

# Discrete Probability Distributions.

P.11

## ① Discrete

(only certain clearly separated values)

- ⓐ Gaps between numbers
- ⓑ Counting

Example: 1, 2, 3. or 1, 2, 3... or 1.1, 1.2...

## ② Continuous

(Infinite possibilities)

- ⓐ Can assume any numerical value in an interval
- ⓑ Depends on how small you want to go. Depends on measurement instrument.

Example: 1  $\dots$  2  Lots of possible numbers

## ③ Random Variable

A random variable is a numerical description of the outcome of an experiment.

or

A quantity resulting from an experiment that, by chance, can assume different values.

### ⓐ Discrete Random Variable "GAPS"

Counting!

Example 1, 2, 3 or 1, 2, 3... or 1.1, 1.2...

### ⓑ Continuous Random Variable "No Gaps"

measuring!

when timing, it depends on measurement instrument

1 or 1.1 or 1.01 or 1.0001

#### ④ Discrete Random Variable "Gaps" P2

Book →

May assume either a finite number of values (1, 2, 3) or an infinite sequence of values (1, 2, 3...).

or

A variable that can only assume certain clearly separated values.

- Examples:
- Roll die . 1, 2, 3, 4, 5, 6
  - Scores for Dancer 0, .1, .2, ..., 9.8, 9.9, 10
  - Product Defective 0 = False 1 = True
  - Inspect to see if 100 Booms fly 0, 1, 2, ..., 99, 100

#### ⑤ Continuous Random Variable "No Gaps"

Book →

May assume any numerical value in an interval or collection of intervals

Examples:

- Time to take timed Test (any time between 0 & 10 min)
- weight of cereal box
- air pressure in air compressor
- money (even though it seems discrete)

## ⑥ Probability Distribution

All outcomes & Prob.  
Random Variable & Prob.

A listing of all the experimental outcomes of an experiment and the probabilities associated with each outcome.  
or

A description of how the probabilities are distributed over the values of the random variable

Description / Listing could be:

- ① Table
  - ② Chart
  - ③ equation
- } see next page →

## ⑦ Discrete Probability Function

A function,  $f(x)$ , that provides the probability that  $X$  assumes a particular value for a discrete random variable. Examples coming...

## ⑧ Required Conditions for a Discrete Probability Distribution

$$P(X) = f(x) \geq 0 \quad \text{any probability} \geq 0$$

$$\sum P(x) = \sum f(x) = 1 \quad \text{All of probability sum to 1}$$

\* outcomes are mutually exclusive & collectively exhaustive.

# Probability Distribution

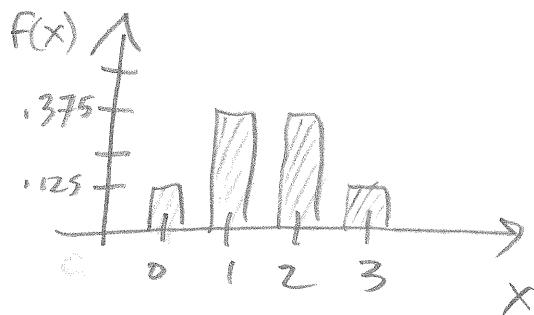
P. 4

① table

Experiment:  
Toss coin  
3 times

X # of Heads	p(x)
0	.125
1	.375
2	.375
3	.125
total	1

② chart



③ equation

(later)  
→

$$f(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{(n-x)}$$

What is convenient about Probability Distributions is that you can then figure out any probability you want:

$$P(X \geq 2) = P(2) + P(3) = .375 + .125 = .5$$

# ① How to Make a Discrete Probability Distr. (25)

Consider random experiment: A coin tossed 3 times

$X$  = random discrete variable = # of Heads

H = Heads

T = Tails

Sample space (list of all possible outcomes) = 55  
 $\{ \text{# of Trials} \} = \text{TOSS} = 3 \text{ times} = 3$   
 Outcomes = 2

2 \* 2 \* 2 = 8 = Count of all outcomes = 55

Count outcomes	Coin toss 5			# of Heads
	1st	2nd	3rd	
1	H	H	H	3
2	H	H	T	2
3	H	T	H	2
4	T	H	H	2
5	H	T	T	1
6	T	H	T	1
7	T	T	H	1
8	T	T	T	0

Possible values of  $x \Rightarrow 0, 1, 2, 3$

Discrete Probability Distribution	
# of Heads	$P(x)$
0	$.1/8 = .125$
1	$3/8 = .375$
2	$3/8 = .375$
3	$.1/8 = .125$
	$\Sigma = 1.00$

