Chapter 9: Hypothesis Testing

Hypothesis

- A statement about a population parameter subject to varification.
- Example:

  An official report claims:
  "The yearly salary of full-time realtor is $85,000."

Hypothesis Testing

(1) A statistical procedure that uses sample evidence & probability theory to determine whether a statement about the value of a population parameter:

"should be rejected" = "Reject"

or

"should NOT be rejected" "Fail to Reject"

AND

(2) Make a concluding statement about the population parameter based on sample evidence.

Example 1

Statement from official Report:

"The yearly salary earned by full-time realtors is \$85,000"

Researcher believes:

Realtors make more than \$85,000

---

1. If we take a random sample & get $\bar{x} = \$88,595$
2. we must decide if sampling Error of $88,595 - 85,000 = 3,595$ is acceptable.

Is the difference \$3,595

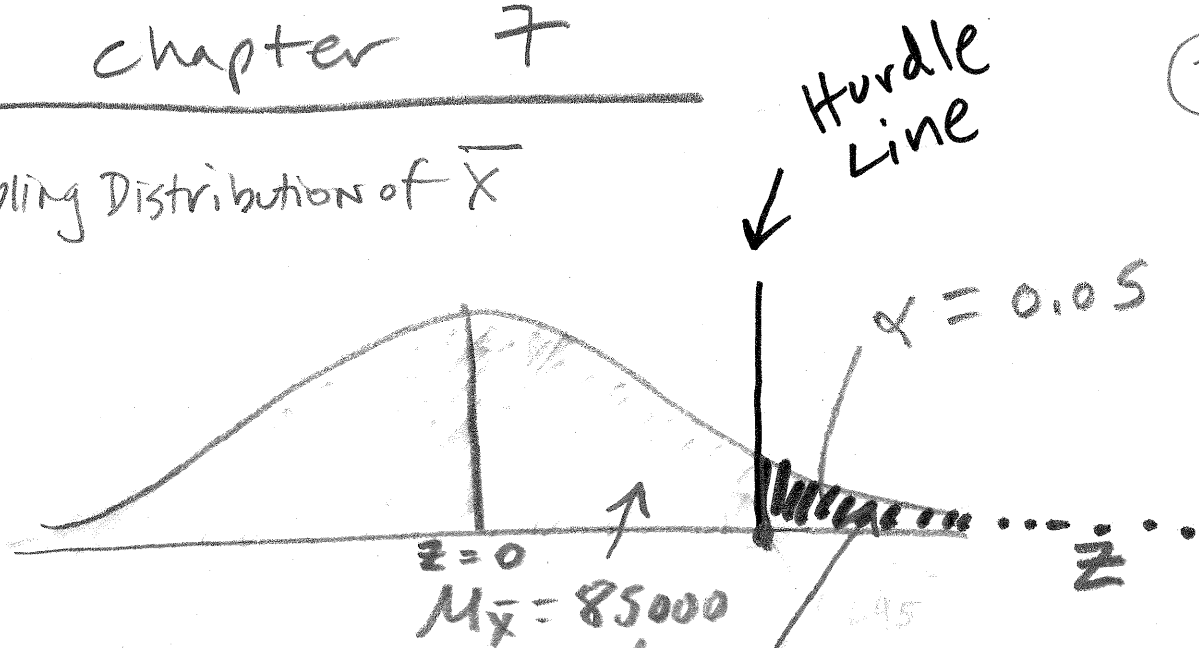
"Statistically significant"

or

"Statistically insignificant"

chapter 7

Sampling Distribution of $\bar{X}$



if our sample evidence provided $\bar{x} = 86,000$

Then original claim of 85,000:

"should not be rejected"

"original claim seems reasonable"

if sample evidence provide $\bar{x} = 88,595$

Then original claim of 85,000:

"should be rejected"

"original claim seems unreasonable"

we will use something called a "Test statistic" (z or t)

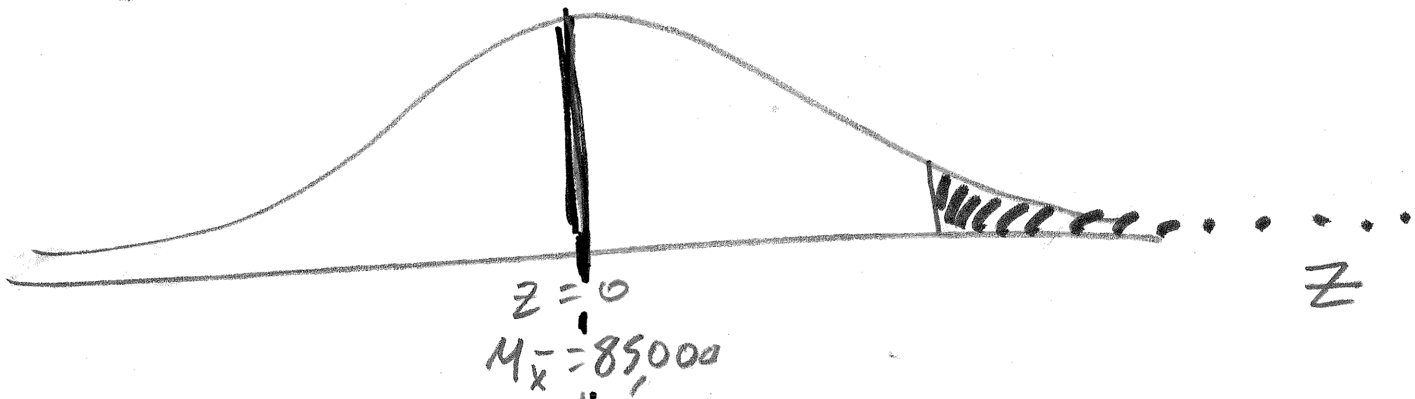
& compare it to our

Hurdle Line.

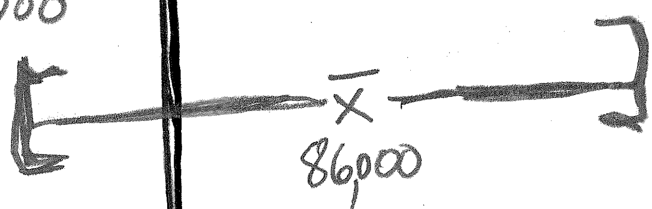
"Test statistic" = # standard Deviations above or below

Chapter 8

sampling Distribution of $\bar{x}$



if our sample evidence provided $\bar{x} = 86,000$



Then because interval contains 85,000, original claim:

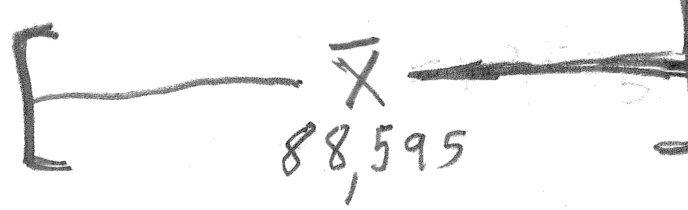
"should Not be rejected"

or

"original claim seems reasonable."

if our sample evidence provided $\bar{x} = 88,595$

Then because interval does not contain 85,000,



original claim:

"should be rejected"

or

"original claim seems unreasonable"

other examples of "statements about a value of a population parameter" that we can test:

Is the new contribution solicitation letter more effective than the old letter, which got 15% contributions?

Is the manufacturer's claim that 16 oz. of catsup is in each bottle?

Is the average wait time in line at Mc Burger's Resturant less than 3 minutes?

Is the new machine faster than the old one?

Steps of Hypothesis Testing

1. Develop Null Hypothesis (H_0) & Alternative Hypothesis (H_1 or H_a)

2. specify the level of significance (α)

3. collect sample Data & compute value of test Statistic (z or t), Draw Picture.

P-value Approach

4. use value of test statistic to compute p-value
5. Reject H_0 if p-value $\leq \alpha$

Critical value Approach

4. use level of significance to determine the critical value and State rejection rule

5. use the value of the test statistic and the rejection rule to determine whether to reject H_0

Notes: ① If population data is Normally distributed, these methods are exact (.99 = CI, α = .01, then 99 intervals contain mu, 1 does Not)

② If pop. data is Not Normal, the bigger the n, the more exact.

Pop Normal = any n can be used
Appox. Normal $n \geq 15$
Not Normal $n \geq 30$
outliers $n \geq 50$

Step 1

Develop Null Hypothesis (H_o) & Alternative H_o (H_a)

Null Hypothesis = H_o

The hypothesis tentatively assumed true in the hypothesis testing procedure. Based on sample evidence we either

"Reject H_o"

or

"Fail to Reject H_o"

Alternative Hypothesis = H_a

Based on sample evidence the hypothesis concluded to be true if the null hypothesis is rejected. we either

"Fail to Reject H_o"

or

"Reject H_o, accept H_a"

Research Hypothesis

Start with alternative hypothesis and make it the conclusion the researcher hopes to support.

Example:

Realtors make more than \$85,000

Validity of a Claim

Assumption that population parameter is true

Example:

Is catsup bottle filled with 16 oz.?

Decision Making

Choose between 2 things.

Example:

Should we accept box of shipped products, yes or no.

Notes about Step 1

- Developing H_0 & H_a can be difficult & takes practice to learn how to do.

- The context, situation, or point of view will help determine the correct H_0 & H_a

(1) Research Hypothesis → usually start with → H_a

Example: "Realtors make more than \$85,000"?

$H_0: \mu \leq 85,000$

$H_a: \mu > 85,000$

(2) Validity of Claim → usually start with → H_0

Example: "Is catsup bottle filled with 16oz."?

Consumer's point of view → Assume bottle is filled with 16oz. But if not, take action

$H_0: \mu \geq 16$ oz.

$H_a: \mu < 16$ oz.

Consumer cares about this

or

Manufacturer's point of view → Manufacture cares about filled too much or too little

$H_0: \mu = 16$ oz.

$H_a: \mu \neq 16$ oz.

(3) Decision Making → (choose between 2 things) → H_0 or H_a

step 1 Develop H_0 & H_a

original statement:

The yearly salary earned by full-time realtor is \$85,000 (sigma = \$12,549)

competing statement:

Researcher believes realtors make more than \$85,000

1st write this:

colon says "Here is Hypothesis"

$H_0 : \mu$

$H_a : \mu$

2nd: Use "more than \$85,000" to determine comparative operator for H_a

$H_0 : \mu$

$H_a : \mu > 85{,}000$

3rd: once you know comparative operator for H_a, put opposite comparative operator <u>and</u> equal sign for H_0.

$H_0 : \leq 85{,}000$

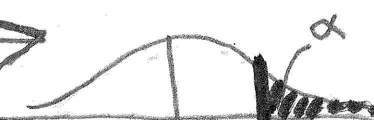
$H_a : > 85{,}000$

4th H_0 ALWAYS get = sign

5th H_a comparative operator

Always tells you which way test is

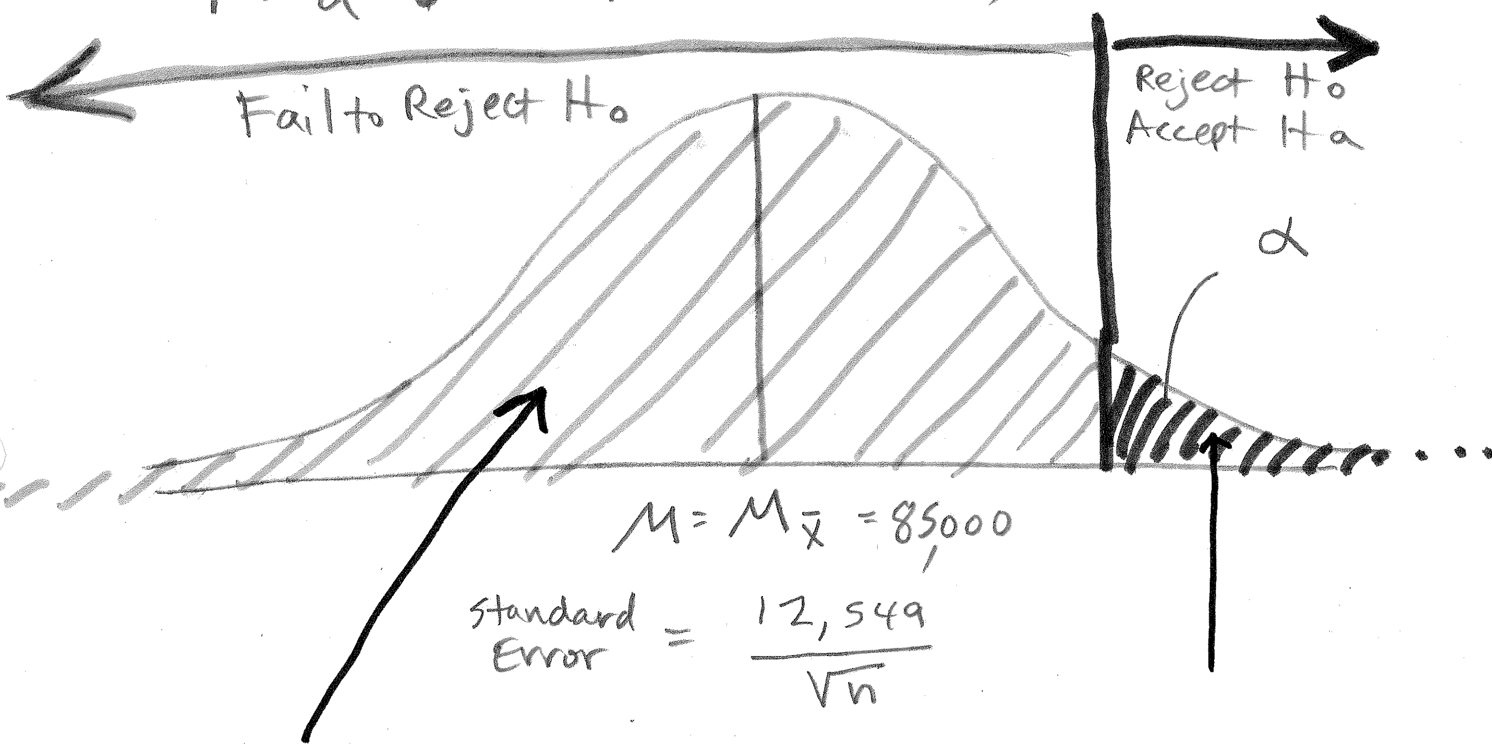
1-tail to Left.



Step 1 Continued..

$$H_0 : \mu \leq \$85,000$$

$$H_a : \mu > \$85,000$$



If we get $\bar{X}$ here we say:

"Based on the sample evidence, we fail to reject Ho. There is little statistical evidence that the mean salary is more than \$85,000." * Don't say Ho is TRUE

If we get $\bar{X}$ here we say:

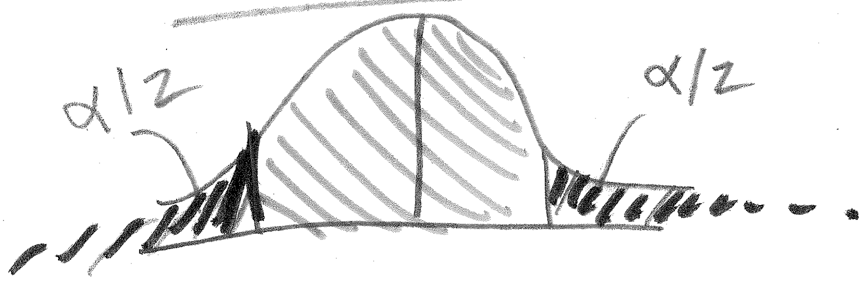
"Based on the sample evidence, we reject Ho and accept Ha. There is statistical evidence that the mean salary is more than \$85,000."

- careful in our language because we are taking samples.
- only two possible outcomes.

Note* Book says "Do not reject Ho". I say "Fail to reject Ho".

3 possible Forms of H_o & H_a

1

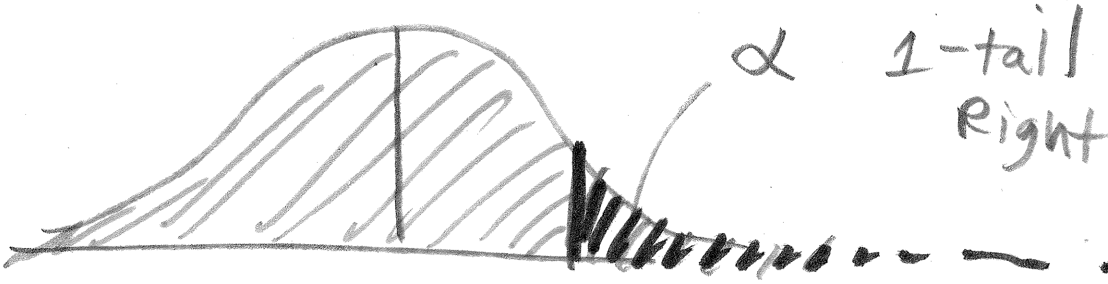


2-tail

$$H_o: \mu = \mu_o$$

$$H_a: \mu \neq \mu_o$$

2



1-tail to Right

$$H_o: \mu \leq \mu_o$$

$$H_a: \mu > \mu_o$$

3

1-tail to Left

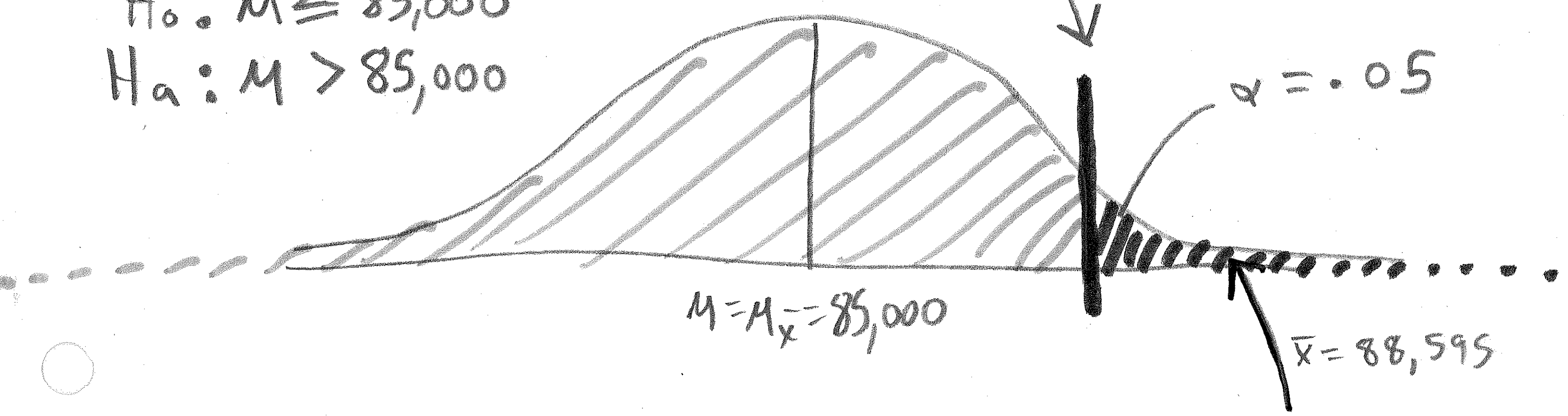
$$H_o: \mu \geq \mu_o$$

$$H_a: \mu < \mu_o$$

$\boxed{\text{Level of significance} = \text{Alpha} = \alpha}$ ← procedural definition

1. α determines the cut off point, which is the threshold used to decide whether the test statistic is statistically significant

$H_0: \mu \leq 85{,}000$
$H_a: \mu > 85{,}000$



If we get $\bar{x} = 88{,}595$ and it is out here, this is statistically significant and we reject H_0 and accept H_a.
$\mu > 85{,}000$.

- If we choose $\alpha = 0.05$, we are taking a 5% risk of rejecting H_0 even though it was true.
- Because we choose α, we can say we are doing a "Significance Test"

Picture Examples for Level of significance where $\mu = 85{,}000$

Alpha

2-tail

If Question was "is $\mu \neq 85{,}000$".

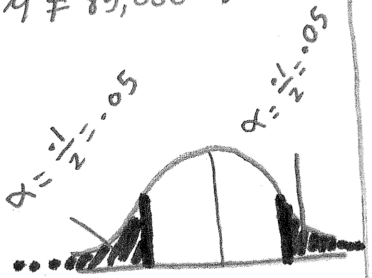
1-tail to Left

If Question was "is $\mu < 85{,}000$".

1-tail to Right

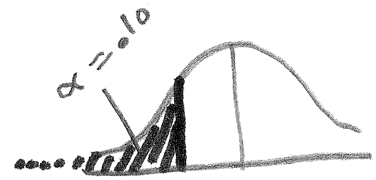
If Question was "is $\mu > 85{,}000$".

$\alpha = .10$



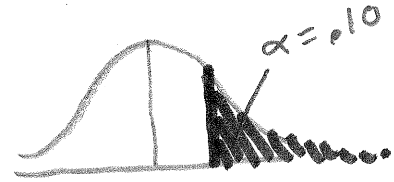
$H_0: \mu = 85000$

$H_a: \mu \neq 85000$



$H_0: \mu \geq 85000$

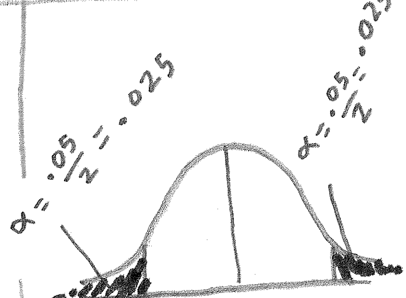
$H_a: \mu < 85000$



$H_0: \mu \leq 85000$

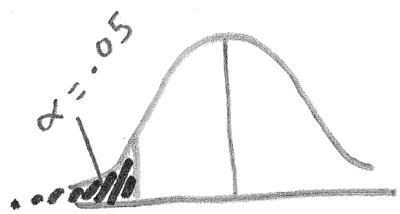
$H_a: \mu > 85000$

$\alpha = .05$



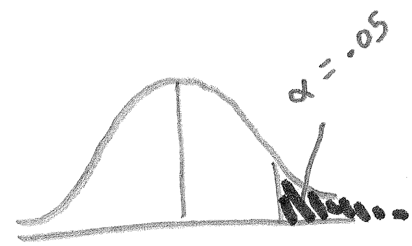
$H_0: \mu = 85000$

$H_a: \mu \neq 85000$



$H_0: \mu \geq 85000$

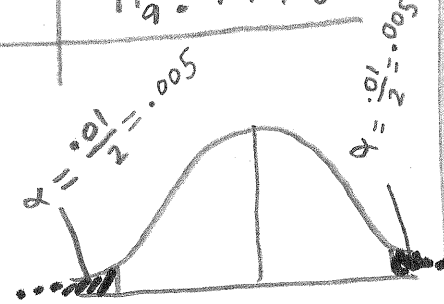
$H_a: \mu < 85000$



$H_0: \mu \leq 85000$

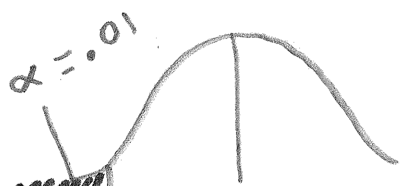
$H_a: \mu > 85000$

$\alpha = .01$



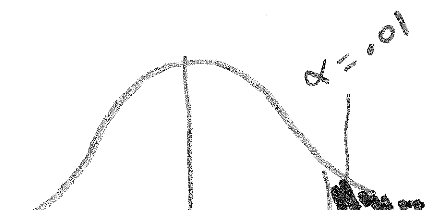
$H_0: \mu = 85000$

$H_a: \mu \neq 85000$



$H_0: \mu \geq 85000$

$H_a: \mu < 85000$



$H_0: \mu \leq 85000$

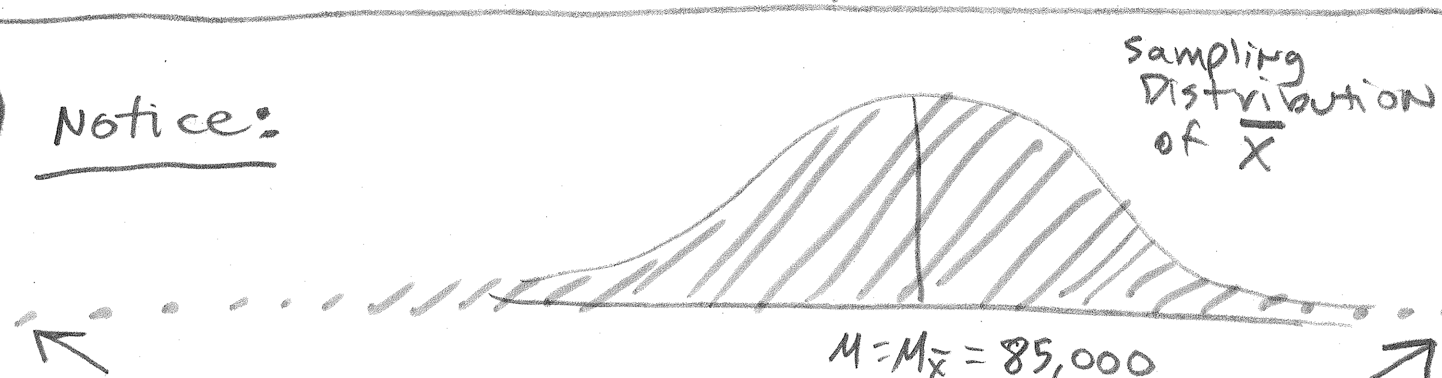
$H_a: \mu > 85000$

These picture examples show the 3 possibilities at 3 different alpha values.

Step 2 Specify Level of Significance (α)

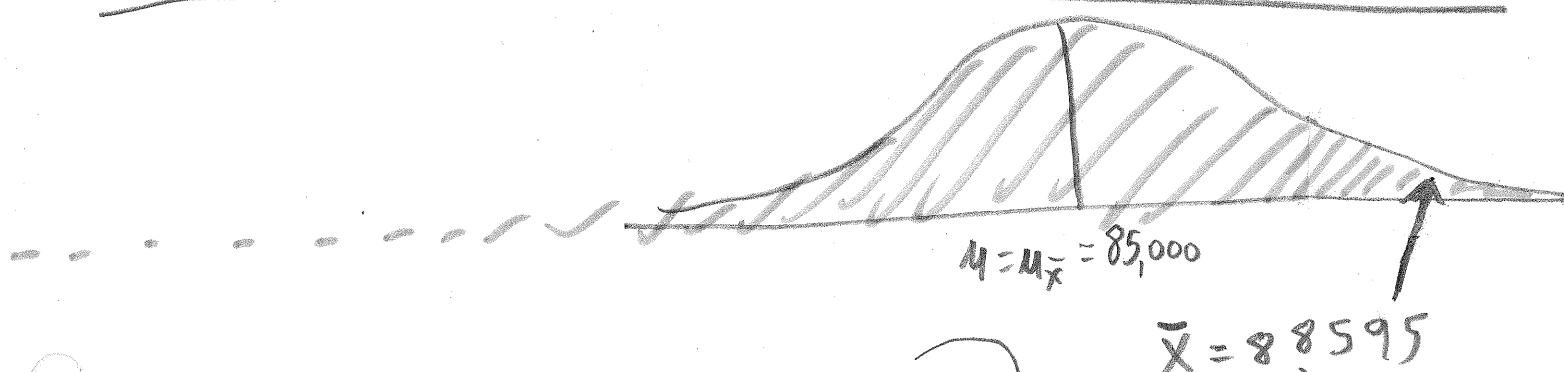
- Because hypothesis Testing is based on Sample Data, we must allow for the possibility of errors.
- unless we test whole population, you run risk of error.

(1) **Notice:**

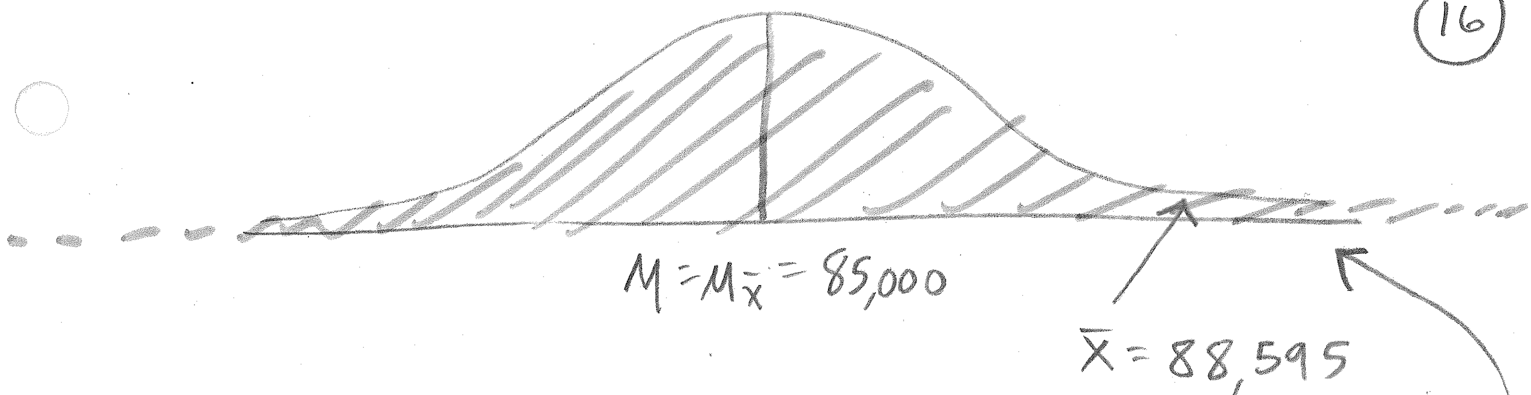


This is entire distribution of possible $\bar{X}$ values

(2) It is possible to take a sample & get an $\bar{X} = 88{,}595$ that is just sample error:



So we could get $\bar{X} = 88{,}595$
But $\mu = 85{,}000$ is still true



It is very unlikely that we could get an $\bar{x}$ out here.

$\boxed{\text{But...}}$

It is a possibility that our particular sample just happened to have a lot of big numbers in it so we got:

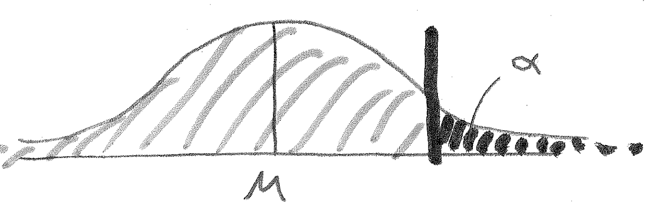
$$\bar{x} = \$88{,}595$$

while the full population mean was still:

$$\mu = \$85{,}000$$

$\boxed{\text{So...}}$

Because we can't usually test the whole population, we have to pick a cut off point and reject the original statement (H_0) if we go beyond that point.



$\boxed{\rightarrow}$ This means we take a risk of an error...

Define:

↙ definition (what it is)

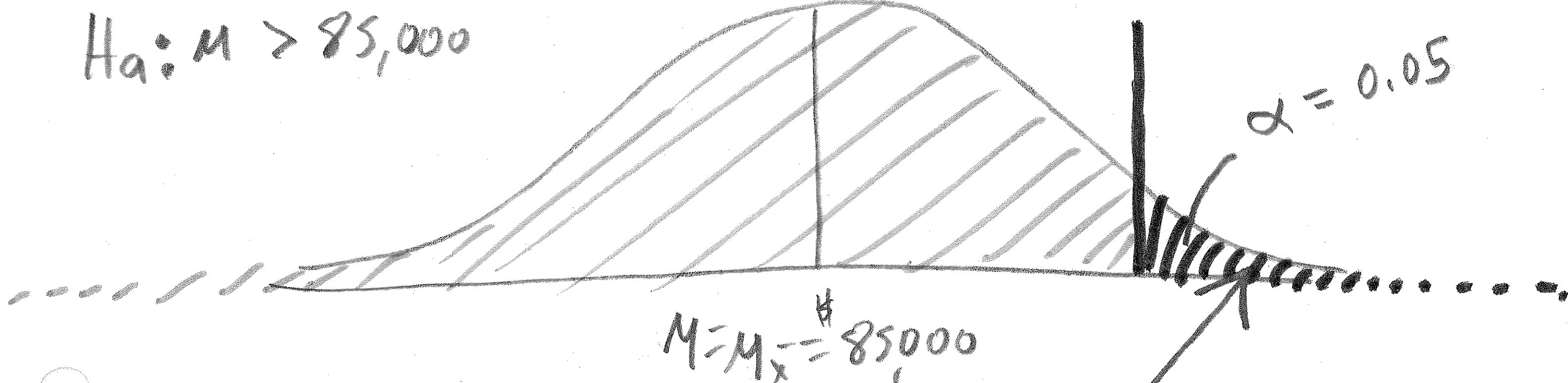
$\boxed{\text{Level of significance} = \text{Alpha} = \alpha = \text{Type I Error}}$

Probability (Risk) of rejecting H_0 even though it is True (as an equality).

example: $\mu = 85,000$

$H_0: \mu \leq 85,000$

$H_a: \mu > 85,000$



If we get $\bar{x} = \$88,595$ & Reject H_0, But H_0 ($\mu = 85,000$) is actually TRUE this is:

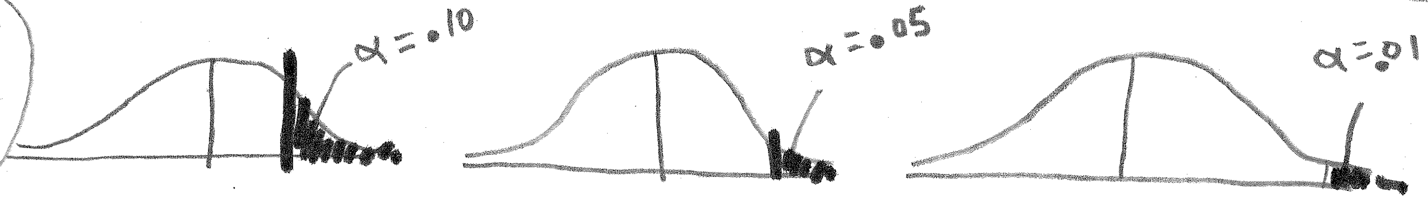
"Type I Error"

"Innocent but found Guilty"

- ✪ Most of the time $\bar{X} = 88,595$ will lead to correct conclusion (about 95 out of 100).
- ✪ "Innocent but found Guilty" Error 5 out of 100 times.

Designer of Hypothesis Test selects alpha and thereby controls the probability of a Type I Error:

especially as n gets big



About: Error 10 out of 100 times | Error 5 out of 100 times | Error 1 out of 100 times

→ As you Move → this way → you reduce α →
→ and Reduce → the risk of Type I Error

Example: ① Drug company may want to set α very small, so they are sure New Drug really works

② Quality Control may want to set α low so they are more sure that the quality is high.

Selecting α

- If cost of making Type I Error is high, choose small α
- If cost of making Type I Error is Not high, choose bigger α

Errors & Correct Conclusions in Hypothesis Testing

		Actual Population Condition	
		H_0 TRUE (H_a FALSE)	H_0 FALSE (H_a TRUE)
Conclusion (based on Sample)	Reject H_0, Accept H_a	Type I Error Alpha	Correct Conclusion
	Fail To Reject H_0	Correct Conclusion	Type II Error Beta

Type I Error

H_0 True,
But we
Reject H_0

Alpha $= \alpha$

"I innocent but found Guilty"

Type II Error

H_0 False,
But we
Fail to Reject H_0

Beta $= \beta$

"Guilty but found innocent"

Because we control for α we can say "Accept H_a" in our conclusion

Because we don't control for β (In this textbook) we <u>can't</u> say "Accept H_0"

Other Wording:

		Actual Population Condition	
		H_0 TRUE (${H_a}$ FALSE)	**H_0 FALSE (H_a TRUE)**
Conclusion (based on Sample)	**Reject H_0 ,Accept H_a**	Type I Error Alpha (Level of Significance) **"False Positive"**	Correct Conclusion **"True Positive"**
	Fail To Reject H_0	Correct Conclusion **"True Negative"**	Type II Error Beta **"False Negative"**

False Positive: Our Alternative (H_a) was selected even though the Null (H_0) was true.

False Negative: Our Alternative (H_a) was not selelcted even though the Null (H_0) was false.

Step 3

Collect sample Data, calculate value of Test statistic (z or t)

Example 1:

Step 1

$H_0: \mu \leq 85{,}000$ Annual Realtor Salary

$H_a: \mu > 85{,}000$ Annual Realtor Salary

Step 2

α = Type I Error = 0.05 = (Cost of error Not too big)

Step 3

we go out & get a sample

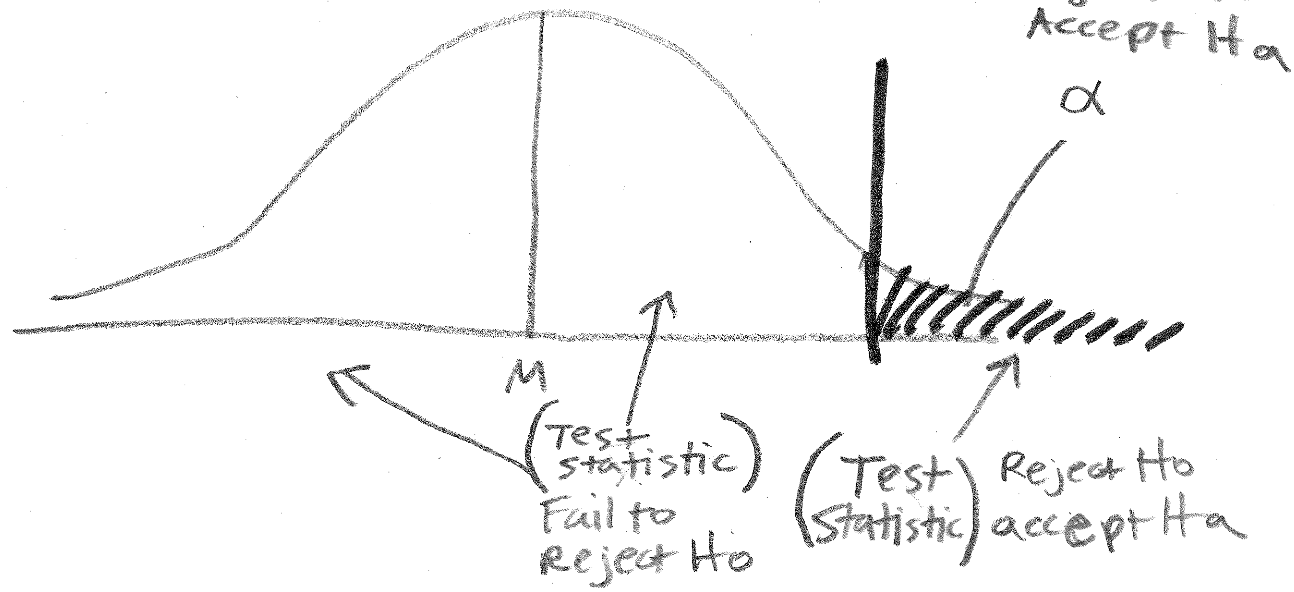
$\bar{X} = 88{,}595$

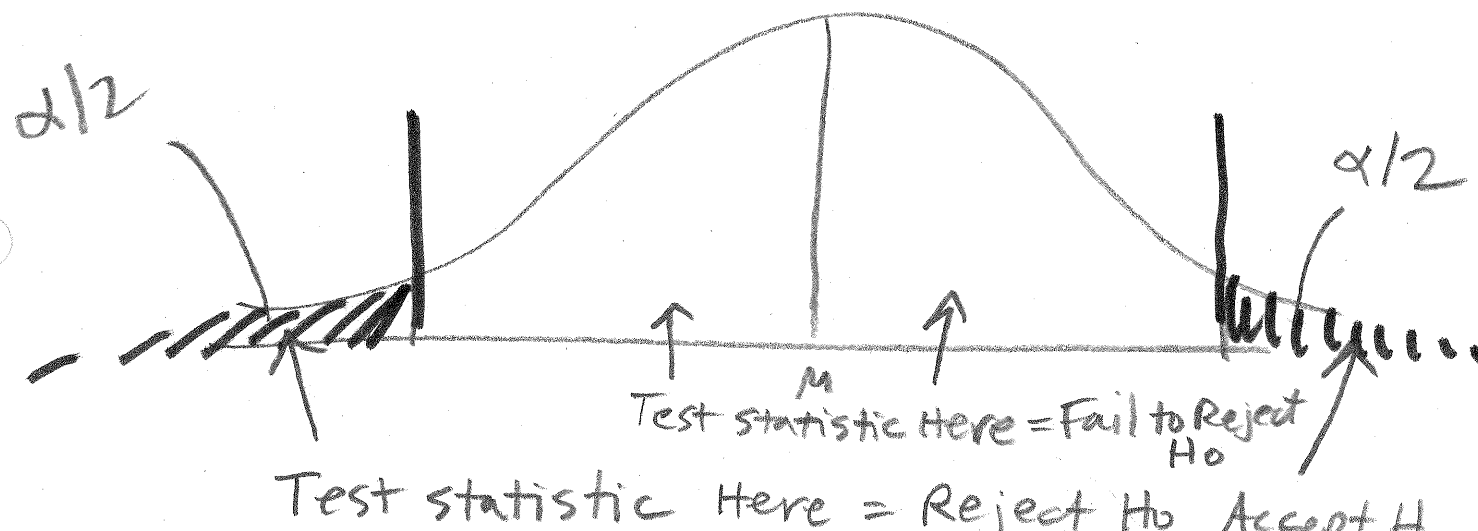
sigma Known = 12,549

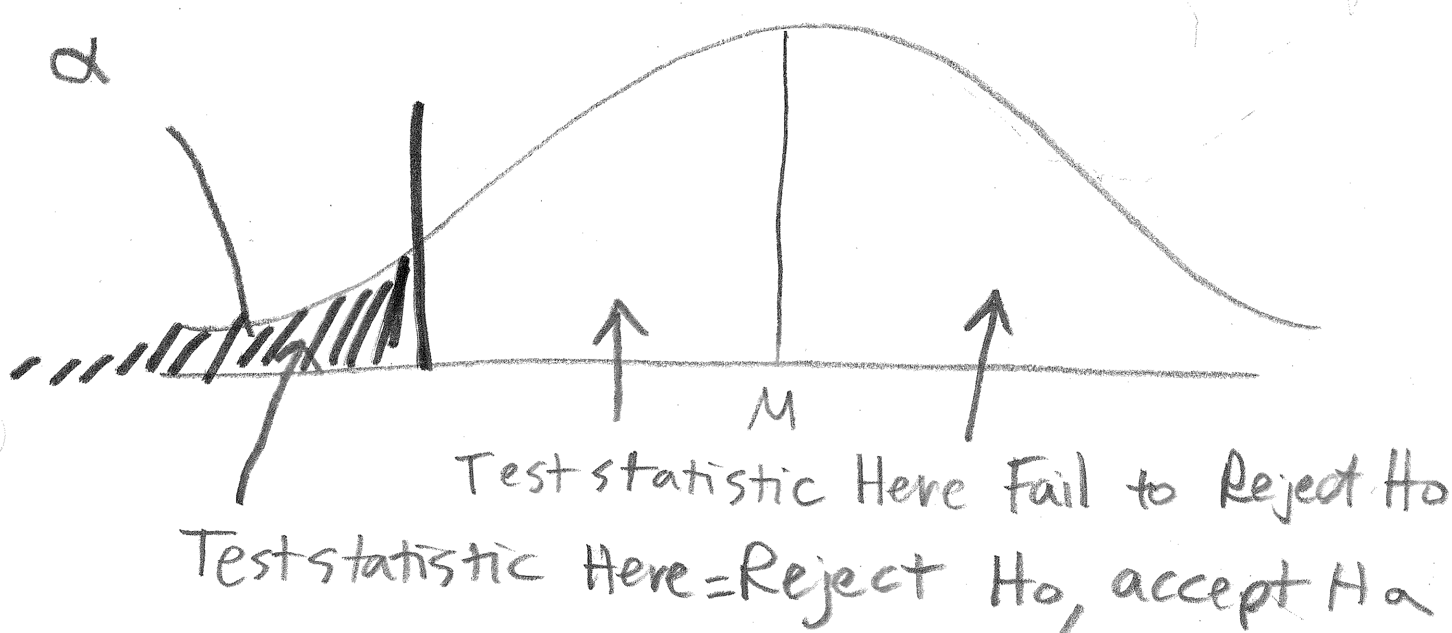
$n = 36$ (Big enough to accomidate some outlier salaries)

But we Need test statistic

1. α determines cut off point
2. test statistic beyond cut off point, then we reject Ho Accept Ha





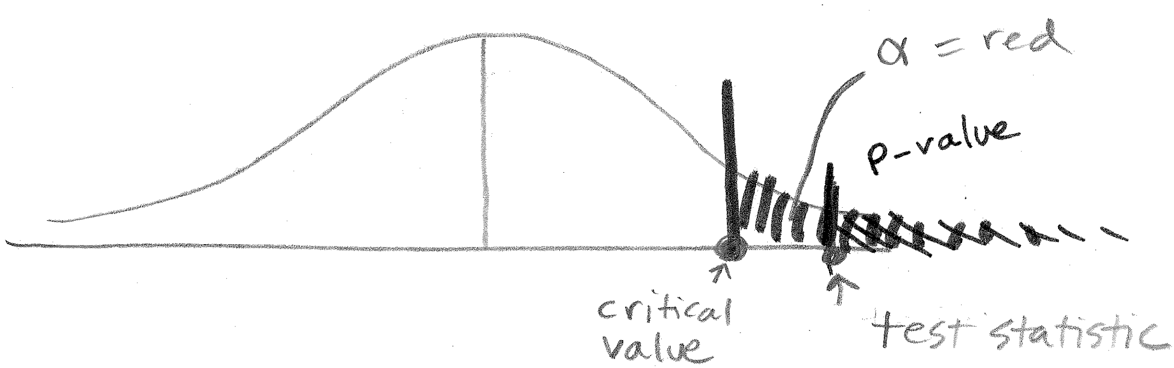


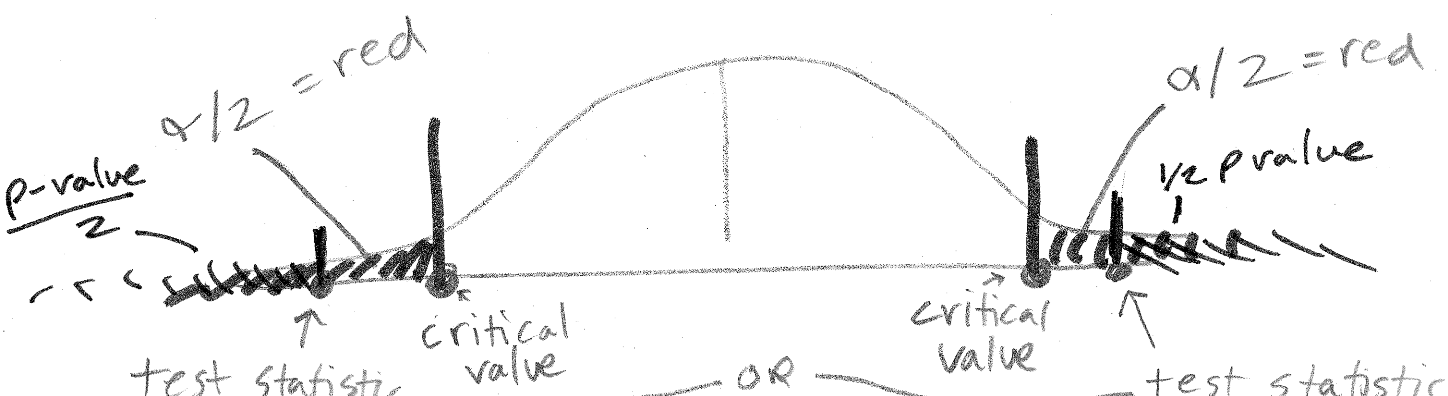
2 Methods for determining whether test statistic is past cut off ("statistically significant")

① p-value : p-value $\leq \alpha$
Reject H_0, Accept H_a

or

② critical value : If test statistic is past critical value
Reject H_0, Accept H_a





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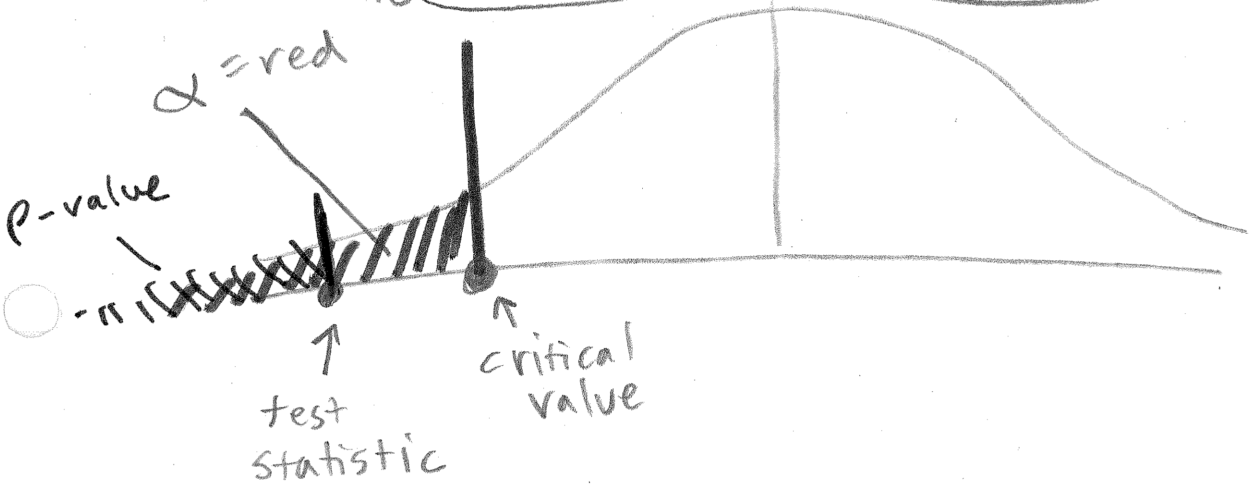
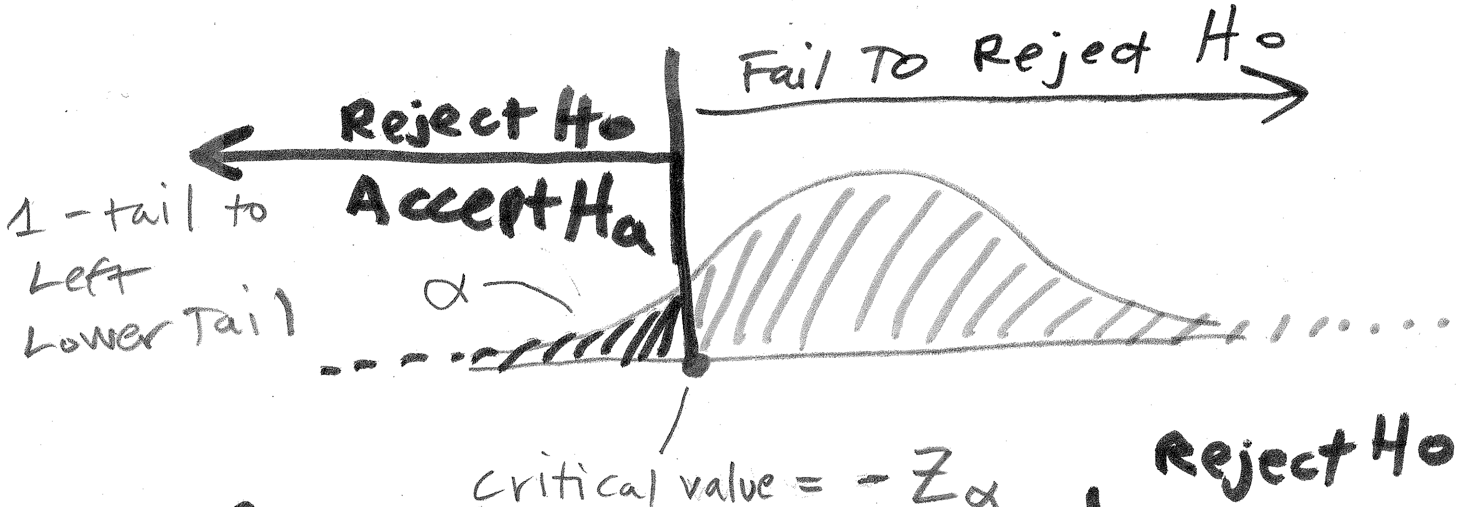
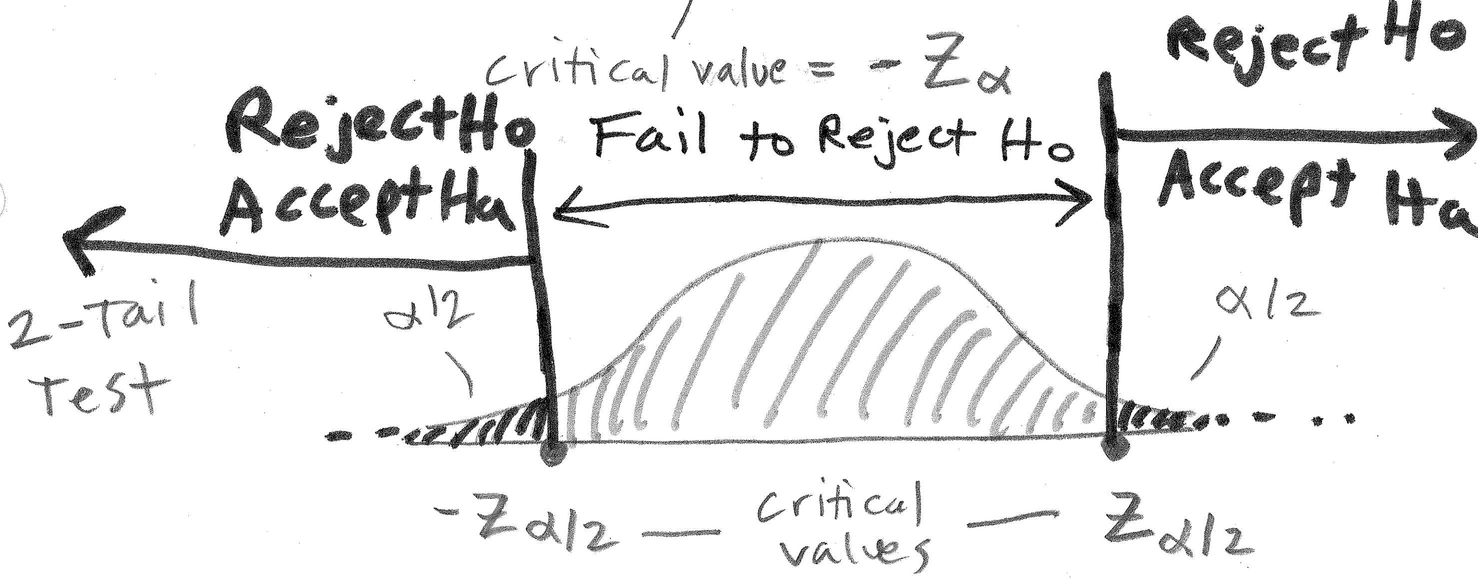
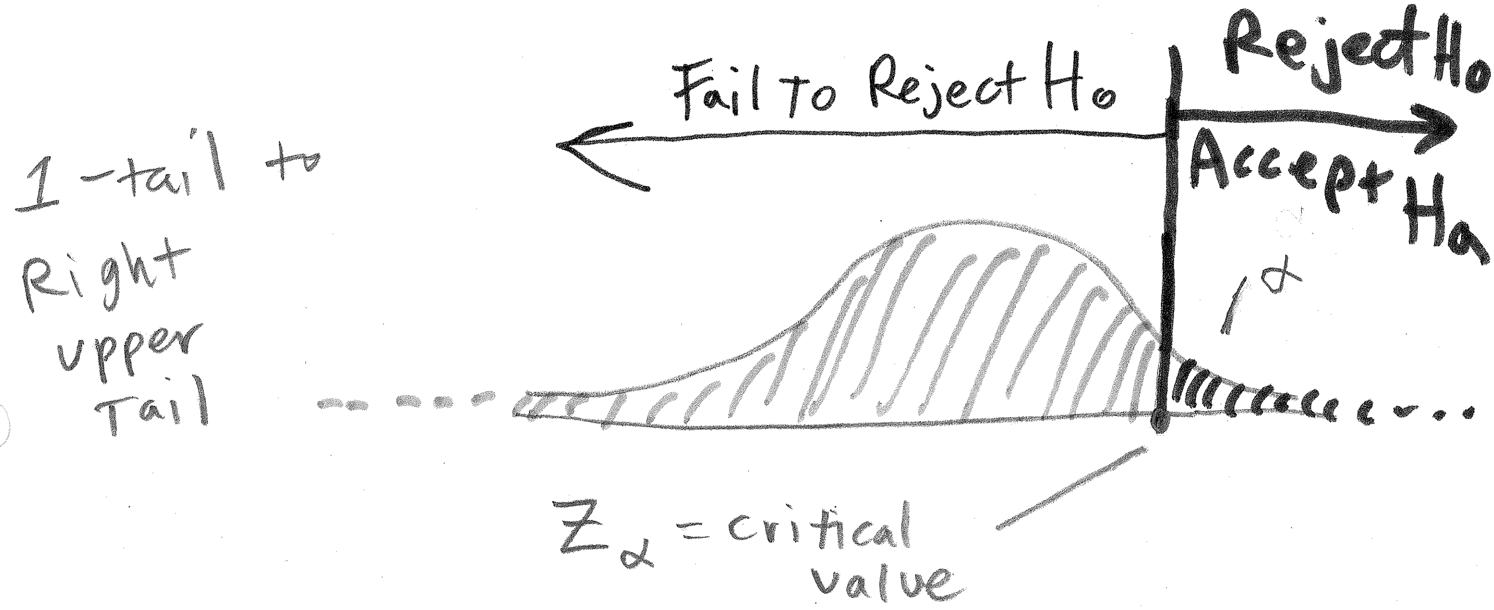


critical value

Hurdle point that determines if the Null Hypothesis is Rejected & the Alternative Hypothesis is Accepted. calculate critical value based on Alpha



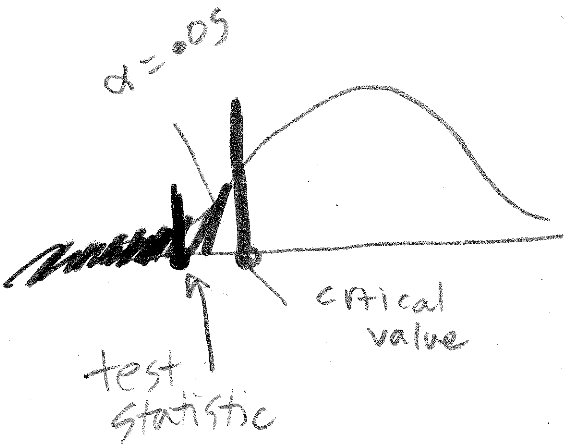




p-value "observed level of significance"

Probability of getting the test statistic value or worse (less or more).

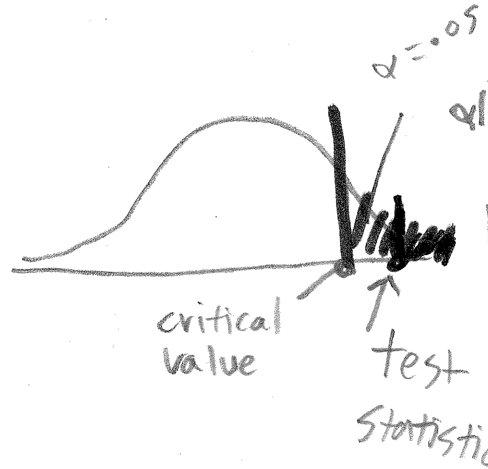
1 tail to Left



p-value = probability of getting test statistic or less

p-value * 1

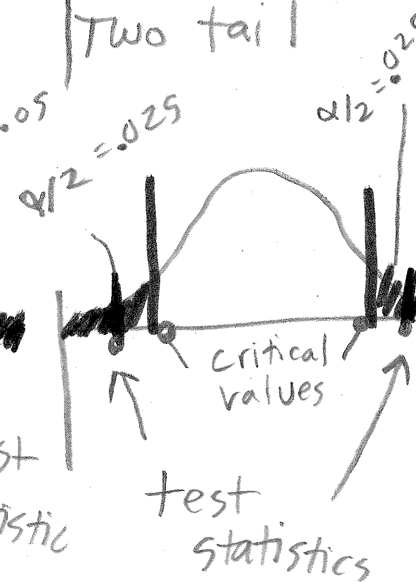
1 tail to Right



p-value = probability of getting test statistic or more

p-value * 1

Two tail



p-value from 1 side = probability of getting Test Statistic or worse (less or more)

then,

p-value * 2

Rejection Rule: p-value $\leq \alpha$, Reject Ho, Accept Ha

Interpreting p-value

p-value > 0.10	Insufficient evidence to say Ha True
0.05 < p-value ≤ 0.10	weak evidence to say Ha True
0.01 < p-value ≤ 0.05	strong evidence to say Ha True
p-value ≤ 0.01	overwhelming evidence to say Ha True

Advantage of p-value (over critical value) is that it tells you how significant the results are:

① what probability of getting a test statistic or worse (less or more)

② what the Type I Error Rate is, like we got \$88,595 for $\bar{x}$ & that value would appear as a True sample mean w/ $\mu = 85{,}000$ 4 in 100 times.

Test statistic (z or t) for Hypothesis Testing About a population mean

σ KNOWN	σ NOT KNOWN
$z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$	$t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$

μ_0 = hypothesized mean

z & t = calculated test statistic, used to determine whether to reject the Null Hypothesis. Compare z or t to critical value to make decision, or used to calculate p-value. z & t = number of Standard Errors above/below Hypothesized mean.

$\bar{x}$ = sample mean

σ = population standard deviation

s = sample standard deviation

n = sample size

Test statistic for Hypothesis Tests About A Population Proportion

$$Z = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0 * (1-p_0)}{n}}}$$

$\bar{p}$ = sample proportion $= \frac{\text{successes}}{n}$

p_0 hypothesized pop. proportion

n = sample size

$$SE = \sigma_{\bar{p}} = \sqrt{\frac{p_0 * (1-p_0)}{n}}$$

must verify:

1. Are there fixed # Trials?
2. Are results Independent?
3. Does each Trial result in success or Failure?
4. p stay same on each trial?
5. $\left.\begin{array}{l} n*p > 5 \\ n*(1-p) > 5 \end{array}\right\}$ text book assumes true for all problems.

* since exact sampling Distribution of $\bar{p}$ (Pbar) is Descrete, Small samples require additional Steps that we will not do in this textbook.

Q: When are we allowed to use t Distribution?

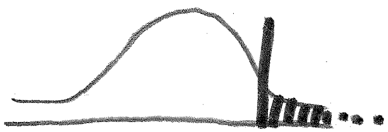
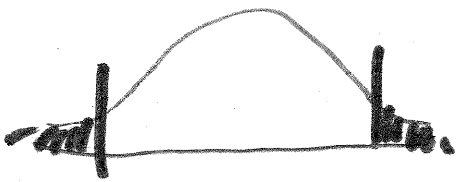
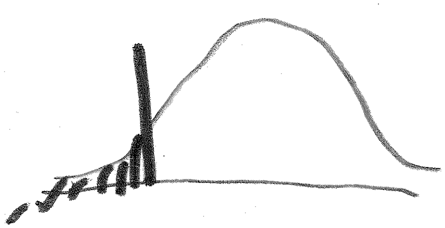
A: When population distribution in normally distributed or near normal, or n is sufficicently large enough

1) If pop distribution is normal or near normal, smaller than 30 sample size may be used
2) If pop distribution is not normal, n >= 30 usually adequate
3) If pop distribution is highly skewed or has outliers, n >= 50 should be used

Notes:

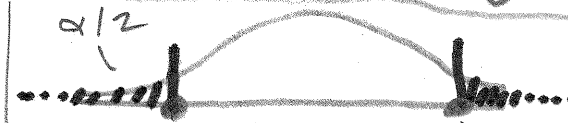
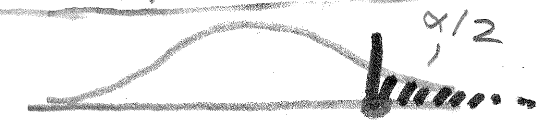
If the population distribution is not known a histogram based on a sample may give you a clue.
Although histogram is not conclusive, sometimes it may be the best clue that you have.
The the histogram shows non-normal or outliers, increasing the sample size and improve the calculations.

Excel Functions

	Z Distribution	**t** Distribution
1 tail to Right upper 	P-value = 1-NORM.S.DIST(Z,1) upper critical value = NORM.S.INV(1 − α)	P-value = 1 − T.DIST(t, df, 1) or = T.DIST.RT(t, df) upper critical value = T.INV(1−α, df)
Two tail 	P-value = NORM.S.DIST(Z,1)*2 Z on Low end Low critical value = NORM.S.INV(α/2) upper = NORM.S.INV(1−α/2) or +/− critical values = +/−NORM.S.INV(α/2)	P-value = T.DIST(lower t, df, 1)*2 or = T.DIST.2T(upper t, df) Low critical value T.INV(α/2, df) upper = T.INV(1 − α/2, df) or +/− critical values = +/− T.INV(α/2, df)
1 tail to Left upper 	P-value = NORM.S.DIST(Z,1) Low critical value = NORM.S.INV(α)	P-value = T.DIST(t, df, 1) Lower critical value = T.INV(α, df)
When to use:	sigma known and proportions, when 4 tests met.	sigma not known

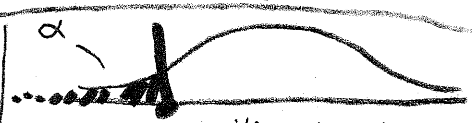
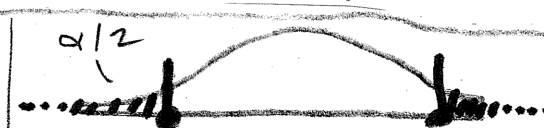
Ⓩ

Hypothesis Testing Z Distribution (Sigma Known)

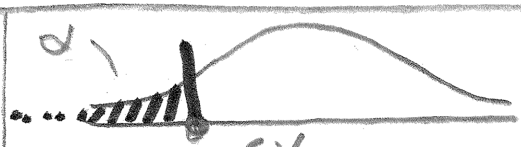
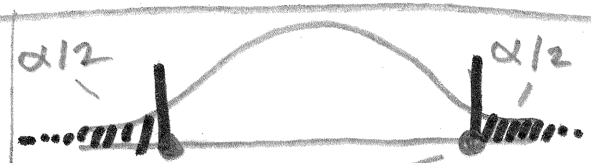
Test Type	 1 Tail Test To Left ← Lower Tail	 ← Two-tail Test →	 1 Tail Test to Right Upper Tail →
Hypothesis	$H_o: \mu >= \mu_o$ $H_a: \mu < \mu_o$	$H_o: \mu = \mu_o$ $H_a: \mu <> \mu_o$ * <> → Not Equal To	$H_o: \mu <= \mu_o$ $H_a: \mu > \mu_o$
Test Statistic	$Z = \dfrac{\bar{X} - \mu_o}{\sigma / \sqrt{n}}$		
p-value Rejection Rule	If: p-value $\leq \alpha$ Then: Reject H_o, Accept H_a		
Excel p-value	=NORM.S.DIST(Z,1)	=NORM.S.DIST(Z,1)*2 Z on Low End	=1-NORM.S.DIST(Z,1)
Critical value Rejection Rule (For 1 tail) Fail to Reject (for 2-Tail)	If: $Z \leq -Z_\alpha$ Then: Reject H_o, Accept H_a $-Z_\alpha$ = critical value (low End)	If: $-Z_{\alpha/2} < Z < Z_{\alpha/2}$ Then: Fail To Reject H_o $-Z_{\alpha/2}$ = Low critical value $Z_{\alpha/2}$ = Upper critical value	If: $Z \geq Z_\alpha$ Then: Reject H_o, Accept H_a
Excel critical value	$-Z_\alpha$ = NORM.S.INV(α)	+/- critical value = = NORM.S.INV(α/2)	Z_α = NORM.S.INV(1-α)

Z

Hypothesis Testing Z Distribution (Proportions)

Test Type	 1 Tail Test To Left ⟵ Lower Tail	 ← Two-tail Test →	 1 Tail Test to Right Upper Tail ⟶
Hypothesis	$H_0: p >= p_0$ $H_a: p < p_0$	$H_0: p = p_0$ $H_a: p <> p_0$	$H_0: p <= p_0$ $H_a: p > p_0$
Test Statistic	{Standard Error} $= SE = \sigma_{\bar{p}} = \sqrt{\frac{p_0 * (1-p_0)}{N}}$	/	$Z = \frac{\bar{p} - p_0}{SE}$
p-value Rejection Rule		If: p-value $\leq \alpha$ Then: Reject Ho, Accept Ha	
Excel p-value	=NORM.S.DIST(z,1)	=NORM.S.DIST(z,1)*2 z on Low End	=1-NORM.S.DIST(z,1)
critical value Rejection Rule (for 1 tail) Fail to Reject (for 2-Tail)	If: $z \leq -z_\alpha$ Then: Reject Ho, Accept Ha $-z_\alpha$ = critical value (low End)	If: $-z_{\alpha/2} < z < z_{\alpha/2}$ Then: Fail TO Reject Ho $-z_{\alpha/2}$ = Low critical value $z_{\alpha/2}$ = Upper critical value	If: $z \geq z_\alpha$ Then: Reject Ho, Accept Ha
Excel critical value	$-z_\alpha$ =NORM.S.INV(α)	+/− critical value = =NORM.S.INV(α/2)	z_α =NORM.S.INV(1−α)

(t) Hypothesis Testing t Distribution (Sigma Not Known)

Test Type	 ← Lower Tail 1 Tail Test To Left	 ← Two-tail Test →	 Upper Tail → 1 Tail Test o Right
Hypothesis	$H_o: \mu >= \mu_o$ $H_a: \mu < \mu_o$	$H_o: \mu = \mu_o$ $H_a: \mu <> \mu_o$	$H_o: \mu >= \mu_o$ $H_a: \mu > \mu_o$
Test Statistic	$t = \dfrac{\bar{X} - \mu_o}{s/\sqrt{n}}$		
p-value Rejection Rule	If: p-value $\leq \alpha$ Then: Reject Ho, Accept Ha		
Excel p-value	= T.DIST(t, df, 1)	= T.DIST.2T(upper t, df) or = T.DIST(lower t, df, 1)*2	= 1 - T.DIST(t, df, 1) or = T.DIST.RT(t, df)
Critical value Rejection Rule (for 1 Tail) Accept Rule (for 2-tail)	If: $t \leq -t_\alpha$ Then: Reject Ho, Accept Ha $-t_\alpha$ = Low Critical Value	If: $-t_{\alpha/2} < t < t_{\alpha/2}$ Then: Fail to Reject Ho $-t_{\alpha/2}$ = Low critical value $t_{\alpha/2}$ = Upper Critical value	If: $t \geq t_\alpha$ Then: Reject Ho, Accept Ha t_α = upper Critical Value
Excel Critical Value	$-t_\alpha$ = T.INV(α, df)	Lower = T.INV(α/2, df) Upper = T.INV(1-α/2, df) or +/- = T.INV(α, df)	t_α = T.INV(1-α, df)

Example of step 3 (collect Data, calculate test Statistic, Draw Picture)

Because we know the population standard deviation $\sigma = 12{,}549$, we can use the test statistic, Z.

$n = 36$

$\bar{x} = \$88{,}595$

$\sigma = \$12{,}549$

$\mu = \mu_{\bar{x}} = 85{,}000$

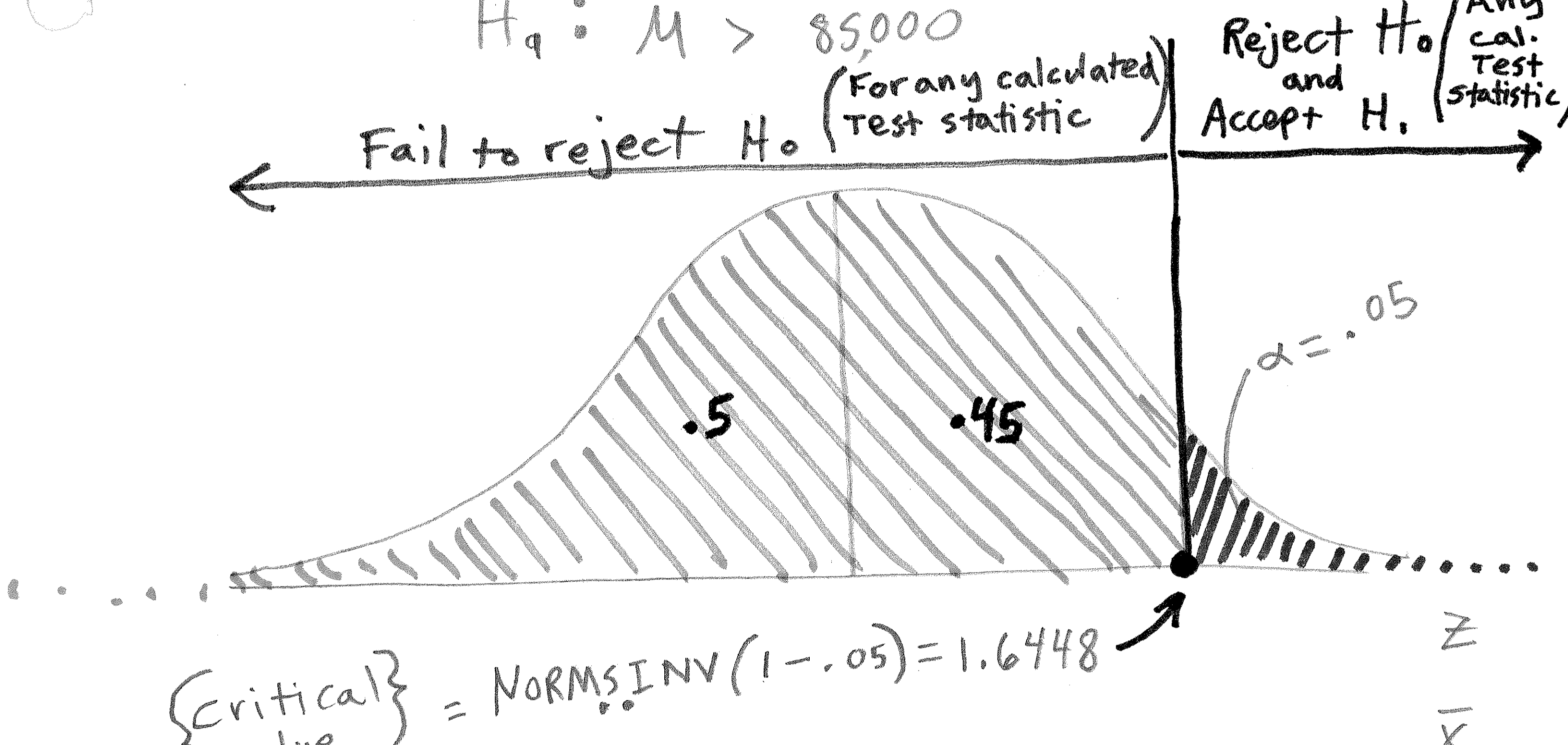
$\sigma_{\bar{x}} = \frac{12549}{\sqrt{36}} = \$2{,}091.50$

$\alpha = .05$

test statistic $= \frac{88595 - 85000}{2{,}091.50} = 1.72$

$H_0: \mu \leq 85{,}000$

$H_a: \mu > 85{,}000$



{critical value} $=$ NORMSINV$(1 - .05) = 1.6448$

$\mu = \mu_{\bar{x}} = 85000$

$\sigma_{\bar{x}} = 2{,}091.50$

Step 4

Decision Rule:

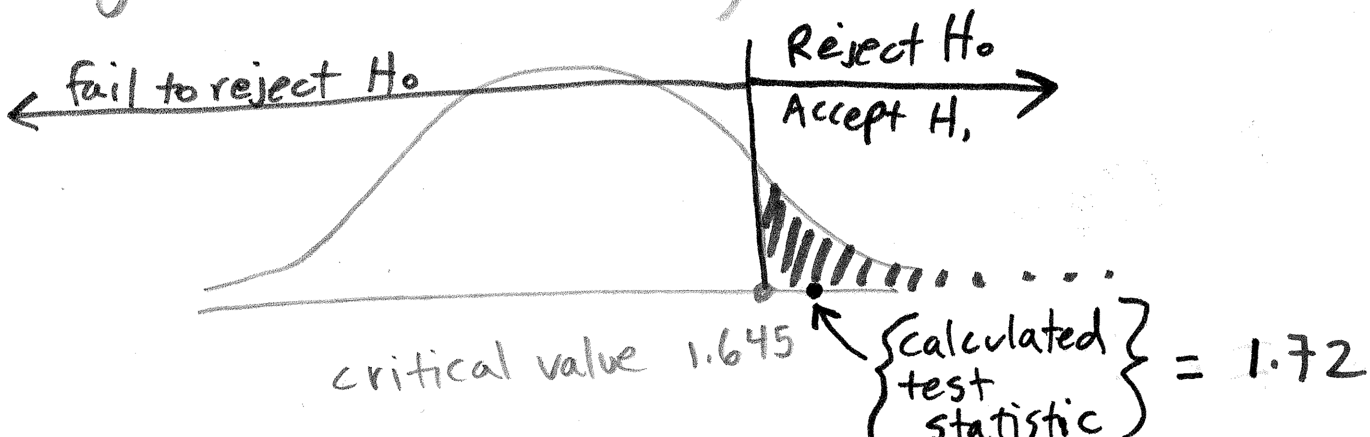
If our test statistic is greater than or equal to 1.6448, we reject Ho and accept Ha.

Step 5 Conclude with critical value & Rejection Rule

$$\left\{\begin{matrix}\text{calculated} \\ \text{test} \\ \text{statistic}\end{matrix}\right\} = \frac{88595 - 85000}{\frac{12549}{\sqrt{36}}} = 1.72$$

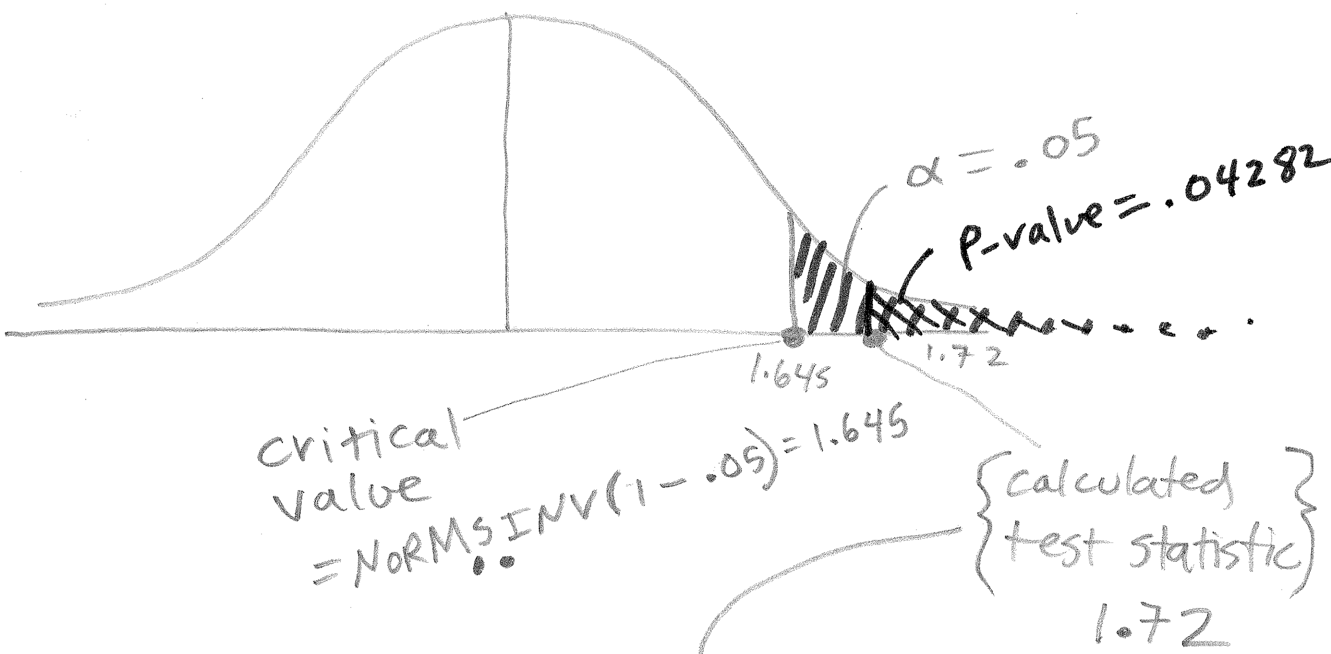
Make Decision:

Because our calculated test statistic is greater than 1.645, we reject H_0 and accept H_1. It is reasonable to assume that the mean salary for real estate agents is greater than \$85000.



Based on the statistical evidence our $\overline{X}$ of 88,595 is statistically significant & provides good evidence that the mean salary for Realtors is more than 85000

step 4 & 5 for p-value



p-value = 1 − NORMSDIST(1.72) = .04282

Because the p-value is less than alpha (.04282 ≤ .05), we reject H_0 & accept H_1. It is reasonable to assume that the mean salary for real estate agents is greater than $85,000

Summary for Real Estate Example:

$\mu = \mu_{\bar{x}} = \mu_0 = 85000$

$\bar{x}$ $\bar{X} = 88,595$

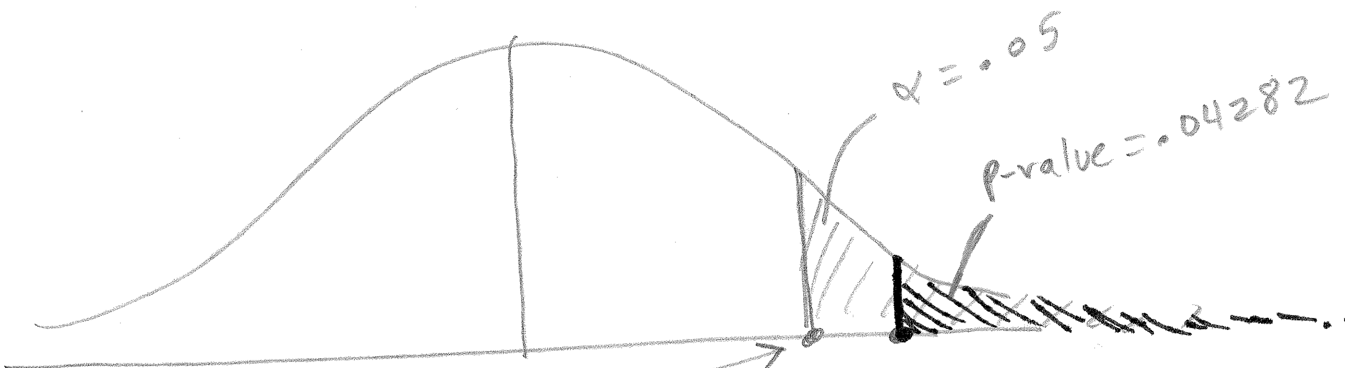
size $n = 36$

sigma $\sigma = \$12,549$

Standard Error $= \sigma_{\bar{x}} = 12549/\sqrt{36} = \$2,091.50$

alpha $\alpha = .05$

test statistic $= \dfrac{88595 - 85000}{2091.5} = 1.72$



Critical Value

Dividing point between the region where the Null Hypothesis is rejected and the region where it is not rejected

upper test

= NORM.S.INV (1 − α)

= NORM.S.INV (1 − .05) = 1.6448

critical value = 1.6448

p-value

Probability of getting test statistic or more = 1 − NORM.S.DIST (z)

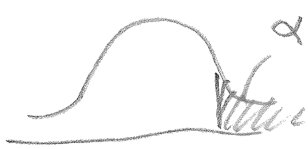
p-value = 1 − NORM.DIST (1.72) = .04282

concluding :

use Z or t to compare to Critical value

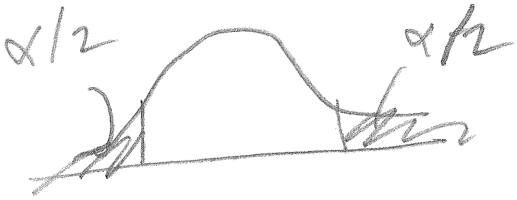
use p-value to compare to alpha α

Critical value



t or Z $\geq$ critical value, Reject H_0, Accept H_a

t or Z $\leq$ critical value, Reject H_0, Accept H_a



-t
-Z $\leq$ critical value $\leq$ t
Z, Fail to Reject H_0

---

p-value

p-value $\leq \alpha$, Reject H_0, Accept H_a

Confidence Interval Hypothesis Testing

If:

$H_0 : \mu = \mu_0$

$H_a : \mu <> \mu_0$

Then:

$\neq$ = Not = <>

Excel Symbol for "Not" or "Not Equal"

μ_0 = hypothesized population mean

① Select a simple random sample from the population and use the value of the sample mean $\bar{X}$ to develop a confidence Interval for the population mean μ.

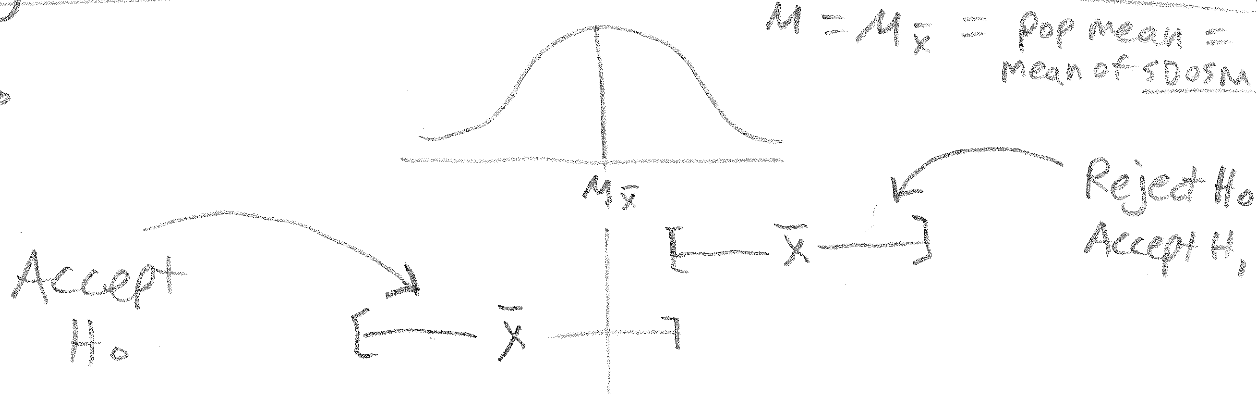
$$\bar{X} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$\bar{X}$ = sample mean
$Z_{\alpha/2}$ = upper Z
σ = pop S.D.
n = sample size

② If the confidence interval contains the hypothesized value (μ_0) μ_0, do not reject H_0, otherwise, Reject H_0 (Reject H_0 if μ_0 is one of the endpoints)

$\mu = \mu_{\bar{x}}$ = pop mean = mean of SDOSM

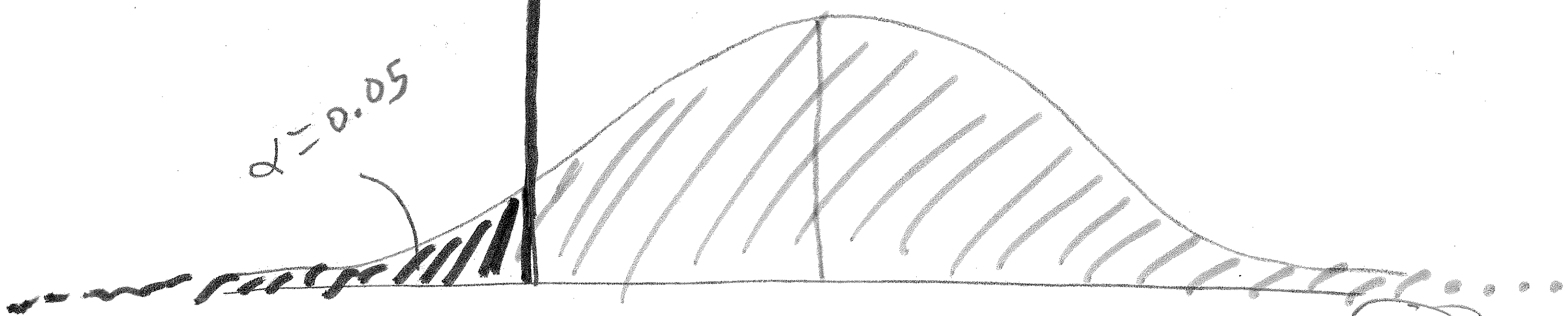
Example:



$H_0: \mu \geq 1056$

$H_a: \mu < 1056$

Reject Ho Accept Ha ← | Fail to Reject Ho →



$\mu_0 = \$1056$

Ave. Tax Refund

step 3

step 4

Area = