) = 0.144)</td>
</tr>
<tr>
<td>2</td>
<td>R, NR, R</td>
<td>((0.4)(0.4)(0.6) = 0.144)</td>
</tr>
</tbody>
</table>

\[ \Sigma = 0.432 \]

\[ P(\text{exactly 2 of 3 selected are Republicans}) = 0.432 \]

b) \[ P(X) = \frac{n!}{x!(n-x)!} \pi^x (1-\pi)^{n-x} \]

\[ P(2) = \frac{3!}{2!(3-2)!} (0.6)^2 (1-0.6)^{3-2} = 3 \cdot (0.36)(0.4) = 0.432 \]

c) \[ P(2) = 0.432 \]