# Steps of Hypothesis Testing

1. Develop Null Hypothesis ($H_0$) & Alternative Hypothesis ($H_1$) or ($H_a$)

2. Specify the level of significance ($\alpha$)

3. Collect sample Data & compute value of test statistic ($z$ or $t$), Draw Picture.

## P-value Approach

4. Use value of test statistic to compute p-value
5. Reject $H_0$ if p-value $\leq \alpha$

## Critical Value Approach

4. Use level of significance to determine the critical value and state rejection rule
5. Use the value of the test statistic and the rejection rule to determine whether to reject $H_0$

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Notes: ① If population data is Normally distributed, these methods are exact ($.99 = CI$, $\alpha = .01$, then 99 intervals contain mu, 1 does Not)

② If pop. data is Not Normal, the bigger the $n$, the more exact.

| | |
|---|---|
| Pop Normal | = any $n$ can be used |
| Appox. Normal | $n \geq 15$ |
| Not Normal | $n \geq 30$ |
| outliers | $n \geq 50$ |

# Test statistic (z or t) for Hypothesis Testing About a population mean

| σ KNOWN | σ **NOT** KNOWN |
| --- | --- |
| $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$ | $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$ |

$\mu_0$ = hypothesized mean

$z$ & $t$ = calculated test statistic, used to determine whether to reject the Null Hypothesis. Compare $z$ or $t$ to critical value to make decision, or used to calculate p-value. $z$ & $t$ = number of Standard Errors above/below Hypothesized mean.

$\bar{x}$ = sample mean

$\sigma$ = population standard deviation

$s$ = sample standard deviation

$n$ = sample size

# Test statistic for Hypothesis Tests About A Population Proportion

$$Z = \frac{\bar{P} - P_0}{\sqrt{\frac{P_0 * (1 - P_0)}{n}}}$$

$\bar{P}$ = sample proportion $= \frac{\text{successes}}{n}$

$P_0$ hypothesized pop. proportion

$n$ = sample size

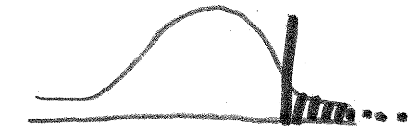
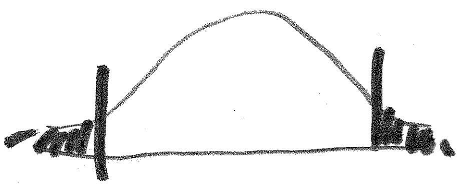
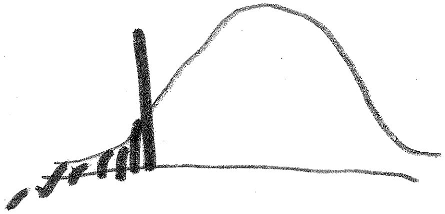
$$SE = \sigma_{\bar{p}} = \sqrt{\frac{P_0 * (1 - P_0)}{n}}$$

must verify:

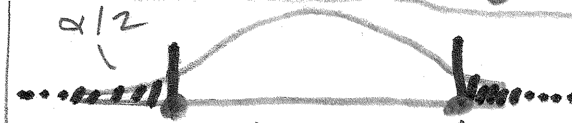
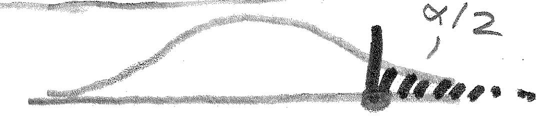
1. Are there fixed # Trials?
2. Are results Independent?
3. Does each Trial result in success or Failure?
4. P stay same on each trial?
5. $n*p > 5$, $n*(1-p) > 5$ } text book assumes true for all problems.

* since exact sampling Distribution of $\bar{P}$ (P bar) is Descrete, Small samples require additional Steps that we will not do in this textbook.

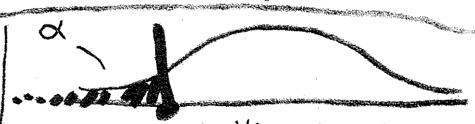
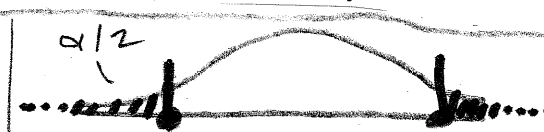
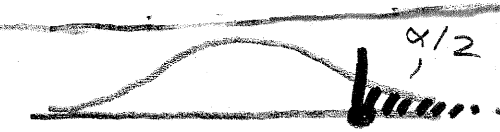
# Excel Functions

| | Z Distribution | t Distribution |
|---|---|---|
| 1 tail to Right upper  | P-value = 1-NORM.S.DIST(Z,1)<br>upper critical value = NORM.S.INV(1-α) | P-value = 1 - T.DIST(t,df,1)<br>or<br>= T.DIST.RT(t,df)<br>upper critical value = T.INV(1-α,df) |
| Two tail  | P-value = NORM.S.DIST(Z,1)*2<br>Z on Low end<br>Low critical value = NORM.S.INV(α/2)<br>upper = NORM.S.INV(1-α/2)<br>or<br>+/- critical values = +/-NORM.S.INV(α/2) | P-value = T.DIST(lower t, df, 1)*2<br>or<br>= T.DIST.2T(upper t, df)<br>Low critical value T.INV(α/2, df)<br>upper = T.INV(1-α/2, df)<br>or<br>+/- critical values = +/- T.INV(α/2, df) |
| 1 tail to Left upper  | P-value = NORM.S.DIST(Z,1)<br>Low critical value = NORM.S.INV(α) | P-value = T.DIST(t,df,1)<br>Lower critical value = T.INV(α,df) |
| When to use: | sigma known and proportions, when 4 tests met. | sigma not known |

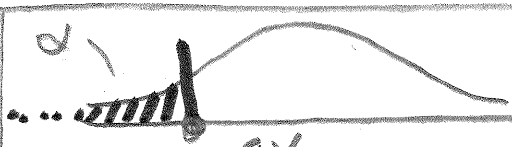
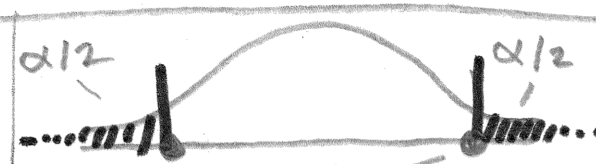
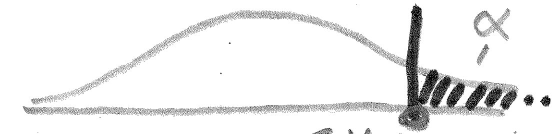
# (Z) Hypothesis Testing Z Distribution (Sigma Known)

| | | | |
|---|---|---|---|
| Test Type |  α, critical value<br>1 Tail Test To Left<br>← Lower Tail |  α/2, critical values<br>← Two-tail Test → |  α/2, critical value<br>1 Tail Test to Right<br>Upper Tail → |
| Hypothesis | $H_0: \mu >= \mu_0$<br>$H_a: \mu < \mu_0$ | $H_0: \mu = \mu_0$<br>$H_a: \mu <> \mu_0$<br>* <> → Not Equal To | $H_0: \mu <= \mu_0$<br>$H_a: \mu > \mu_0$ |
| Test statistic | | $Z = \dfrac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$ | |
| p-value Rejection Rule | | If: p-value $\leq \alpha$<br>Then: Reject Ho, Accept Ha | |
| Excel p-value | =NORM.S.DIST(Z,1) | =NORM.S.DIST(Z,1)*2<br>Z on Low End | =1-NORM.S.DIST(Z,1) |
| Critical value Rejection Rule (For 1 tail) Fail to Reject (for 2-Tail) | If: $Z \leq -Z_\alpha$<br>Then: Reject Ho, Accept Ha<br>$-Z_\alpha$ = critical value (low End) | If: $-Z_{\alpha/2} < Z < Z_{\alpha/2}$<br>Then: Fail To Reject Ho<br>$-Z_{\alpha/2}$ = Low critical value<br>$Z_{\alpha/2}$ = Upper critical value | If: $Z \geq Z_\alpha$<br>Then: Reject Ho, Accept Ha |
| Excel critical value | $-Z_\alpha$ = NORM.S.INV($\alpha$) | +/- critical value = =NORM.S.INV($\alpha$/2) | $Z_\alpha$ = NORM.S.INV(1-$\alpha$) |

# Z Hypothesis Testing Z Distribution (Proportions)

| | | | |
|---|---|---|---|
| Test Type |  1 Tail Test To Left ← Lower Tail |  ← Two-tail Test → |  1 Tail Test to Right Upper Tail → |
| Hypothesis | Ho: $p >= p_0$<br>Ha: $p < p_0$ | Ho: $p = p_0$<br>Ha: $p <> p_0$ | Ho: $p <= p_0$<br>Ha: $p > p_0$ |
| Test Statistic | {Standard Error} $= SE = \sigma_{\bar{p}} = \sqrt{\frac{p_0 * (1-p_0)}{N}}$ | / | $Z = \frac{\bar{p} - p_0}{SE}$ |
| p-value Rejection Rule | If: p-value $\leq \alpha$<br>Then: Reject Ho, Accept Ha | | |
| Excel p-value | =NORM.S.DIST(z,1) | =NORM.S.DIST(z,1)*2<br>z on Low End | =1-NORM.S.DIST(z,1) |
| critical value Rejection Rule (for 1 tail) Fail To Reject (for 2-Tail) | If: $z \leq -z_\alpha$<br>Then: Reject Ho, Accept Ha<br>$-z_\alpha$ = critical value (low End) | If: $-z_{\alpha/2} < z < z_{\alpha/2}$<br>Then: Fail To Reject Ho<br>$-z_{\alpha/2}$ = Low critical value<br>$z_{\alpha/2}$ = Upper critical value | If: $z \geq z_\alpha$<br>Then: Reject Ho, Accept Ha |
| Excel critical value | $-z_\alpha$ = NORM.S.INV(α) | +/- critical value = =NORM.S.INV(α/2) | $z_\alpha$ = NORM.S.INV(1-α) |

# $t$ Hypothesis Testing $t$ Distribution (Sigma Not Known)

| | | | |
|---|---|---|---|
| Test Type | <br>← Lower Tail<br>1 Tail Test To Left | <br>← Two-tail Test → | <br>Upper Tail →<br>1 Tail Test o Right |
| Hypothesis | $H_0: M >= M_0$<br>$H_a: M < M_0$ | $H_0: M = M_0$<br>$H_a: M <> M_0$ | $H_0: M >= M_0$<br>$H_a: M > M_0$ |
| Test Statistic | $t = \dfrac{\bar{X} - M_0}{S/\sqrt{n}}$ | | |
| P-value Rejection Rule | If: P-value $\leq \alpha$<br>Then: Reject Ho, Accept Ha | | |
| Excel P-value | = T.DIST($t$, df, 1) | = T.DIST.2T(upper $t$, df)<br>or<br>= T.DIST(lower $t$, df, 1)*2 | = 1 − T.DIST($t$, df, 1)<br>or<br>= T.DIST.RT($t$, df) |
| Critical value Rejection Rule (for 1 Tail) Accept Rule (for 2-tail) | If: $t \leq -t_\alpha$<br>Then: Reject Ho, Accept Ha<br>$-t_\alpha$ = Low Critical Value | If: $-t_{\alpha/2} < t < t_{\alpha/2}$<br>Then: Fail to Reject Ho<br>$-t_{\alpha/2}$ = Low critical value<br>$t_{\alpha/2}$ = Upper Critical value | If: $t \geq t_\alpha$<br>Then: Reject Ho, Accept Ha<br>$t_\alpha$ = Upper Critical Value |
| Excel Critical Value | $-t_\alpha$ = T.INV($\alpha$, df) | Lower = T.INV($\alpha/2$, df)<br>Upper = T.INV($1-\alpha/2$, df)<br>or<br>+/− = T.INV($\alpha$, df) | $t_\alpha$ = T.INV($1-\alpha$, df) |