(1) Random Variable X

(a) A numerical value resulting from a Random Experiment that, by chance, can assme different values.

(b) A numerical description of the outcome of a Random Experiment.

Remember:

Random Experiment

1. well defined outcomes
2. Only 1 outcome on each Trial
3. outcome occurs by chance

(c) Examples:

customers coming into store 0, 1, 2, 3...
weight of cereal Box 10 oz., 10.21 oz.
Defect = 1 Not Defect = 0

(2) Discrete Random Variable X (usually counting)

(a) Discrete = "Gaps" between numbers 1, 2, 3 or 1.1, 1.2, 1.3

(b) May assume either:

A finite number of values like: 1, 2, 3
An infinite sequence of numbers like: 1, 2, 3...

(c) Examples:

customers coming into a store like: 0, 1, 2, 3...
scores for a Dancer like: 0, 0.1, 0.2, ... 9.8, 9.9, 10
product Quality like Defect = 1 Not Defect = 0

(3) Continuous Random Variable

1 2 { Lots of possible numbers

(a) May assume any numerical value in an interval or collection of intervals. Depends on measurement instrument.

(b) Examples:

weight of cereal Box 10.1 oz., 10.11 oz., 10.112 oz. $0 \le X \le 12$
Time between customers in line at Disneyland $X \geq 0$ min.
% score on Test $0\% \le X \le 100\%$
Money (even though it seems Discrete)

(4) Probability Distribution

A description / presentation of how the probabilities are distributed over the values of the random variable.

(5) Discrete Probability Distribution

(a) A description / presentation of how the probabilities are distributed over the values of a discrete random variable.

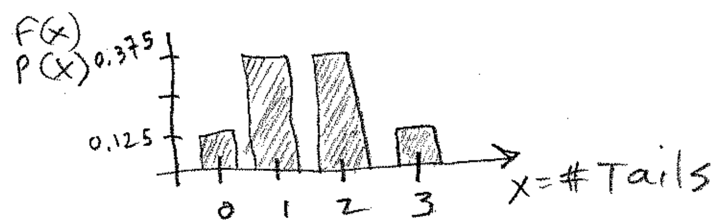
(b) Description / presentation can be:

(1) Table like: Experiment: Flip coin 3 times

# of Tails	f(x) or P(x)
0	0.125
1	0.375
2	0.375
3	0.125
$\Sigma =$	1

Each $P(x) \geq 0$

(2) Chart like:



(3) Discrete Probability Function like: $f(x) = P(x) = \frac{n!}{x!(n-x)!} * p^x * (1-p)^{n-x}$

(6) Discrete Probability Function f(x) or P(x)

A probability function, f(x) or P(x), that provides the probability for each value of the discrete random variable.

(7) Requirements for f(x) or P(x)

$\boxed{1}$ $f(x) = P(x) \geq 0$ AND $\boxed{2}$ $\Sigma f(x) = \Sigma P(x) = 1$

(8)

Methods for assigning Probabilities that are useful for creating Discrete Probability Distributions are: classical, Relative & subjective.

8 Examples for creating Discrete Probability Distr.:

(a) classical

Experiment: Roll 1 die, X = 1, 2, 3, 4, 5, 6

Roll number X	f(x), P(x)
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6
Σ	1

Each P(x) ≥ 0 ✓

Discrete Uniform Probability Function

$f(x) = P(x) = 1/n$

n = # of values random variable may assume

(b) Relative (from past data)

Experiment = count # Restaurant Banquet Rooms Used in 1 day

Relative Frequency method based on large data sets is called "Empirical Discrete Distribution"

X = # of Rooms Used	# of Days Frequency	Relative Frequency f(x) P(x)
0	2	0.02
1	21	0.21
2	42	0.41
3	27	0.27
4	8	0.08
Total	100	Σ = 1.00

(c) Subjective (Little past data & Not equally likely outcomes)

Experiment: # of compressor sales in 1 day

# of compressor sales X	f(x), P(x)
0	0.10
1	0.35
2	0.40
3	0.12
4 or more	0.03
Σ	1

P(x) ≥ 0 ✓

Sales manager just estimated from memory. Did not directly analyze past data. First year of operation, No past data.

9 Advantage of Probability Distribution?

Easy to calculate probability of a variety of events.

Example:

# of compressor sales X	$f(x)$, $P(x)$
0	0.10
1	0.35
2	0.40
3	0.12
4 or more	0.03
Σ	1 ✓

$P(x) \geq 0$ ✓

$P(X \leq 1) = 0.35 + 0.1 = 0.45$

$P(X \geq 2) = 0.40 + 0.12 + 0.03 = 0.55$

10 Steps for building a Discrete Probability Distribution

a) Steps:
1. Define Random Variable
2. Build Frequency Distribution
3. Calculate Relative Frequency $f(x) = P(x)$
4. Check Requirements: $P(x) \geq 0$ AND $\Sigma P(x) = 1$
5. Create column chart (if desired) to visually portray Distribution (Discrete = columns NOT Touch)
6. Make predictions

b) Examples next two pages:

How to Make a Discrete Probability Distr.

consider random experiment: A coin tossed 3 times

X = random discrete variable = # of Heads

H = Heads

T = Tails

Experimental outcomes

sample space (list of all possible sample points) = SS

{# of Trials} = TOSS = 3 times = 3

outcomes = 2

$2 * 2 * 2 = 8$ = count of all sample Points = SS

count sample points	coin toss 1st	2nd	3rd	# of Heads
1	H	H	H	3
2	H	H	T	2
3	H	T	H	2
4	T	H	H	2
5	H	T	T	1
6	T	H	T	1
7	T	T	H	1
8	T	T	T	0

possible values of x ⇒ 0, 1, 2, 3

Discrete Probability Distribution

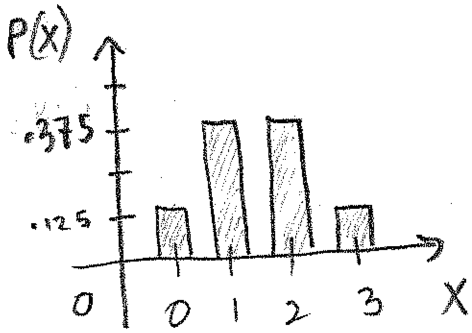
# of Heads	P(x)
0	1/8 = .125
1	3/8 = .375
2	3/8 = .375
3	1/8 = .125
	Σ = 1.00

P(0 Heads in 3 toss) = .125

P(1 Head in 3 toss) = .375

P(2 Heads in 3 toss) = .375

P(3 Heads in 3 toss) = .125



2nd method

# of Heads	P(x)
0	.5 * .5 * .5 * 1 = .125
1	.5 * .5 * .5 * 3 = .375
2	.5 * .5 * .5 * 3 = .375
3	.5 * .5 * .5 * 1 = .125

Example 2:

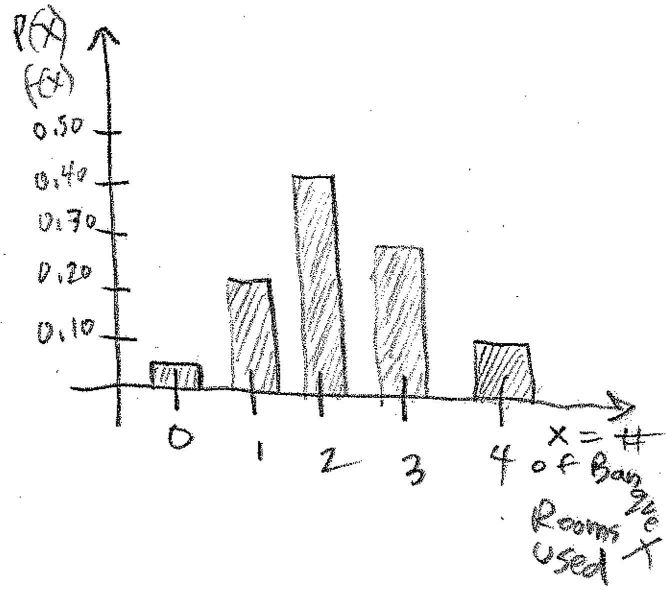
create Discrete Probability Distribution

I saac's Italian Restaurant has 4 banquet rooms. over the past year they collected the following data for weekend room usage: on two days 0 rooms were used,
on 21 days 1 room was used,
on 42 days 2 rooms were used,
on 27 days 3 rooms were used,
on 4 days 8 rooms were used.

create a Discrete Probability Distribution.

Let Discrete Random variable = X = # of banquet rooms used during day

X = # Rooms used	# of Days Frequency	Relative Frequency f(x), P(x)
0	2	2/100 = 0.02
1	21	21/100 = 0.21
2	42	42/100 = 0.42
3	27	27/100 = 0.27
4	8	8/100 = 0.08
	Σ = 100	Σ = 1.00



$P(X \leq 1) = 0.02 + 0.21 = 0.23$

$P(X = 2 \text{ OR } X = 3) = 0.42 + 0.27 = 0.69$

$P(X > 0) = 1 - 0.02 = 0.98$

Conclusion:
- people like rooms
- Maybe we don't need 4 rooms
- staffing should anticipate 1-3 rooms being used.

11 Expected Value for Discrete Random Variable

$$E(x) = \mu = \sum X * f(x) = \sum X * P(x)$$

1. mean
2. weighted mean
3. Expected Value
4. Long-Run Ave
5. Does not have to be a value that Random variable can assume
6. central location

* yes, Greek Letters are used for Discrete Random variables.

☆ Remember from ch. 3?

X # of Rooms	f(x) P(X)	X * P(X)
0	0.02	0*0.02 = 0
1	0.21	1 * 0.21 = 0.21
2	0.42	2* 0.42 = 0.84
3	0.27	3 * 0.27 = 0.81
4	0.08	4*0.08 = 0.32

$\sum P(x)$ 1

$\sum X*P(x) = 2.18 = E(X)$

Just a weighted mean

*In Excel use: SUMPRODUCT

12 Standard Deviation for Discrete Random Variable

$$\sigma = \sqrt{\sum (x - E(x))^2 * P(X)}$$

1. measures variation
2. How fairly mean represent data points

Random variable

Expected value weighted mean

Probability for Random variable

X # of Rooms	X − E(X)	$(X - E(x))^2$	$(X - E(x))^2 * P(x)$
0	0 − 2.18 = −2.18	4.7524	0.095048
1	1 − 2.18 = −1.18	1.3924	0.292404
2	2 − 2.18 = −0.18	0.0324	0.013608
3	3 − 2.18 = 0.82	0.6724	0.181548
4	4 − 2.18 = 1.82	3.3124	0.264992

$\Sigma = -0.9$ $\qquad$ $\Sigma = 0.8476$ = Var.

Not add up to Zero BECAUSE we don't have ALL Raw Data (All Numbers)

Standard Deviation $= \sigma = \sqrt{0.8476} = 0.920651943$

Conclusion: Average about 2 rooms used w/ SD of about 1 room.

Back to chapter 4: Adding & Multiplying

An insurance agent has appointments with 4 prospective clients tomorrow. From the past she knows that the probability of making a sale on any one appointment is 1 in 5. What is likelihood that she will sell 3 policies in 4 tries?

Event = sell 3 policies in 4 tries
number of steps or Trials = 4
number of outcomes on any sale attempt = 2
success = sale = S
failure = Not sale = NS
Each attempt at a sale is an independent event
P (sale) = 0.2 = P
P (Not sale) = 1 − 0.2 = 0.8 = (1−P)
X = Discrete Random variable = # of sales in 4 tries
x = 0, 1, 2, 3, 4

# sample points	sample points	Multiply independent events to get P(s.p.)
1	S, S, S, NS	.2 * .2 * .2 * .8 = .0064
2	S, S, NS, S	.2 * .2 * .8 * .2 = .0064
3	S, NS, S, S	.2 * .8 * .2 * .2 = .0064
4	NS, S, S, S	.8 * .2 * .2 * .2 = .0064
		P(3 sales in 4 Tries) = .0256

Define: Probability of Event: Add Prob. of all sample Points!!

Add to get Probability of event

Whole Distribution

14 Build probability Distribution (Discrete) Pg 9

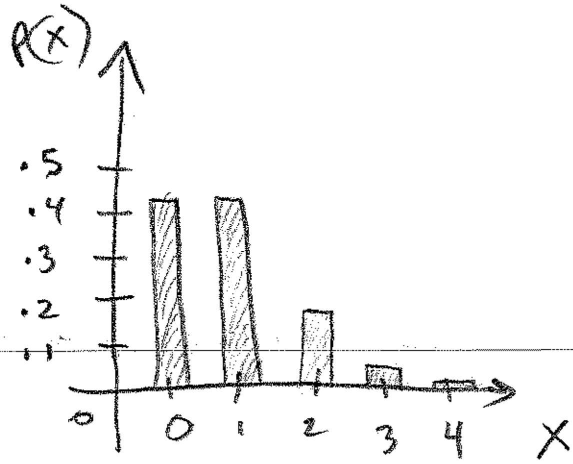
ss = 2*2*2*2 = 16

Possible outcomes sample points	Attempt sale 1st	2nd	3rd	4th	# of sales	Probability of occurrence	
1	S	S	S	S	4	.2*.2*.2*.2 =	.0016
2	S	S	S	NS	3	.2*.2*.2*.8 =	.0064
3	S	S	NS	S	3	.2*.2*.8*.2 =	.0064
4	S	NS	S	S	3	.2*.8*.2*.2 =	.0064
5	NS	S	S	S	3	.8*.2*.2*.2 =	.0064
6	S	S	NS	NS	2	.2*.2*.8*.8 =	.0256
7	S	NS	NS	S	2	.2*.8*.8*.2 =	.0256
8	NS	NS	S	S	2	.8*.8*.2*.2 =	.0256
9	S	NS	S	NS	2	.2*.8*.2*.8 =	.0256
10	NS	S	S	NS	2	.8*.2*.2*.8 =	.0256
11	NS	S	NS	S	2	.8*.2*.8*.2 =	.0256
12	S	NS	NS	NS	1	.2*.8*.8*.8 =	.1024
13	NS	S	NS	NS	1	.8*.2*.8*.8 =	.1024
14	NS	NS	S	NS	1	.8*.8*.2*.8 =	.1024
15	NS	NS	NS	S	1	.8*.8*.8*.2 =	.1024
16	NS	NS	NS	NS	0	.8*.8*.8*.8 =	.4096

$\Sigma = 1$

# of Sales x Random variable	P(x)	P(x)	
0	P(0) =	.4096*1 =	.4096
1	P(1) =	.1024*4 =	.4096
2	P(2) =	.0256*6 =	.1536
3	P(3) =	.0064*4 =	.0256
4	P(4) =	.0016*1 =	.0016

$\Sigma = 1$



Discrete Probability Distributions with n = 4 pi = .2

But there must be an easier way!!

For our sales agent problem we have 16 total possible sample points but we needed only 4 of them:

2	S, S, S, NS
3	S, S, NS, S
4	S, NS, S, S
5	NS, S, S, S

4 total ⇓

How can we calculate this 4?

"sample points"

(15) Number of Experimental outcomes that provide exactly X successes in n Trials

$$\left\{\begin{array}{l}\text{\# experimental outcomes} \\ \text{that have X successes} \\ \text{in n Trials}\end{array}\right\} = \frac{n!}{X!(n-X)!}$$

X = # successes of Random Discrete variable

* we use X instead of n, because X = successes in n trials

n = # Fixed Trials

* we use n instead of N because n = # of Fixed Trials

sample points

$n = 4$
$x = 3$

$$\left\{\begin{array}{l}\text{\# of experimental} \\ \text{outcomes that} \\ \text{have x successes} \\ \text{in n trials}\end{array}\right\} = \frac{4!}{3!(4-3)!} = \frac{\cancel{1}*\cancel{2}*\cancel{3}*4}{\cancel{1}*\cancel{2}*\cancel{3}\,(1)!} = \frac{4}{1} = 4$$

* earlier formula $= \dfrac{N!}{n!(N-n)!}$

N = count of all objects = pop size
n = size of subset = sample size

16 Also Notice from sales agent problem

# sample point Experimental outcomes	sample point	multiply Independent Events to get P(sample point)	
1	S, S, S, NS	.2 * .2 * .2 * .8	= 0.0064
2	S, S, NS, S	.2 * .2 * .8 * .2	0.0064
3	S, NS, S, S	.2 * .8 * .2 * .2	0.0064
4	NS, S, S, S	.8 * .2 * .2 * .2	0.0064

P(3 in 4 Try) = 0.0256
= 0.0256

multiplication can be done in any order

$p = 0.2$ = probability of success

$(1-p) = 0.8$ = probability of Not success

$x = 3$ = # of successes

$n = 4$ = # of Trials

$$4 * 0.2 * 0.2 * 0.2 * 0.8$$

$$4 * 0.2^3 * 0.8^1$$

$$\frac{n!}{x!(n-x)!} \quad p^x * (1-p)^{n-x}$$

Binomial Probability Function

$$\frac{4!}{3!(4-3)!} * 0.2^3 * (1-0.2)^{4-3} = \frac{\cancel{1}*\cancel{2}*\cancel{3}*4}{\cancel{1}*\cancel{2}*\cancel{3}*(1)!} * 0.008 * 0.8^1$$

$$= 4 * 0.008 * 0.8 = 4 * 0.0064 = 0.0256$$

17 Binomial Probability Distribution (Discrete P.D.)

List of all outcomes for a Binomial Experiment (common multi-step experiment that has many useful applications) and the probabilities associated with each experimental outcome (sample point). (P.12)

18 Requirements for Binomial Experiment

These 3 met = Bernoulli process

1. The experiment consists of a sequence of n identical Trials. (Random variable counts the # of successes in a Fixed # of Trials. Fixed # of Trials = n)
2. 2 outcomes are possible on each Trial. one is defined as a "success" and the other is "Not success" or "failure." S or F.
3. Probability of success, denoted as "p" or the greek letter π ("pi"), remains the same on each Trial. $1-p$ does Not change. (Stationary Assumption)
4. The Trials are independent (one does Not affect Next).

Think of sales person loosing enthusiasm... p.221

Examples:

1. Make 4 sales calls. ① $n=4$, ② S = sale, ③ $p=.2$ ④ yes
2. Test with 15 T/F questions. ① $n=15$, ② S = get correct, ③ $p=.5$ ④ yes
3. Flip coin 3 times. ① $n=3$, ② S = H, ③ $p=.5$ ④ yes
4. Drive across Bridge 7 times During Rush Hour Traffic ① $n=7$ ② S = stuck in Traffic ③ $p=.15$ ④ yes
5. Air flight from Oak. to Seattle, 6 Flights per day. ① $n=6$ ② S = late ③ $p=.1$ ④ yes

19 Binomial Probability Function

$$P(x) = f(x) = \frac{n!}{x!(n-x)!} * p^x * (1-p)^{(n-x)}$$

Equals height of column when you plot on column chart

$P(x) = f(x) =$ Probability of x successes in n Trials

$n =$ # of Fixed Trials

$p =$ Probability of success on any 1 Trial

$1-p =$ Probability of failure on any 1 Trial

20 Excel Binomial Probability Function

= BINOM.DIST (number_s, trials, probability_s, cumulative)

number_s = X = Discrete Random variable count # successes

Trials = n = # of Fixed Trials

Probability_s = p = Probability of success.

cumulative = 0 for exactly X $\boxed{P(X=2)}$

1 for less than or equal to $\boxed{P(X \le 2)}$

Example:

For our sales Agent Problem, what is probability of making exactly 3 sales in 4 Attempts, $p = .2$?

$n = 4$
$x = 3$
$p = .2$

$$f(3) = P(3) = \frac{4!}{3!(4-3)!} * .2^3 * .8^{(4-3)} =$$

$$= 4 * .008 * .8 = .0256$$

or

= BINOMDIST (3, 4, .2, 0) = .0256

Binomial?

1) Fixed # Trials? yes.
2) 2 outcomes on each Trial? yes.
3) Probability same each Trial? yes.
4) Events independent? yes.

If $p = .2$, $n = 4$

$\boxed{\text{Find } P(X \le 2) = f(X \le 2)}$

$X = 0$ or $X = 1$ or $X = 2$

ⓐ $P(0 \text{ or } 1 \text{ or } 2) = f(0 \text{ or } 1 \text{ or } 2) = f(0) + f(1) + f(2) =$

$$= \frac{4!}{0!(4-0)!} * .2^0 * .8^{(4-0)} + \frac{4!}{1!(4-1)!} * .2^1 * .8^{(4-1)} + \frac{4!}{2!(4-2)!} * .2^2 * .8^{(4-2)}$$

$$= .4096 + .4096 + .1536$$

$$= .9728 = P(x \le 2) = f(x \le 2)$$

ⓑ $= \text{BINOMDIST}(2, 4, .2, 1) = .9728$

Example:

Find $f(X \ge 3) = P(X \ge 3)$

$p = .2$

$n = 4$

$X = 3$ or $X = 4$

ⓐ $P(X = 3 \text{ or } X = 4) = P(3) + P(4) =$

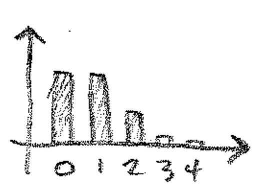
$$= \frac{4!}{3!(4-3)!} * .2^3 * .8^{(4-3)} + \frac{4!}{4!(4-4)!} * .2^4 * .8^{(4-4)}$$

$$= .0256 + .0016$$

$$= .0272 = P(X \ge 3) = f(x \ge 3$$

ⓑ $= 1 - \text{BINOMDIST}(3-1, 4, .2, 1) = .0272$

To go 1 below so 3 is included.



* Excel function always does cumulative from Low End up → And All area = 1

Expected Value & Standard Deviation for the Binomial Distribution

$$E(X) = \mu = \text{Mean} = n * p$$

$$\sigma = \text{Standard Deviation} = \sqrt{n * p * (1-p)}$$

n = # of Fixed Trials
p = Probability of Success

Example:

For our sales Agent problem, what is the mean number of sales she will make in 4 attempts, and what is the standard Deviation?

$n = 4$
$p = .2$

$$E(X) = \text{mean} = 4 * .2 = .8$$

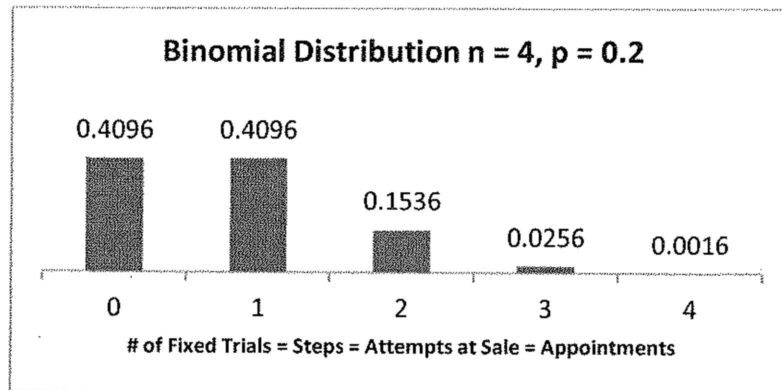
"For every 4 calls she can expect to sell .8 policies. If she has 40 calls planned, she can expect to sell

$$\frac{40}{4} * .8 = 8 \text{ policies}$$

$$\text{Standard Deviation} = \sqrt{.8 * (1-.2)} = .8$$

measures dispersion & could be used to compare to other sales people.

	A	B	C	D	E	F	G	H	I	J	K	L
1	An insurance agent has appointments with 4 clients tomorrow.											
2	From past data, the chance of making a sale is 1 in 5. What is likelihood that she will sell 3 policies in 4 tries?											
3	**Binomial Experiment?**											
4	1	Fixed # of Identical Trials = n			Yes							
5	2	Each trial only results in S or F			Yes							
6	3	p remains the same for each trial			Yes							
7	4	All events are independent			Yes							
8												
9	n = # of Fixed Trials = Appointments				4							
10	p = Prob of Success =				0.2							
11	Random Discrete Variable = x = # Sales in 4 Tries											
12	Mean = μ = E(x) = Expected Value =				0.8		=SUMPRODUCT(A18:A22,B18:B22)					
13	Mean = μ = E(x) = Expected Value =				0.8		=E9*E10					
14	Standard Deviation =				0.64		=SUMPRODUCT((A18:A22-E12)^2,B18:B22)					
15	Standard Deviation =				0.64		=E9*E10*(1-E10)					
16												
17	X	P(x) = f(x)										
18	0	0.4096		=BINOM.DIST(A18,E9,E10,0)								
19	1	0.4096		=BINOM.DIST(A19,E9,E10,0)								
20	2	0.1536		=BINOM.DIST(A20,E9,E10,0)								
21	3	0.0256		=BINOM.DIST(A21,E9,E10,0)								
22	4	0.0016		=BINOM.DIST(A22,E9,E10,0)								
23	Total	1		=SUM(B18:B22)								
24												
25	P(x) = f(x)	P(x) = f(x)	SUM	X								
26	P(x = 3)	0.0256	0.0256	3		=BINOM.DIST(D26,E9,E10,0)				=SUM(B21)		
27	P(x <= 3)	0.9984	0.9984	3		=BINOM.DIST(D27,E9,E10,1)				=SUM(B18:B21)		
28	P(x > 3)	0.0016	0.0016	3		=1-BINOM.DIST(D28,E9,E10,1)				=SUM(B22)		
29	P(x >= 3)	0.0272	0.0272	3		=1-BINOM.DIST(D29-1,E9,E10,1)				=SUM(B21:B22)		
30	P(x < 3)	0.9728	0.9728	3		=BINOM.DIST(D30-1,E9,E10,1)				=SUM(B18:B20)		



p. 16

Binomial Experiment Eample 1:

22

① A flight from Oakland to Seattle occurs 6 times per day. The probability that any one flight is late is 0.1. What is the probability that exactly 2 planes are late? What is the probability that less than 2 planes are late? Is this a binomial experiment? Mean? SD?

Binomial Experiment?

① Fixed # of trials (each count S/F)?
yes ✓ $n = 6$

② Each Trial Independent? yes ✓ (more or less)

③ S/F each time? yes late or Not late

④ Probability of success same each trial?
yes $p = .1$

Variables

$p = .1$ = success = late
$1 - p = 1 - .1 = .9$ = Not late
$n = 6$ = Fixed # of trials

$x = 2$
$x < 2$

$P(2) = \frac{6!}{(6-2)!\,2!} * (.1)^2 * (1-.1)^{(6-2)}$

$P(2) = \frac{6 \cdot 5}{2} * (.01) * (.9)^4$

$P(2) = 15 * .01 * .6561$

$P(2) = .098415$

Probability of exactly 2 flights late is .098415

Excel:
=BINOMDIST(x,n,P,0)

$P(x<2) = P(1) + P(0)$

$P(x<2) = .3543 + .5314$

$P(x<2) = .8857$

The Probability that less than 2 flight will be late is .8857

Excel:
=BINOMDIST(x,n,P,1)

$\mu = np$

$\mu = 6 * .1$

$\mu = .6$

The mean amount late per day is .6 flights

$\sigma = \sqrt{n\pi * (1-\pi)}$

$\sigma = \sqrt{6 * .1 * (1-.1)}$

$\sigma = \sqrt{.6 * .9}$

$\sigma = \sqrt{.54} = .7348$

The Standard Deviation is .7348

Binomial Experiment Example 2:

The probability of sitting in traffic on the West Seattle Bridge during rush hour is .15. During your next 7 rush hour bridge crossings, what is the probability that you will sit in traffic 3 times? 5 or more times? Mean? SD?

Binomial?

Fixed # of trials? yes $n = 7$

$* \uparrow = p$

Independent? yes

S/F? stuck in traffic/Not stuck in traffic

P same each time? yes $P = .15$

$(1-P) = .85$

$$P(3) = \frac{7!}{(7-3)!\,3!} * .15^3 (1-.15)^{(7-3)} = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2} * .003375 * .85^4 =$$

$$= 35 * .003375 * .5220625 = .061662$$

$$P(x \geq 5) = P(7) + P(6) + P(5) = .0011522 + .0000678 + .0000017 = .0012217$$

$$\mu = .15 * 7 = 1.05$$

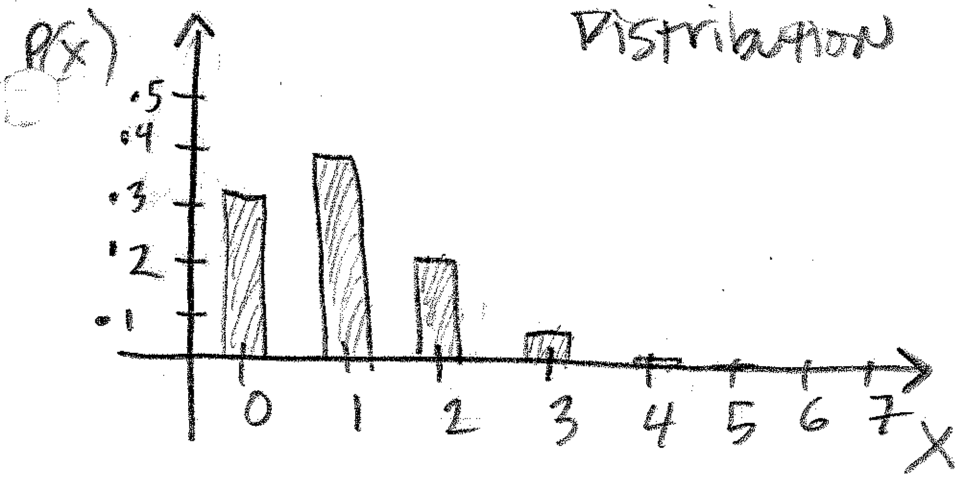
$$\sigma = \sqrt{.15 * 7 * (1-.15)} = .94472181$$

Example:

Binomial Probability Distribution

$n = 7$

$\pi = .15$



# of successes
of "stuck in traffic"

② Plot Table with column chart

① Use BINOM.DIST function to create whole table

X # of times stuck in traffic	P(x)
0	.3206
1	.3960
2	.2097
3	.0617
4	.0109
5	.0012
6	.0001
7	.0000

(23) For Binomial Distribution :

(1) As p (π) approaches $.5$, the Distribution becomes symmetrical

(2) As n gets larger, the Distribution becomes symmetrical.

Exercise 6.2

Check your answers against those in the ANSWER section.

It is known that 60 percent of all registered voters in the 42nd Congressional District are Republicans. Three registered voters are selected at random from the district. Compute the probability that exactly 2 of the 3 selected are Republicans, using:

a. The rules of probability **b.** The binomial formula. c. table

Binomial ?

Fixed trials ? yes $n = 3$

Independent ? yes

S or F ? R or Not yes

π same each time ? yes $\pi = .6$

$n = 3$

$\pi = .6$

$x = 2$

$1 - \pi = .4$

a)

location NR	order of occurance	Probability of occurance	" "
3	R, R, NR	$(.6)(.6)(.4) =$	.144
1	NR, R, R	$(.4)(.6)(.6) =$	.144
2	R, NR, R	$(.6)(.4)(.6) =$	.144

$\Sigma = .432$

P(exactly 2 of 3 selected are Republicans) = .432

b) $P(x) = \frac{n!}{x!(n-x)!} \pi^x (1-\pi)^{n-x}$

$P(2) = \frac{3!}{2!(3-2)!} (.6)^2 (1-.6)^{3-2} = 3(.36)(.4) = .432$

c)

$P(2) = .432$

Poisson Probability Distribution

ⓐ Discrete Probability Distribution used to estimate the number of occurrences over a specified interval of time or space.

- ✪ Good for number of arrivals in a waiting line situation over a certain time period
  - like how many people arrive to stand in line @ Disneyland or at Dick's Hamburgers in Seattle
- ✪ Good for number of repairs needed over a distance of road or pipe.

ⓑ Properties or Requriements for Poisson Experiment

1. Actual Relative Frequency Pattern from past data "fits" Poisson Pattern
2. Variance is about equal to mean
3. Probability of an occurrence is same for any two intervals of equal length.
4. The occurrence or nonoccurrence in any interval is independent of the occurrence/non-occurrence in any other interval.

ⓒ Formula:

$$P(x) = \frac{\mu^x * e^{-\mu}}{x!}$$

x = # occurances
μ = mean (calculated from past data)
e = 2.71828 ← constant

Excel: =POISSON.DIST(x, mean, cumulative)

cumulative = 0 for exact x
1 less than or equal to x

* x = # of occurrences, No upper limit, 0, 1, 2...
But for occurrences like people arriving to get in line...

(d) Poisson Example:

The mean number of people arriving at Dick's Drive In Hamburgers in Seattle at Saturday noon lunch period (Noon to 1 PM) during a 1 minute period is 3.88 people.
Looking at past Data:

1. The mean & variance are equal
2. Probability of a person arriving is the same for any two time periods of equal length.
3. Arrival / Non-arival of a person in any time period is idependent of the arrival / nonarrival of a person in any other time period.
4. M = mean = 3.88 people.

$$P(x) = \frac{M^x * e^{-M}}{x!}$$

$$P(\text{4 arrivals in 1 minute}) = \frac{3.88^4 * 2.71828^{-3.88}}{4!}$$

$= 0.19501$

= BINOM.DIST(4,3.87969,0)

= 0.0206572

P (3 or 4 or 5 arrivals in 1 minute) POISSON.DIST(5,3.87969,1)

− POISSON.DIST(3−1,3.87969,1)

= 0.5473729

Subtract 1:
Have to go back to 2 so you can include 3

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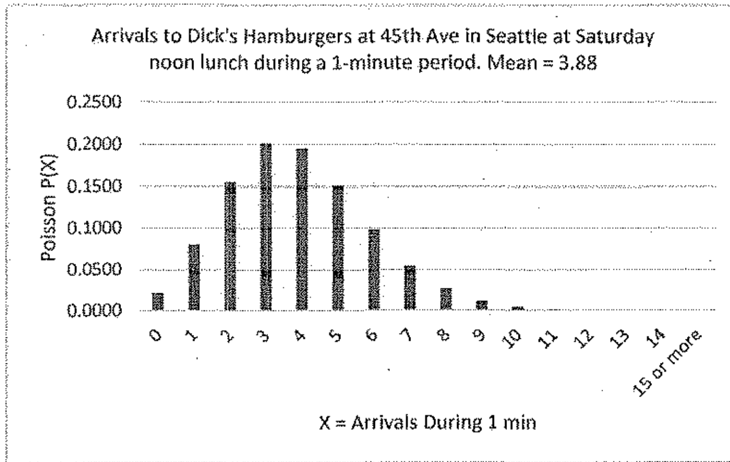
Arrivals to Dick's Hamburgers at 45th Ave in Seattle at Saturday noon lunch during a 1-minute period. Mean = 3.88

Mean	3.88	
Variance	3.75	

X = Arrivals During 1 min	Poisson P(X)	
0	0.0207	=POISSON.DIST(A7,B3,0)
1	0.0801	=POISSON.DIST(A8,B3,0)
2	0.1555	=POISSON.DIST(A9,B3,0)
3	0.2011	=POISSON.DIST(A10,B3,0)
4	0.1950	=POISSON.DIST(A11,B3,0)
5	0.1513	=POISSON.DIST(A12,B3,0)
6	0.0978	=POISSON.DIST(A13,B3,0)
7	0.0542	=POISSON.DIST(A14,B3,0)
8	0.0263	=POISSON.DIST(A15,B3,0)
9	0.0113	=POISSON.DIST(A16,B3,0)
10	0.0044	=POISSON.DIST(A17,B3,0)
11	0.0016	=POISSON.DIST(A18,B3,0)
12	0.0005	=POISSON.DIST(A19,B3,0)
13	0.0001	=POISSON.DIST(A20,B3,0)
14	0.0000	=POISSON.DIST(A21,B3,0)
15 or more	0.0000	=POISSON.DIST(A22,B3,0)
Total	1.0000	=SUM(B7:B22)



$$P(X) = \frac{\mu^X * e^{-\mu}}{X!}$$

Low X	High X		SUM	POISSON	Formula			
	0	P(0)	0.020657	0.020657	0.020657	=SUM(B7)	=POISSON.DIST(B27,B3,0)	=B3^B27*EXP(1)^-B3/FACT(B27)
3	5	P(3 <= No. Arvls. <= 5)	0.547373	0.547373		=SUM(B10:B12)	=POISSON.DIST(B28,B3,1)-POISSON.DIST(A28-1,B3,1)	
2		P(No. Arvls > 2)	0.743730	0.743733		=SUM(B10:B22)	=1-POISSON.DIST(A29,B3,1)	

	A	B	C	D	E	F	G	H	I	J	K	L
1	**Hypergeometric Distribution**											
2	Similar to Binomial Distribution except: 1) the trails are not independent AND 2) the probability of success changes from trial to trial.											
3	**In Excel use: HYPGEOM.DIST(x,n,population_s,number_pop,cumulative)**											
4	x = Number of Successes in Sample = Discrete Random Variable = **sample_s**											
5	n = Trials/Steps in Experiment = Sample Size = **number_sample**											
6	Number Successes in Pop. = **population_s**											
7	Population Size = **number_pop**											
8	**cumulative** = FALSE = 0 = probability mass function. Use to calculate exactly x											
9	**OR**											
10	**cumulative** = TRUE = 1 = probability mass function. Use to calculate <= x (everything from smallest up to and including x)											
12	**Probability of pulling 2 Face Cards in 5 tries?**											
13	Success = Face card						Face Card (Not Ace)					
14	Event = pull 2 face cards (not an ace) in 5 tries											
15	x = Number of Successes in Sample = Discrete Random Variable = **sample_s**						2					
16	n = Trials/Steps in Experiment = Sample Size = **number_sample**						5					
17	Number Successes in Pop. = **population_s**						12					
18	Population Size = **number_pop**						52					
19	**cumulative** = FALSE = 0 = probability mass function. Use to calculate exactly x						0					
20	Number of Sample Points						10		=COMBIN(G16,G15)			
22		**Probability of pulling 2 Face Cards in 5 tries**					0.2509		=SUM(G26:G35)			
23		**Probability of pulling 2 Face Cards in 5 tries**					0.2509		=HYPGEOM.DIST(G15,G16,G17,G18,0)			
25	**Sample Point**	**Step 1**	**Step 2**	**Step 3**	**Step 4**	**Step 5**	**Prob.**					
26	Sample Point 1	F	F	NF	NF	NF	0.02509		=12/52*11/51*(50-10)/50*(49-10)/49*(48-10)/48			
27	Sample Point 2	F	NF	F	NF	NF	0.02509		=12/52*(51-11)/51*11/50*(49-10)/49*(48-10)/48			
28	Sample Point 3	F	NF	NF	F	NF	0.02509		=12/52*(51-11)/51*(50-11)/50*11/49*(48-10)/48			
29	Sample Point 4	F	NF	NF	NF	F	0.02509		=12/52*(51-11)/51*(50-11)/50*(49-11)/49*11/48			
30	Sample Point 5	NF	F	F	NF	NF	0.02509		=(52-12)/52*12/51*11/50*(49-10)/49*(48-10)/48			
31	Sample Point 6	NF	F	NF	F	NF	0.02509		=(52-12)/52*12/51*(50-11)/50*11/49*(48-10)/48			
32	Sample Point 7	NF	F	NF	NF	F	0.02509		=(52-12)/52*12/51*(50-11)/50*(49-11)/49*11/48			
33	Sample Point 8	NF	NF	F	F	NF	0.02509		=(52-12)/52*(51-12)/51*12/50*11/49*(48-10)/48			
34	Sample Point 9	NF	NF	F	NF	F	0.02509		=(52-12)/52*(51-12)/51*12/50*(49-11)/49*11/48			
35	Sample Point 10	NF	NF	NF	F	F	0.02509		=(52-12)/52*(51-12)/51*(50-12)/50*12/49*11/48			

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	A	B	C	D	E	F	G	H	I	J
1	**Example 2:**									
2	Success = Face card				Face Card (Not Ace)					
3	Event = pull 2 face cards (not an ace) in 5 tries									
4	x = Number of Successes in Sample = Discrete Random Variable = sample_s				2					
5	n = Trials/Steps in Experiment = Sample Size = number_sample				5					
6	Number Successes in Pop. = population_s				12					
7	Population Size = number_pop				52					
8	cumulative = FALSE = 0 = probability mass function. Use to calculate exactly x				0					
9										
10	X = Pull Face in 5 tries	P(x)								
11	0	0.25318127	=HYPGEOM.DIST(A11,E5,E6,E7,0)							
12	1	0.42196879	=HYPGEOM.DIST(A12,E5,E6,E7,0)							
13	2	0.25090036	=HYPGEOM.DIST(A13,E5,E6,E7,0)							
14	3	0.06602641	=HYPGEOM.DIST(A14,E5,E6,E7,0)							
15	4	0.00761843	=HYPGEOM.DIST(A15,E5,E6,E7,0)							
16	5	0.00030474	=HYPGEOM.DIST(A16,E5,E6,E7,0)							
17	Total	1	=SUM(B11:B16)							
18										
19	Calculate the probability of getting 0 Face cards OR 1 face card OR 2 face cards in 5 tries:									
20	X	P(X)	SUM							
21	2	P(x<=2)	0.92605042	=SUM(B11:B13)						
22			HYPGEOM.DIST							
23			0.92605042	=HYPGEOM.DIST(A21,E5,E6,E7,1)						
24	**Example 3:**									
25	During the financial crisis of 2008, of the ten biggest banks, only three increase lending after they were given TARP funds of about 2 Billion dolars were given out.									
26	If you took at random sample of 4 of the ten biggest banks at that time, what is the probability that 1 of them would have increased lending, all 3?									
27										
28	Population size =	10								
29	success in pop	3								
30	Sample size	4								
31	Success =	Did lend $								
32	x = Number	# that did lend in sample								
33	x	P(x)								
34	1	0.5								
35	3	0.03333333								

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