$P(0 \text{ Heads in 3 toss}) = .125$

$P(1 \text{ Head in 3 toss}) = .375$

$P(2 \text{ Heads in 3 toss}) = .375$

$P(3 \text{ Heads in 3 toss}) = .125$



2nd method

# of Heads	$P(x)$
0	$.5 * .5 * .5 * 1 = .125$
1	$.5 * .5 * .5 * 3 = .375$
2	$.5 * .5 * .5 * 3 = .375$
3	$.5 * .5 * .5 * 1 = .125$

# 9.6 steps for creating Probability Distributions

- ① Define Random Variable
- ② Build Frequency Distribution
- ③ Calculate Relative Frequency  $P(x) = f(x)$
- ④ Check Requirement 1  $f(x) \geq 0$
- ⑤ Check Requirement 2  $\sum f(x) = 1$
- ⑥ Create column chart to visually portray Distribution  
(Discrete = columns do not touch)
- ⑦ Make predictions

(10)

# Frequency Tables & Relative Frequency (Chapter 2) give us Probabilities based on the Relative Frequency Method

Example: over last 20 days the number of operating rooms used at TG Hospital were:

- on 3 days only 1 was used
- on 5 days 2 were used
- on 8 days 3 were used
- on 4 days all (4) were used

Let: Discrete Random variable =  $X$   
 $= \# \text{ of operating rooms used for 1 day}$

Rooms used	# Days	$P(X)$	each $f(x) > 0?$	
1	3	$\frac{3}{20} = .15$	$.15 > 0$ T	✓
2	5	$\frac{5}{20} = .25$	$.25 > 0$ T	✓
3	8	$\frac{8}{20} = .40$	$.40 > 0$ T	✓
4	4	$\frac{4}{20} = .20$	$.20 > 0$ T	✓
$\Sigma = 20$		$\Sigma = 1$		

$$\sum f(x) = 1$$

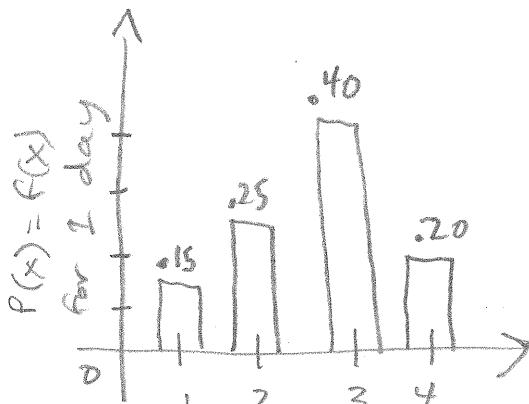
T ✓

$$P(\text{3 rooms used in 1 day}) = f(3) = .4$$

$$P(X \geq 2) = f(2) + f(3) + f(4) = .85$$

$$P(X = 1) = f(1) = .15$$

$$P(X = 0) = 0$$



*Columns Not touch w/ discrete!!*

(11)

## Discrete Uniform Probability Function

equation

$$f(x) = \frac{1}{n}$$

$n$  = # of values the random variable can assume

Example:

Roll Die

$$S = \{1, 2, 3, 4, 5, 6\}$$

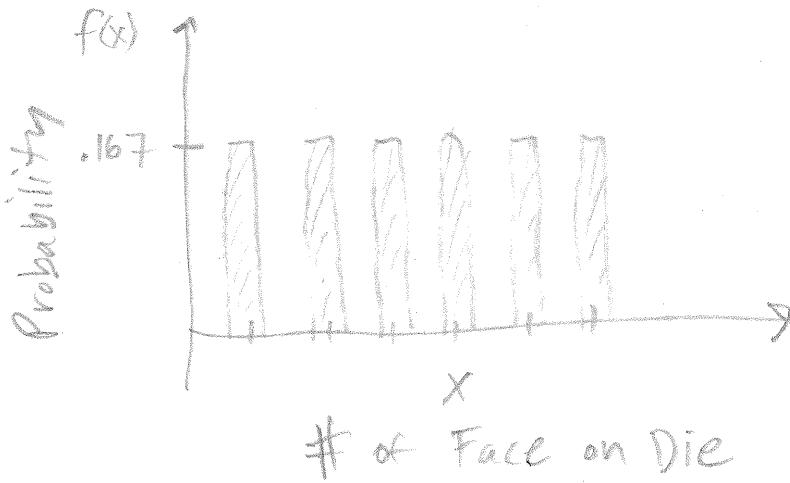
$$\text{Probability} = f(x) = \frac{1}{6}$$

$$x = 1, 2, 3, 4, 5, 6 = \begin{matrix} \text{top} \\ \text{face} \end{matrix}$$

table

outcome x	$f(x)$
1	$\frac{1}{6} = .167$
2	$\frac{1}{6} = .167$
3	$\frac{1}{6} = .167$
4	$\frac{1}{6} = .167$
5	$\frac{1}{6} = .167$
6	$\frac{1}{6} = .167$

$$\sum f(x) = 1$$

chart

# Mean & Standard deviation for Discrete Random Variable or Discrete Probability Distribution

Greek Letters are used

mean

$$E(X) = M = \sum X f(x)$$

standard deviation

$$\sigma = \sqrt{\sum (x - M)^2 f(x)}$$

$M$  = mean

$\sigma$  = standard Deviation

$X$  = random variable

$f(x) = P(x)$  = probability of taking on a particular value of  $X$

$\Sigma$  = add

\* used often in Finance.

Example:

Discrete Prob. Distr.

Rooms used $X$	$f(x) = P(x)$
1	.15
2	.25
3	.40
4	.20

① Expected Value

② long-run average of Random Var.

③ weighted Average (value \* Prob.)

④ Does not have to be a value that the random variable can assume (why it's called long run Ave.)

① Amount of Spread in Data

② Amount of Variability or Dispersion

③ Does mean Fairly Represent Data?

④ Each Deviation squared is multiplied by prob. (weight), then all are added, then take square root.

$$P(2) = .25$$

$$P(X > 2) = .6$$

But what is a typical number of rooms used?  
what is variation?

$$E(X) = M = \text{Mean} = 1 * .15 + 2 * .25 + 3 * .4 + 4 * .2 = 2.65$$

on any typical day, 2.65 rooms will be used. we won't get that exactly, but the number can be used for planning.

$x$	$x - \mu$	$(x - \mu)^2$	$(x - \mu)^2 * f(x)$
1	$1 - 2.65 = -1.65$	2.7225	.408375
2	$2 - 2.65 = -.65$	1.4225	.105625
3	$3 - 2.65 = .35$	.1225	.049
4	$4 - 2.65 = 1.35$	1.8225	.3645
	$\Sigma = -.6$		$\Sigma = 1.9275$

not equal to zero because we did not use all

Raw Data!

$$\sigma = (1.9275)^{\frac{1}{2}} = .96306801$$



Used to compare our typical value 2.65 to other hospitals with 2.65 or close to 2.65 to tell which mean more fairly represents its data points.

Back to chapter 4:

P.11

## Probability Rules for Adding & Multiplying

Example:

An insurance agent has appointments with 4 prospective clients tomorrow. From the past she knows that the probability of making a sale on any one appointment is 1 in 5 ( $\frac{1}{5} = .2$ ), what is likelihood that she will sell 3 policies in 4 tries?

Event = sell 3 policies in 4 tries

# of outcomes in 2 sales attempt = 2

Sale = S      (Assume 4 attempts are independent)  
No Sale = NS

Probability of success =  $P_S = \pi_S$  ("pi") = .2      (same each time)

Probability of Not success =  $P_{NS} = \pi_{NS} = 1 - .2 = .8$

# of Trials (steps) = n = 4

X = Discrete Random Variable = # Sales = # successes = 3

(1st): use multiplication rule (events are independent)

$$P(NS, S_1, S_2, S_3) = P(NS) * P(S_1) * P(S_2) * P(S_3) \leftarrow (\text{example})$$

# Sample Point

Each is Mutually Exclusive

	Sample Points	Probability of Sample Point
SP <sub>1</sub>	NS, S, S, S	.8 * .2 * .2 * .2 = .0064
SP <sub>2</sub>	S, NS, S, S	.2 * .8 * .2 * .2 = .0064
SP <sub>3</sub>	S, S, NS, S	.2 * .2 * .8 * .2 = .0064
SP <sub>4</sub>	S, S, S, NS	.2 * .2 * .2 * .8 = .0064

$$P(\text{Event} = 3 \text{ sales in 4 Attempts}) = .0256$$

Let's create whole distribution

(2nd) use  
Adding Rule

Prob. of Event

Sum of all  
sample points

↓  
mutually Exclusive

$$= P(NS \text{ or } S_1 \text{ or } S_2 \text{ or } S_3)$$

$$= P(NS) + P(S_1)$$

$$+ P(S_2) + P(S_3)$$

Build

## probability Distribution (Discrete) P.12

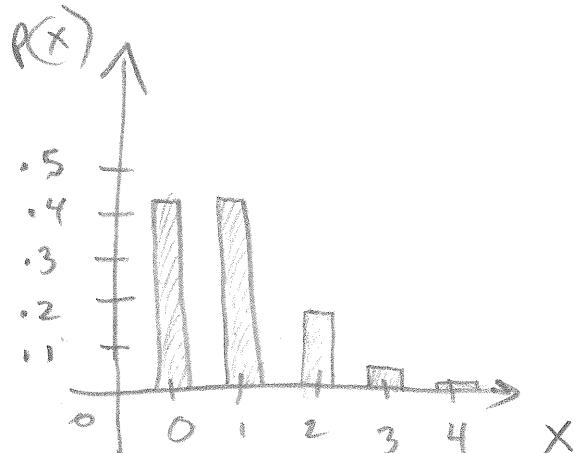
$$SS = 2 * 2 * 2 * 2 = 16$$

Possible outcomes sample points	Attempt sale				# of sales	Probability of occurrence
	1st	2nd	3rd	4th		
1	S	S	S	S	4	.2 * .2 * .2 * .2 = .0016
2	S	S	S	NS	3	.2 * .2 * .2 * .8 = .0064
3	S	S	NS	S	3	.2 * .2 * .8 * .2 = .0064
4	S	NS	S	S	3	.2 * .8 * .2 * .2 = .0064
5	NS	S	S	S	3	.8 * .2 * .2 * .2 = .0064
6	S	S	NS	NS	2	.2 * .2 * .8 * .8 = .0256
7	S	NS	NS	S	2	.2 * .8 * .8 * .2 = .0256
8	NS	NS	S	S	2	.8 * .8 * .2 * .2 = .0256
9	S	NS	S	NS	2	.2 * .8 * .2 * .8 = .0256
10	NS	S	S	NS	2	.8 * .2 * .2 * .8 = .0256
11	NS	S	NS	S	2	.8 * .2 * .8 * .2 = .0256
12	S	NS	NS	NS	1	.2 * .8 * .8 * .8 = .0256
13	NS	S	NS	NS	1	.8 * .2 * .8 * .8 = .0256
14	NS	NS	S	NS	1	.8 * .8 * .2 * .8 = .0256
15	NS	NS	NS	S	1	.8 * .8 * .8 * .2 = .0256
16	NS	NS	NS	NS	0	.8 * .8 * .8 * .8 = .4096

$$\sum = 1$$

# of Sales Random variable	P(x)	P(x)
0	$P(0) = .4096 * 1 = .4096$	
1	$P(1) = .1024 * 4 = .4096$	
2	$P(2) = .0256 * 6 = .1536$	
3	$P(3) = .0064 * 4 = .0256$	
4	$P(4) = .0016 * 1 = .0016$	

$$\sum = 1$$



Discrete Probability Distributions with  $n=4$   $p_i = .2$

But there must be an easier way!!



### 13) Binomial Probability Distribution (Discrete P.D.)

List of all outcomes for a Binomial Experiment (common multi-step experiment that has many useful applications) and the probabilities associated with each experimental outcome (sample point). P.13

### 14) Requirements for Binomial Experiment

- ① The experiment consists of a sequence of  $n$  identical Trials. (Random variable counts the # of successes in a Fixed # of Trials. Fixed # of Trials =  $n$ )
- ② 2 outcomes are possible on each Trial. one is defined as a "success" and the other is "Not success" or "Failure". S or F.
- ③ Probability of success, denoted as " $P$ " or the greek letter  $\pi$  ("pi"), remains the same on each Trial.  $1 - P$  does not change. (stationary)  
Assumption  
think of sales person losing a passed customer
- ④ The Trials are independent (one does not affect next)

Examples:

- ① Make 4 sales calls. ①  $n=4$ , ②  $S=\text{sale}$ , ③  $P=.2$  ④ yes
- ② Test with 15 T/F questions. ①  $n=15$ , ②  $S=\text{correct}$ , ③  $P=.5$  ④ yes
- ③ Flip coin 3 times. ①  $n=3$ , ②  $S=H$ , ③  $P=.5$  ④ yes
- ④ Drive across bridge 7 times ①  $n=7$  ②  $S=\begin{matrix} \text{get stuck in} \\ \text{Traffic} \end{matrix}$  ③  $P=.15$  ④ yes  
During Rush Hour Traffic
- ⑤ Air flight from Oak. to Seattle. ①  $n=6$  ②  $S=\text{late}$  ③  $P=.1$  ④ yes  
6 flights per day.

For our sales agent problem we have 16 total possible sample points but we needed only 4 of them:

P. 14

2 S, S, S, NS  
3 S, S, NS, S  
4 S, NS, S, S  
5 NS, S, S, S

4 total  $\downarrow$

How can we calculate this?  
 ↙ "sample points"

(15) Number of Experimental outcomes that provide exactly  $X$  successes in  $n$  Trials

$$\left\{ \begin{array}{l} \text{\# experimental outcomes} \\ \text{that have } X \text{ successes} \\ \text{in } n \text{ trials} \end{array} \right\} = \frac{n!}{X!(n-X)!}$$

$X$  = # successes of Random Discrete Variable

\* we use  $X$  instead of  $N$ , because  
 $X$  = successes in  $n$  trials

$n$  = # Fixed Trials

\* we use  $n$  instead of  $N$  because  
 $n$  = # of Fixed Trials

$$n = 4 \\ X = 3$$

$$\left\{ \begin{array}{l} \text{\# of experimental} \\ \text{outcomes that} \\ \text{have } X \text{ successes} \\ \text{in } n \text{ trials} \end{array} \right\} = \frac{4!}{3!(4-3)!} = \frac{1*2*3*4}{1*2*3(1)!} = \frac{4}{1} = 4$$

\* earlier formula :

$$\frac{N!}{n!(N-n)!}$$

$N$  = count of all objects = pop size

$n$  = size of subset = sample size

## ⑯ Binomial Probability Function

$$P(X) = f(x) = \frac{n!}{x!(n-x)!} * p^n * (1-p)^{(n-x)}$$

$P(X) = f(x)$  = Probability of  $x$  successes in  $n$  Trials

$n$  = # of Fixed Trials

$p$  = Probability of success on any 1 Trial

$1-p$  = Probability of failure on any 1 Trial

## ⑰ Excel Binomial Probability Function

= BINOMDIST (number\_s, trials, probability\_s, cumulative)

number\_s =  $X$  = Discrete Random Variable count # successes

trials =  $n$  = # of Fixed Trials

probability\_s =  $p$  = Probability of success.

cumulative = 0 for exactly  $X$   $P(X=2)$   
1 for less than or equal to

$P(X \leq 2)$

### Example:

For our sales Agent Problem, what is probability of making exactly 3 sales in 4 Attempts,  $p=.2$ ?

$$n=4$$

$$x=3$$

$$p=.2$$

$$f(3) = P(3) = \frac{4!}{3!(4-3)!} * .2^3 * .8^{(4-3)} =$$

$$= 4 * .008 * .8 = .0256 \checkmark$$

or

$$= \text{BINOMDIST}(3, 4, .2, 0) = .0256$$

### Binomial?

- ① Fixed # Trials? yes.
- ② 2 outcomes on each Trial?  
yes.
- ③ probability same each Trial?  
yes.
- ④ Events independent? yes.

Example:

If  $p = .2$   
 $n = 4$

$x = 0 \text{ or } x = 1 \text{ or } x = 2$

Find  $P(x \leq 2) = f(x \leq 2)$

(P.16)

a)  $P(0 \text{ or } 1 \text{ or } 2) = f(0 \text{ or } 1 \text{ or } 2) = f(0) + f(1) + f(2) =$

 $= \frac{4!}{0!(4-0)!} * .2^0 * .8^{(4-0)} + \frac{4!}{1!(4-1)!} * .2^1 * .8^{(4-1)} + \frac{4!}{2!(4-2)!} * .2^2 * .8^{(4-2)}$ 
 $= .4096 + .4096 + .1536$ 
 $= .9728 = P(x \leq 2) = f(x \leq 2)$

b)  $= \text{BINOMDIST}(2, 4, .2, 1) = .9728$

Example:

$p = .2$

$n = 4$

$x = 3 \text{ or } x = 4$

Find  $f(x \geq 3) = P(x \geq 3)$

a)  $P(x = 3 \text{ or } x = 4) = P(3) + P(4) =$

$= \frac{4!}{3!(4-3)!} * .2^3 * .8^{(4-3)} + \frac{4!}{4!(4-4)!} * .2^4 * .8^{(4-4)}$

$= .0256 + .0016$

$= .0272 = P(x \geq 3) = f(x \geq 3)$

b)  $= 1 - \text{BINOMDIST}(3-1, 4, .2, 1) = .0272$

To 3 is included.  
below 50



\* Excel function always does cumulative from  
Low End up → And All area = 1

18

# Expected Value & Standard Deviation for the Binomial Distribution

P.17

$$E(X) = \mu = \text{Mean} = n * p$$

$$\sigma = \text{Standard Deviation} = \sqrt{n * p * (1-p)}$$

$n$  = # of Fixed Trials

$p$  = Probability of Success

## Example:

For our sales Agent Problem, What is the mean number of sales she will make in 4 attempts, and what is the standard deviation?

$$n = 4$$

$$p = .2$$

$$E(X) = \text{mean} = 4 * .2 = .8$$

"For every 4 calls she can expect to sell .8 policies. If she has 40 calls planned, she can expect to sell

$$\frac{40}{4} * .8 = 8 \text{ policies}$$

$$\text{Standard Deviation} = \sqrt{.8 * (.1-.2)} = .8$$

measures dispersion & could be used to compare to other sales people.

# Binomial Experiment Examples:

P. 18

- ① A flight from Oakland to Seattle occurs 6 times per day. The probability that any one flight is late is 0.1. What is the probability that exactly 2 planes are late? What is the probability that less than 2 planes are late? Is this a binomial experiment? Mean? SD?

## Binomial Experiment?

- ① Fixed # of trials (each count S/F)?

yes ✓  $n = 5$

- ② Each trial independent? yes ✓ (more or less)

- ③ S/F each time? yes late or not late

- ④ Probability of success same each trial?

yes  $\pi = .1$

## Variables

$$\pi = .1 = \text{success} = \text{late} \quad x = 2$$

$$1 - \pi = 1 - .1 = .9 = \text{Not late} \quad x < 2$$

$$n = 6 = \text{Fixed # of trials}$$

$$P(0) = \frac{6!}{(6-0)!0!} * (.1)^0 * (.9)^{6-0}$$

$$P(2) = \frac{6!}{2!4!} * (.1)^2 * (.9)^4$$

$$P(2) = 15 * .01 * .6561$$

$$P(2) = .098415$$

Probability of exactly 2 flights late is .098415

Excel:  
 $=BINOMDIST(x, n, \pi, 0)$

$$P(x < 2) = P(1) + P(0)$$

$$P(x < 2) = .3543 + .5314$$

$$P(x < 2) = .8857$$

The probability that less than 2 flight will be late is .8857  
Excel:

$=BINOMDIST(x, n, \pi, 1)$

$$M = n\pi$$

$$M = 6 * .1$$

$$M = .6$$

The mean amount late per day is .6 flights

$$\sigma = \sqrt{n\pi * (1-\pi)}$$

$$\sigma = \sqrt{6 * .1 * (1 - .1)}$$

$$\sigma = \sqrt{.6 * .9}$$

$$\sigma = \sqrt{.54} = .7348$$

The Standard Deviation is .7348

# Binomial Experiment Example 2:

The probability of sitting in traffic on the West Seattle Bridge during rush hour is .15. During your next 7 rush hour bridge crossings, what is the probability that you will sit in traffic 3 times? 5 or more times? Mean? SD?

Binomial?

Fixed # of trials? yes  $n = 7$

Independent? yes

S/I/F? stuck in traffic/not stuck in traffic

$\pi$  same each time? yes  $\pi = .15$   
 $(1-\pi) = .85$

$$\ast \pi = p$$

$$P(3) = \frac{7!}{(7-3)!3!} * .15^3 (1-.15)^{(7-3)} = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2} * .003375 * .85^4 =$$

$$= 35 * .003375 * .5220625 = .061662$$

$$P(x \geq 5) = P(5) + P(6) + P(7) = .0011522 + .0000678 + .0000017 = .0012217$$

$$M = .15 * 7 = 1.05$$

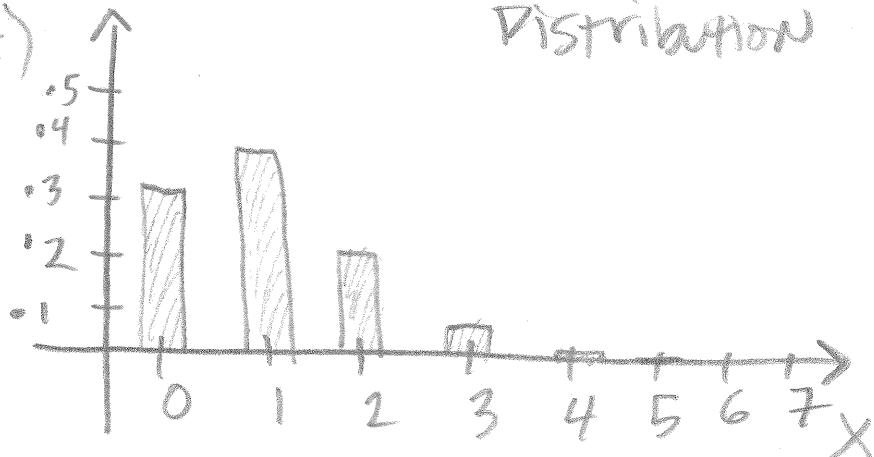
$$\sigma = \sqrt{.15 * 7 * (1-.15)} = .944722181$$

# Binomial Probability

Distribution

$$n = 7$$

$$\pi = 0.15$$



# of successes  
# of "stuck in traffic"

X	P(X)
0	.3200
1	.3960
2	.2097
3	.0617
4	.0109
5	.0012
6	.0001
7	.0000

## For Binomial Distribution :

P.21  
.22

- ① As  $p(\pi)$  approaches .5, the Distribution becomes symmetrical
- ② As  $n$  gets larger, the Distribution becomes symmetrical.

**Problem 2**

fixed # trials ✓  
independent ✓  
S or F ✓

$\pi$  stays same ✓

23

An insurance representative has appointments with four prospective clients tomorrow. From past experience she knows that the probability of making a sale on any appointment is 1 in 5 or 0.20. Use the rules of probability to determine the likelihood that she will sell a policy to 3 of the 4 prospective clients.

$$\pi = .2$$

$$\# \text{ of trials} = n = 4$$

$$x = 3$$

Let NS = No Sale

$$\pi_{NS} = 1 - .2 = .8$$

S = Sale

$$\pi_S = .2$$

Multiplying  
and then  
Adding  
Rules

Location of NS	order of occurrence	Probability of occurrence
1	NS, S, S, S	$(.8)(.2)(.2)(.2) = .0064$
2	S, NS, S, S	$(.2)(.8)(.2)(.2) = .0064$
3	S, S, NS, S	$(.2)(.2)(.8)(.2) = .0064$
4	S, S, S, NS	$(.2)(.2)(.2)(.8) = .0064$

$$\sum = .0256$$

$$P(\text{of exactly 3 sales in 4 tries}) = .0256$$

**Problem 3**

Now let's use formula [6-3] for the binomial distribution to compute the probability that the sales representative in Problem 2 will sell a policy to exactly 3 out of the 4 prospective clients.

$$P(x) = \frac{n!}{x!(n-x)!} \pi^x (1-\pi)^{n-x}$$

$$P(3) = \frac{4!}{3!(4-3)!} (.2)^3 (1-.2)^{4-3} = 4 (.008)(.8) = .0256$$

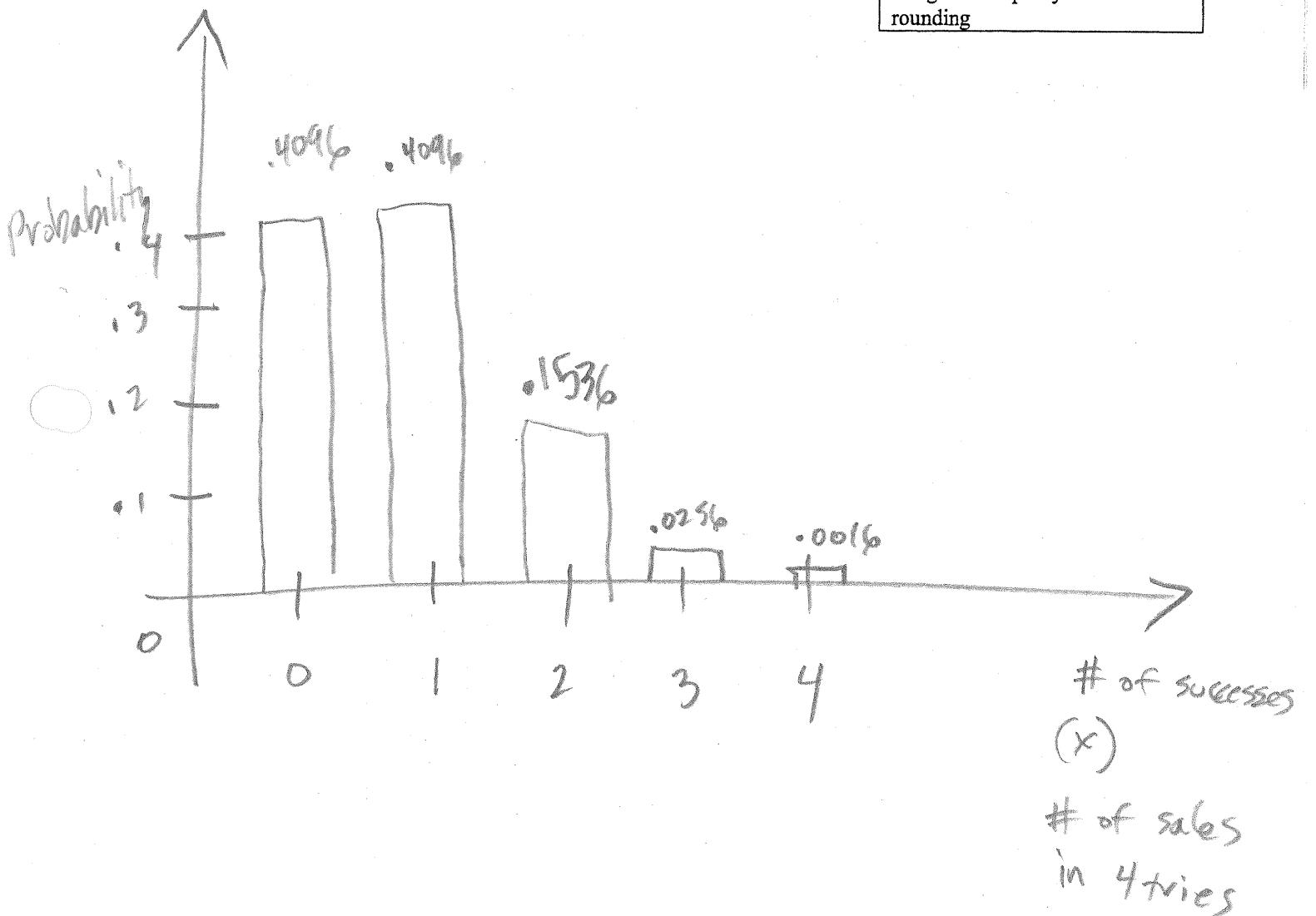
$$\text{Table } P(3) = .0256$$

### Problem 4

In Problems 2 and 3 the probability of 3 sales resulting from 4 appointments was computed using both the rules of addition and multiplication and the binomial formula. A more convenient way of arriving at the probabilities for 0, 1, 2, 3, or 4 sales out of 4 appointments is to refer to a binomial table. We use the binomial table to determine the probabilities for all possible outcomes.

Binomial Probability Distribution	
$n = 4$	$\pi = 0.20$
Number of Successes (x)	Probability
0	0.410
1	0.410
2	0.154
3	0.026
4	0.002
	*1.002

\*Slight discrepancy due to rounding



**Problem 5**

Use the information regarding the insurance representative, where  $n = 4$  and  $\pi = 0.20$ , to compute the probability that the representative sells more than two policies. Also determine the mean and variance of the number of policyholders.

$$P(X > 2) = .0272$$

$$M = n * \pi = .8$$

$$\sigma = \sqrt{n * \pi (1 - \pi)} = \sqrt{(.8)(.8)} = .8$$

**Exercise 6.3**

Check your answers against those in the ANSWER section.

Labor negotiators estimate that 30 percent of all major contract negotiations result in a strike. During the next year, 12 major contracts must be negotiated. Determine the following probabilities using Appendix A:

- a. no major strikes    b. at least 5    c. between 2 and 4 (that is 2, 3, or 4).

$$\pi \text{ of strike} = .3$$

$$1 - \pi = .7$$

$$n = 12$$

$$P(0) = .0134$$

$$P(X \geq 5) = .2763$$

$$P(2 \leq X \leq 4) = .6386$$

Trials fixed ✓  $n = 12$   
 Independent ✓  
 S/F ✓  $\pi = .3$   
 $\pi$  stay same ✓

### Exercise 6.2

Check your answers against those in the ANSWER section.

It is known that 60 percent of all registered voters in the 42<sup>nd</sup> Congressional District are Republicans. Three registered voters are selected at random from the district. Compute the probability that exactly 2 of the 3 selected are Republicans, using:

- a. The rules of probability      b. The binomial formula.

c. table

Binomial?

Fixed trials? Yes     $n = 3$

Independent? Yes

S or F? R or Not    YES

$\pi$  same each time? YES     $\pi = .6$

a)

location	order of occurrence	Probability of occurrence	
NR			
3	R, R, NR	$(.6)(.6)(.4) =$	.144
1	NR, R, R	$(.4)(.6)(.6) =$	.144
2	R, NR, R	$(.6)(.4)(.6) =$	.144
$\Sigma$			.432

$$P(\text{exactly 2 of 3 selected are Republicans}) = .432$$

b)  $P(X) = \frac{n!}{x!(n-x)!} \pi^x (1-\pi)^{n-x}$

$$P(2) = \frac{3!}{2!(3-2)!} (.6)^2 (1-.6)^{3-2} = 3 (.36) (.4) = .432$$

c)  $P(2) = .432$