Chapter 9: Hypothesis Testing

Hypothesis
• A statement about a population parameter subject to verification.
• Example:
  An official report claims:
  "The yearly salary of full-time realtor is $85,000."

Hypothesis Testing
1. A statistical procedure that uses sample evidence & probability theory to determine whether a statement about the value of a population parameter:
   "should be rejected" = "Reject"
   or
   "should not be rejected" = "Fail to Reject"
   AND

2. Make a concluding statement about the population parameter based on sample evidence.
Statement from official Report:
"The yearly salary earned by full-time realtors is $85,000"

Researcher believes:
Realtors make more than $85,000

1. If we take a random sample & get \( \bar{x} = \$88,595 \)
2. We must decide if sampling Error of
\( 88,595 - 85,000 = 3,595 \) is acceptable.

Is the difference $3,595

"Statistically Significant"
or
"Statistically insignificant"
Chapter 7

Sampling Distribution of \( \bar{X} \)

If our sample evidence provided \( \bar{X} = 86,000 \)
Then original claim of 85,000:
"should not be rejected"
"original claim seems reasonable"

If sample evidence provide \( \bar{X} = 88,595 \)
Then original claim of 85,000:
"should be rejected"
"original claim seems unreasonable"

We will use something called a "Test statistic" (\( Z \) or \( t \))
& compare it to our:
Hurdle Line.
"Test statistic" = \# standard deviations above or below
Chapter 8

Sampling distribution of $\bar{X}$

If our sample evidence provided $\bar{X} = 86,000$

Then because interval contains 85,000, original claim:
"Should not be rejected" or
"Original claim seems reasonable."

if our sample evidence provided $\bar{X} = 88,595$

Then because interval does not contain 85,000,
"Original claim: should be rejected" or
"Original claim seems unreasonable."
other examples of "statements about a value of a population parameter" that we can test:

Is the new contribution solicitation letter more effective than the old letter, which got 15% contributions?

Is the manufacturer's claim that 16 oz. of catsup is in each bottle?

Is the average wait time in line at McBurger's Restaurant less than 3 minutes?

Is the new machine faster than the old one?
Steps of Hypothesis Testing

1. Develop Null Hypothesis (H₀) & Alternative Hypothesis (H₁) or (Hₐ)
2. Specify the level of significance (α)
3. Collect sample data & compute value of test statistic (Z or T), draw picture.

**P-value Approach**

4. Use value of test statistic to compute p-value
5. Reject H₀ if p-value ≤ α

**Critical Value Approach**

4. Use level of significance to determine the critical value and
5. Use the value of the test statistic and the rejection rule to determine whether to reject H₀

Notes:
1. If population data is normally distributed, these methods are exact (99% CI, α = 0.01, then 99% intervals contain μ₁, I does not)
2. If population data is not normal, the bigger the n, the more exact. Pop normal = any n can be used
   Approx. Normal n ≥ 15
   Not Normal n ≥ 30
   Outliers n ≥ 50
Step 1

Develop Null Hypothesis (Ho) & Alternative Ho (Ha)

Null Hypothesis = Ho

The hypothesis tentatively assumed true in the hypothesis testing procedure. Based on sample evidence we either
"Reject Ho"
or
"Fail to Reject Ho"

Alternative Hypothesis = Ha

Based on sample evidence the hypothesis concluded to be true if the null hypothesis is rejected. We either
"Fail to Reject Ho"
or
"Reject Ho, accept Ha"
Research Hypothesis

Start with alternative hypothesis and make this conclude the researcher hopes to support.

Example:
Realtors make more than $85,000

Validity of a Claim

Assumption that population parameter is true

Example:
Is catsup bottle filled with 16 oz?

Decision Making

Choose between 2 things.

Example:
Should we accept box of shipped products, yes or no.
Notes about Step 2

1. Developing $H_0$ & $H_a$ can be difficult & takes practice to learn how to do.

2. The context, situation, or point of view will help determine the correct $H_0$ & $H_a$

1. Research Hypothesis → usually start with → $H_a$
   Example: "Real estate agents make more than $85,000?"
   $H_0: M \leq 85,000$
   $H_a: M > 85,000$

2. Validity of Claim → usually start with → $H_0$
   Example: "Is cat's up bottle filled with 16 oz?"
   Assume bottle is filled with 16 oz.
   But if not, take action
   $H_0: M \geq 16$ oz.
   $H_a: M < 16$ oz.

   or

   Manufacturer's point of view
   $H_0: M = 16$ oz.
   $H_a: M \neq 16$ oz.

3. Decision Making → (choose between) → $H_0$ or $H_a$
Step 1: Develop \( H_0 \) & \( H_a \)

Original Statement:
The yearly salary earned by full-time realtor is \( $85,000 \) (\( \sigma = 12,549 \))

Competing Statement:
Researcher believes realtors make more than \( $85,000 \)

1st write this: (color says "Here is Hypothesis")

\[
\begin{align*}
H_0 : & \quad \mu \\
H_a : & \quad \mu
\end{align*}
\]

2nd: Use "more than \( $85,000 \)" to determine comparative operator for \( H_a \)

\[
\begin{align*}
H_0 : & \quad \mu \\
H_a : & \quad \mu > 85,000
\end{align*}
\]

3rd: Once you know comparative operator for \( H_a \), put opposite comparative operator and equal sign for \( H_0 \).

\[
\begin{align*}
H_0 : & \quad \leq 85,000 \\
H_a : & \quad > 85,000
\end{align*}
\]

4th: \( H_0 \) ALWAYS get = sign

5th: \( H_a \) comparative operator

Always tells you which way test is
Step 1 Continued...

\[ H_0: M \leq $85,000 \]
\[ H_a: M > $85,000 \]

- If we get \( \bar{x} \) here we say:
  "Based on the sample evidence, we fail to reject \( H_0 \). There is little statistical evidence that the mean salary is more than $85,000." *Don't say \( H_0 \) is true

- If we get \( \bar{x} \) here we say:
  "Based on the sample evidence, we reject \( H_0 \) and accept \( H_a \). There is statistical evidence that the mean salary is more than $85,000."

* careful in our language because we are taking samples.
* only two possible outcomes.
3 possible forms of $H_o$ & $H_a$

1. Two-tail
   
   $H_o: M = M_0$
   $H_a: M \neq M_0$

2. 1-tail to Right
   
   $H_o: M \leq M_0$
   $H_a: M > M_0$

3. 1-tail to Left
   
   $H_o: M \geq M_0$
   $H_a: M < M_0$
Level of Significance = Alpha = $\alpha$

1. $\alpha$ determines the cut off point, which is the threshold used to decide whether the test statistic is statistically significant.

$H_0: M \leq 85,000$

$H_a: M > 85,000$

If we get $\bar{x} = 88,595$ and it is out here, this is statistically significant and we reject $H_0$ and accept $H_a$.

$M = M_{\bar{x}} = 85,000$

$\bar{x} = 88,595$

- If we choose $\alpha = 0.05$, we are taking a 5% risk of rejecting $H_0$ even though it was true.
- Because we choose $\alpha$, we can say we are doing a "Significance Test"
### Picture Examples for Level of Significance where \( M = 85,000 \)

<table>
<thead>
<tr>
<th>Alpha</th>
<th>2-tail</th>
<th>1-tail to Left</th>
<th>1-tail to Right</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha = .10 )</td>
<td>( H_0: M = 85,000 ) ( H_a: M \neq 85,000 )</td>
<td>( H_0: M &gt; 85,000 ) ( H_a: M &lt; 85,000 )</td>
<td>( H_0: M \leq 85,000 ) ( H_a: M &gt; 85,000 )</td>
</tr>
<tr>
<td>( \alpha = .05 )</td>
<td>( H_0: M = 85,000 ) ( H_a: M \neq 85,000 )</td>
<td>( H_0: M &gt; 85,000 ) ( H_a: M &lt; 85,000 )</td>
<td>( H_0: M \leq 85,000 ) ( H_a: M &gt; 85,000 )</td>
</tr>
<tr>
<td>( \alpha = .01 )</td>
<td>( H_0: M = 85,000 ) ( H_a: M \neq 85,000 )</td>
<td>( H_0: M &gt; 85,000 ) ( H_a: M &lt; 85,000 )</td>
<td>( H_0: M \leq 85,000 ) ( H_a: M &gt; 85,000 )</td>
</tr>
</tbody>
</table>

These picture examples show the 3 possibilities at 3 different alpha values.
Step 2: Specify Level of Significance ($\alpha$)

- Because hypothesis testing is based on sample data, we must allow for the possibility of errors.
- Unless we test the whole population, you run risk of error.

1. Notice:
   - This is entire distribution of possible $\bar{x}$ values.

2. It is possible to take a sample & get an $\bar{x} = 88,595$ that is just sample error:
   - So we could get $\bar{x} = 88,595$ but $M = 85,000$ is still true.
It is very unlikely that we could get an \( \bar{x} \) out here.

**But...**

It is a possibility that our particular sample just happened to have a lot of big numbers in it so we got:

\[
\bar{x} = \$88,595
\]

while the full population mean was still:

\[
\mu = \$85,000
\]

**So...**

Because we can't usually test the whole population, we have to pick a cut off point and reject the original statement (\( H_0 \)) if we go beyond that point.

\[ \rightarrow \text{This means we take a risk of an error...} \]
Define:

- Level of significance = \( \text{Alpha} = \alpha = \text{Type I Error} \)
- Probability (Risk) of rejecting \( H_0 \) even though it is true (as an equality).

\[ H_0: M \leq 85,000 \]
\[ H_a: M > 85,000 \]

\( \bar{x} = 88,595 \) and we reject \( H_0 \).

But \( H_0 \) (\( M = 85,000 \)) is actually true this is:

- "Type I Error"
- "Innocent but Found Guilty"

Most of the time \( \bar{x} = 88,595 \) will lead to correct conclusion (about 95 out of 100).

"Innocent but Found Guilty" Error 5 out of 100 times.
Designer of Hypothesis Test selects \( \alpha \) and thereby controls the probability of a Type I Error:

- As you move this way, you reduce \( \alpha \) and reduce the risk of Type I Error.

Example:

1. Drug company may want to set \( \alpha \) very small, so they are sure New Drug really works.
2. Quality Control may want to set \( \alpha \) low so they are more sure that the quality is high.

Selecting \( \alpha \):

- If cost of making Type I Error is high, choose small \( \alpha \).
- If cost of making Type I Error is not high, choose bigger \( \alpha \).
## Errors & Correct Conclusions in Hypothesis Testing

### Actual Population Condition

<table>
<thead>
<tr>
<th>Conclusion (based on Sample)</th>
<th>$H_0$ TRUE ($H_a$ FALSE)</th>
<th>$H_0$ FALSE ($H_a$ TRUE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reject $H_0$, Accept $H_a$</td>
<td>Type I Error</td>
<td>Correct Conclusion</td>
</tr>
<tr>
<td>Accept $H_a$</td>
<td>Alpha</td>
<td>Correct Conclusion</td>
</tr>
<tr>
<td>Fail To Reject $H_0$</td>
<td>Correct Conclusion</td>
<td>Type II Error</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Beta</td>
</tr>
</tbody>
</table>

### Type I Error

- $H_0$ True, but we reject $H_0$
- $\alpha = \alpha$
- "I innocent but found guilty"

### Type II Error

- $H_0$ False, but we fail to reject $H_0$
- $\beta = \beta$
- "Guilty but found innocent"

Because we don't control for $\beta$ (In this textbook) we can't say "Accept $H_0$"
Other Wording:

<table>
<thead>
<tr>
<th>Conclusion (based on Sample)</th>
<th>Actual Population Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$H_0$ TRUE ($H_a$ FALSE)</td>
</tr>
<tr>
<td>Reject $H_0$, Accept $H_a$</td>
<td>Type I Error</td>
</tr>
<tr>
<td></td>
<td>Alpha (Level of Significance)</td>
</tr>
<tr>
<td></td>
<td>&quot;False Positive&quot;</td>
</tr>
<tr>
<td>Fail To Reject $H_0$</td>
<td>Correct Conclusion</td>
</tr>
<tr>
<td></td>
<td>&quot;True Negative&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**False Positive**: Our Alternative ($H_a$) was selected even though the Null ($H_0$) was true.

**False Negative**: Our Alternative ($H_a$) was not selected even though the Null ($H_0$) was false.
Step 3: collect sample data, calculate value of Test statistic ($z$ or $t$)

**Example 1:**

**Step 1**
- $H_0: M \leq 85,000$ Annual Realtor Salary
- $H_a: M > 85,000$ Annual Realtor Salary

**Step 2**
- $\alpha = \text{Type I Error} = 0.05$ (Cost of error not too big)

**Step 3**
- We go out & get a sample
- $\bar{X} = 88,595$
- $\sigma$ known $= 12,549$ (Big enough to accommodate some outlier salaries)
- $n = 36$

But we need test statistic
Test statistic Here = Fail to Reject Ho

Test statistic Here = Reject Ho

Test statistic Here = Fail to Reject Ho

Test statistic Here = Reject Ho

α/2

α/2

α

α

a determines cut off point, then we reject Ho. Accept Ho.

Test statistic beyond cut off point, then we reject Ho. Accept Ho.
2 Methods for determining whether test statistic is past cut off ("Statistically Significant")

1. **p-value**: $p\text{-value} \leq \alpha$
   - Reject $H_0$, Accept $H_a$
   - or

2. **Critical value**: If test statistic is past critical value
   - Reject $H_0$, Accept $H_a$

---

![Diagram](image)
**critical value**

Hurdle point that determines if the Null Hypothesis is Rejected & the Alternative Hypothesis is Accepted. Calculate critical value based on Alpha.

1 tail to Left Lower Tail

1 tail to Right Upper Tail

2 Tail Test

\[ \text{critical value} = -Z_{\alpha} \]

\[ -Z_{\alpha/2} - \text{critical values} - Z_{\alpha/2} \]
**P-value**  "observed level of significance"

Probability of getting the test statistic value or worse (less or more).

1 tail to Left

- \( \alpha = 0.05 \)
- Test Statistic
- Critical Value
- \( p\text{-value} = \) probability of getting test statistic or less
- \( p\text{-value} \times 1 \)

1 tail to Right

- \( \alpha = 0.05 \)
- Test Statistic
- Critical Value
- \( p\text{-value} = \) probability of getting test statistic or more
- \( p\text{-value} \times 1 \)

Two tail

- \( \alpha_1 = 0.025 \)
- \( \alpha_2 = 0.025 \)
- Critical Values
- Test Statistics
- \( p\text{-value} = \) probability of getting test statistic or worse (less or more)
- then
- \( p\text{-value} \times 2 \)

**Rejection Rule:**  \( p\text{-value} \leq \alpha \), Reject \( H_0 \), Accept \( H_a \)
### Interpreting p-value

<table>
<thead>
<tr>
<th>p-value</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>p-value &gt; 0.10</td>
<td>Insufficient evidence to say $H_a \text{ True}$</td>
</tr>
<tr>
<td>0.05 ≤ p-value ≤ 0.10</td>
<td>Weak evidence to say $H_a \text{ True}$</td>
</tr>
<tr>
<td>0.01 ≤ p-value ≤ 0.05</td>
<td>Strong evidence to say $H_a \text{ True}$</td>
</tr>
<tr>
<td>p-value ≤ 0.01</td>
<td>Overwhelming evidence to say $H_a \text{ True}$</td>
</tr>
</tbody>
</table>

The advantage of p-value (over critical value) is that it tells you how significant the results are:

1. What probability of getting a test statistic or worse (less or more)

2. What the Type I Error Rate is, like we got 88.595 for $\bar{x}$ & that value would appear as a true sample mean w/ $M = 85,000$ 4 in 100 times.
Test statistic (\(z\) or \(t\)) for Hypothesis Testing

About a Population Mean

\[ \begin{align*}
\text{\( \sigma \) Known} & \quad \text{\( \sigma \) Not Known} \\
Z &= \frac{\bar{X} - M_0}{\sigma / \sqrt{n}} & t &= \frac{\bar{X} - M_0}{s / \sqrt{n}}
\end{align*} \]

- \( \bar{X} \) = Sample mean
- \( \sigma \) = Population standard deviation
- \( s \) = Sample standard deviation
- \( n \) = Sample size
- \( M_0 \) = Hypothesized mean

Z or t = calculated test statistic, used to determine whether to reject the Null Hypothesis. Compare Z or t to critical value to make decision or used to calculate \( p \)-value. \( Z \pm t \) = number of Standard Errors above/below Hypothesized mean.
Test statistic for Hypothesis Tests About A Population Proportion

\[
Z = \frac{\bar{p} - p_0}{\sqrt{p_0 \times (1 - p_0)}} \times n
\]

\(\bar{p}\) = sample proportion
\(p_0\) = hypothesized pop. proportion
\(n\) = sample size

\[\text{SE} = \sigma_{\bar{p}} = \sqrt{\frac{p_0 \times (1 - p_0)}{n}}\]

Must verify:

1. Are there fixed # Trials?
2. Are results independent?
3. Does each trial result in success or failure?
4. \(p\) stay same on each trial?
5. \(n \times p > 5\) \(n \times (1 - p) > 5\) 

\text{textbook assumes true for all problems.}

* since exact sampling distribution of \(\bar{p}\) (Pear) is discrete, small samples require additional steps that we will not do in this textbook.
Q: When are we allowed to use t Distribution?
A: When population distribution is normally distributed or near normal, or n is sufficiently large enough
   1) If pop distribution is normal or near normal, smaller than 30 sample size may be used
   2) If pop distribution is not normal, n >= 30 usually adequate
   3) If pop distribution is highly skewed or has outliers, n >= 50 should be used

Notes:
If the population distribution is not known a histogram based on a sample may give you a clue.
Although histogram is not conclusive, sometimes it may be the best clue that you have.
The histogram shows non-normal or outliers, increasing the sample size and improve the calculations.
<table>
<thead>
<tr>
<th>Excel Functions</th>
<th>( Z ) Distribution</th>
<th>( T ) Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>One tail to Right</strong>&lt;br&gt;Upper</td>
<td>( p\text{-value} = 1 - \text{NORM.S.DIST}(Z, 1) )&lt;br&gt;Upper critical value ( = \text{NORM.S.INV}(1 - \alpha) )</td>
<td>( p\text{-value} = 1 - T\text{.DIST}(t, df, 1) )&lt;br&gt;Upper critical value ( = T\text{.INV}(1 - \alpha, df) )</td>
</tr>
<tr>
<td><strong>Two tails</strong></td>
<td>( p\text{-value} = \text{NORM.S.DIST}(Z, 1) \times 2 )&lt;br&gt;Lower critical value ( = \text{NORM.S.INV}(\alpha/2) )&lt;br&gt;Upper critical value ( = \text{NORM.S.INV}(1 - \alpha/2) ) or ( \pm \text{critical values} = \pm \text{NORM.S.INV}(\alpha/2) )</td>
<td>( p\text{-value} = T\text{.DIST}(\text{lower}, df, 1) \times 2 )&lt;br&gt;Lower critical value ( = T\text{.DIST.2T}(\text{upper}, df) )&lt;br&gt;Upper critical value ( = T\text{.INV}(1 - \alpha/2, df) ) or ( \pm \text{critical values} = \pm 1 - T\text{.INV}(\alpha/2, df) )</td>
</tr>
<tr>
<td><strong>One tail to Left</strong>&lt;br&gt;Upper</td>
<td>( p\text{-value} = \text{NORM.S.DIST}(Z, 1) )&lt;br&gt;Low critical value ( = \text{NORM.S.INV}(\alpha) )</td>
<td>( p\text{-value} = T\text{.DIST}(t, df, 1) )&lt;br&gt;Lower critical value ( = T\text{.INV}(\alpha, df) )</td>
</tr>
<tr>
<td><strong>When to use</strong>: Sigma known and proportions, when 4 tests met.</td>
<td>Sigma not known</td>
<td></td>
</tr>
</tbody>
</table>
## Hypothesis Testing

### $Z$ Distribution (Sigma Known)

<table>
<thead>
<tr>
<th>Test Type</th>
<th>Hypothesis</th>
<th>Test Statistic</th>
<th>$p$-value Rejection Rule</th>
<th>Excel $p$-value</th>
<th>Critical Value Rejection Rule</th>
<th>Excel Critical Value</th>
</tr>
</thead>
</table>
| 1 Tail Test to Left | $H_0: M \geq M_0$  
$H_a: M < M_0$ | $z = \frac{X - M_0}{\sigma / \sqrt{n}}$ | $IF: p-value \leq \alpha$  
$THEN: Reject H_0, Accept H_a$ | $= NORM.S.DIST(z, 1)$ | If: $z \leq -Z_\alpha$  
Then: Reject H_0, Accept H_a  
$-Z_\alpha = \text{critical value (Low End)}$ | $-Z_\alpha = \text{NORM.S.INV}(\alpha)$ |
| 2 Tail Test | $H_0: M = M_0$  
$H_a: M < M_0$ \newline $\text{or} \newline H_a: M > M_0$ | $z = \frac{X - M_0}{\sigma / \sqrt{n}}$ | $IF: p-value \leq \alpha$  
$THEN: Reject H_0, Accept H_a$ | $= NORM.S.DIST(z; 1) \times 2$  
$z \text{ on Low End}$ | $-Z_{\alpha/2} < Z < Z_{\alpha/2}$  
Then: Fail To Reject H_0  
$-Z_{\alpha/2} = \text{low critical value}$  
$Z_{\alpha/2} = \text{upper critical value}$ | $Z_\alpha = \text{NORM.S.INV}(1 - \alpha)$  
$+/- \text{ critical value} = NORM.S.INV(\alpha/12)$ |
<table>
<thead>
<tr>
<th>Test Type</th>
<th>Hypothesis</th>
<th>Test Statistic</th>
<th>p-value Rejection Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Test Type</strong></td>
<td><strong>Hypothesis</strong></td>
<td><strong>Standard Error</strong></td>
<td><strong>Rejection Rule</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( SE = \sigma_p = \sqrt{\frac{p_0(1-p_0)}{n}} )</td>
<td>( \text{If: } p\text{-value} \leq \alpha ) Then: Reject ( H_0 ), Accept ( H_a )</td>
</tr>
<tr>
<td><strong>Hypothesis</strong></td>
<td></td>
<td>( Z = \frac{\bar{p} - p_0}{SE} )</td>
<td></td>
</tr>
<tr>
<td>One Tail Test to Left</td>
<td>( H_0: p \geq p_0 )</td>
<td>( p \text{-value} = \text{NORM.S.DIST}(Z, 1) )</td>
<td>( Z \leq -Z_{\alpha} ) Then: Reject ( H_0 ), Accept ( H_a ) ( -Z_{\alpha} = \text{critical value (low end)} )</td>
</tr>
<tr>
<td></td>
<td>( H_a: p &lt; p_0 )</td>
<td>( \text{NORM.S.DIST}(Z, 1) \times 2 )</td>
<td>( Z \geq Z_{\alpha} ) Then: Reject ( H_0 ), Accept ( H_a ) ( Z_{\alpha} = \text{NORM.S.INV}(1-\alpha) )</td>
</tr>
<tr>
<td>Two-Tail Test</td>
<td>( H_0: p = p_0 )</td>
<td>( 1 - \text{NORM.S.DIST}(Z, 1) )</td>
<td>( Z \leq -Z_{\alpha/2} ) ( Z_{\alpha/2} = \text{lower critical value} ) Then: Fail To Reject ( H_0 ) ( Z_{\alpha/2} = \text{upper critical value} )</td>
</tr>
<tr>
<td></td>
<td>( H_a: p \neq p_0 )</td>
<td>( \text{NORM.S.INV}(1-\alpha/2) )</td>
<td>( Z \geq Z_{\alpha/2} )</td>
</tr>
<tr>
<td></td>
<td>( H_a: p &gt; p_0 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Hypothesis Testing

#### $t$ Distribution (Sigma Not Known)

<table>
<thead>
<tr>
<th>Test Type</th>
<th>Hypothesis</th>
<th>Test Statistic</th>
<th>$t$-value</th>
<th>Critical Value Rejection Rule</th>
<th>Excel Critical Value</th>
<th>Excel $p$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lower Tail Test To Left</strong></td>
<td>$H_0: M \geq M_0$</td>
<td>$H_a: M &lt; M_0$</td>
<td>$t = \frac{\bar{X} - M_0}{S/\sqrt{n}}$</td>
<td>If $t \leq -t_\alpha$ Then: Reject $H_0$, Accept $H_a$</td>
<td>$-t_\alpha = \text{Low Critical Value}$</td>
<td>$= \text{T.DIST}(t, df, 1)$</td>
</tr>
<tr>
<td><strong>Two-tail Test</strong></td>
<td>$H_0: M = M_0$</td>
<td>$H_a: M \neq M_0$</td>
<td>$t = \frac{\bar{X} - M_0}{S/\sqrt{n}}$</td>
<td>If $-t_{\alpha/2} &lt; t &lt; t_{\alpha/2}$ Then: Fail to Reject $H_0$</td>
<td>$t = \text{T.DIST}._2T(t, df)$ or $= \text{T.DIST}(t, df, 1)*2$</td>
<td></td>
</tr>
<tr>
<td><strong>Upper Tail Test To Right</strong></td>
<td>$H_0: M \leq M_0$</td>
<td>$H_a: M &gt; M_0$</td>
<td>$t = \frac{\bar{X} - M_0}{S/\sqrt{n}}$</td>
<td>If $t \geq t_\alpha$ Then: Reject $H_0$, Accept $H_a$</td>
<td>$t_\alpha = \text{Upper Critical Value}$</td>
<td>$= \text{T.DIST.RT}(t, df)$</td>
</tr>
</tbody>
</table>
Example of step 3: (collect data, calculate test statistic, draw picture)

Because we know the population standard deviation $\sigma = 12,549$, we can use the test statistic $Z$.

$$n = 36$$
$$\bar{X} = \frac{\sum X}{n} = 88,595$$
$$\sigma = 12,549$$

$$M = \bar{X} = 85,000$$

$$\sigma_x = \frac{\sigma}{\sqrt{n}} = \frac{12,549}{\sqrt{36}} = 2,091.50$$

$$\alpha = 0.05$$

$$H_0: M \leq 85,000$$
$$H_a: M > 85,000$$

**Decision Rule:** If our test statistic is greater than or equal to 1.6448, we reject $H_0$ and accept $H_a$.

$$\text{Test statistic} = \frac{88,595 - 85,000}{2,091.50} = 1.72$$

$$\{\text{Critical value}\} = \text{NORMS.INV}(1 - 0.05) = 1.6448$$

**For any calculated test statistic:**

- Reject $H_0$ and accept $H_a$. 
- Fail to reject $H_0$. 

$\bar{X}$ is the sample mean, $M$ is the hypothesized mean, $\sigma_x$ is the standard error of the mean.
Steps 5: 

conclude with critical value & Rejection Rule

\[
\text{Calculated test statistic} = \frac{88,595 - 85,000}{12549} = 1.72
\]

Make Decision:

Because our calculated test statistic is greater than 1.645, we reject Ho and accept H1. It is reasonable to assume that the mean salary for real estate agents is greater than $85,000.

Based on the statistical evidence our \( \bar{x} \) of 88,595 is statistically significant and provides good evidence that the mean salary for real estate agents is more than 85,000.
Step 4 & 5 for p-value

Critical value
= NORMSINV(1 - .05) = 1.645

\[ \alpha = .05 \]

\{calculated test statistic\}
= 1.72

\[ \text{P-value} = 1 - \text{NORMSDIST}(1.72) = .04282 \]

Because the p-value is less than alpha \((.04282 \leq .05)\), we reject \(H_0\) and accept \(H_1\). It is reasonable to assume that the mean salary for real estate agents is greater than $85,000.
Summary for Real Estate Example:

\[ M = M_0 = 85,000 \]
\[ \bar{X} = 88,595 \]

Size \[ n = 36 \]

Standard Error \[ \delta = \frac{12,549}{\sqrt{36}} = 2,091.50 \]

\[ \alpha = 0.05 \]

Test Statistic \[ \frac{88,595 - 85,000}{2,091.5} = 1.72 \]

Critical Value

Dividing point between the region where the null hypothesis is rejected and the region where it is not rejected.

\[ p-value = 1 - \text{NORMSDIST}(z) \]
\[ p-value = 1 - \text{NORMSDIST}(1.72) = 0.04282 \]
Concluding:

Use $z$ or $t$ to compare to critical value

Use $p$-value to compare to alpha $\alpha$

**Critical value**

- $t$ or $z \geq$ critical value, Reject $H_0$, Accept $H_a$
- $t$ or $z \leq$ critical value, Reject $H_0$, Accept $H_a$
- $-t \leq t \leq z$, Fail to Reject $H_0$

**$p$-value**

- $p$-value $\leq \alpha$, Reject $H_0$, Accept $H_a$
**Confidence Interval Hypothesis Testing**

If:

- $H_0: M = M_0$
- $H_a: M \neq M_0$

Then:

- $M_0$ = hypothesized population mean

1. Select a simple random sample from the population and use the value of the sample mean $\bar{X}$ to develop a confidence interval for the population mean $M$.

   \[ \bar{X} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \]

   - $\bar{X}$ = sample mean
   - $Z_{\alpha/2}$ = upper $Z$ value
   - $\sigma$ = pop. s.d.
   - $n$ = sample size

2. If the confidence interval contains the hypothesized value ($M_0$), do not reject $H_0$, otherwise, reject $H_0$ (Reject $H_0$ if $M_0$ is one of the endpoints).

**Example:**

- $M = M_\bar{X} = \text{pop. mean} = \text{mean of } 500 \text{ doses}$

Accept $H_0$  

Reject $H_0$
Step 3

Ho: 
Ha: 

Fail to Reject Ho

Reject Ho
Accept Ha

$\alpha = 0.05 = \text{Risk of Rejecting Ho when it is True}$

$M = \$83,000$

$\alpha$ determined Hurdle. Beyond this point we will say sample error is statistically significant.

Step 4

$\alpha \downarrow$
$p$-value $0.043 \leq 0.05$

So

Reject Ho, Accept Ha

Test statistic $1.72 \geq 1.645 \text{ c.v.}$

So

Reject Ho, Accept Ha

$Z$ Test statistic = $\frac{\text{# standard Errors}}{\text{# standard Deviations for sampling Dist. } \bar{X}} = 1.72$

$p$-value = probability of getting $Z$ of $1.72$ or more

$= 0.043$
Steps

Reject Ho
Accept Ha

$\alpha = 0.05$
Risk of rejecting $H_0$ when it is true

Fail to Reject $H_a$

$H_0: M \geq 16oz$
$H_a: M < 16oz$

To determine hurdle. Beyond this point we will say sample error is statistically significant.

Step 4

$p$-value = Probability is area that represents probability of getting $-0.8$ or less
$p$-value = 0.21

$z$-test statistic = $-0.8$

Because $-0.8 > -1.645$, we fail to reject $H_0$

Because $0.21 > 0.05$, we fail to reject $H_0$
Step 3

Reject Ho
Accept Ha

$\alpha/2 = 0.025$
Risk of Rejecting Ho, when it is True

$\alpha$ determines Hurdle Point at which we say sample error is Statistically Significant

P-value for Two-Tail = probability of 1.44 or more TIMES 2 !!!

$P$-value = $0.075 \times 2 = 0.15$

Test Statistic = 1.44
-1.96 - critical values 1.96
**Step 3**

- **H₀**: \( M \leq 250 
- **Hₐ**: \( M > 250 

- **Fail to Reject H₀**: α determines critical value; cv is hurdle
- **Reject H₀**: \( \alpha = 0.01 \)
  - risk of rejecting H₀ when it is true

**Step 4**

- p-value = probability of getting \( t \geq 3.87 \) or more
- p-value = 0.0008 = very strong evidence

- Critical value = 2.62
- t-test statistic = 3.87

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Step 3
Reject Ho
Accept Ha
\[ \alpha = 0.05 \]

Fail to Reject Ho

Step 4

p-value = Probability of getting
-0.944 or less

Test statistic = -0.944
P-value = 0.175
Critical value = -1.68

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Step 3

Reject $H_0$
Accept $H_a$

$p \leq 0.005$

Fail to Reject $H_0$

Step 4

$\frac{1}{2} p\text{-value} = \text{Probability of getting 1.52 or more}$
MUST DOUBLE

Test statistic $= 1.52$

$\pm$-critical value $= 1.99$

$p\text{-value} = 0.0663 \times 2 = 0.1326$

Reject $H_0$
Accept $H_a$

$p \leq 0.005$
3) Decision Making → Choose between 2 things → Ho or H₁

Example:
Should we accept box of 20 meter boomerangs?
(if they fly too short, can't be used in competition)
(if they fly too long, times will not be fast enough)

Choice #1 → H₀: M = 20 meters → [accept Box]
Choice #2 → H₀ or H₁: M ≠ 20 meters → [reject Box]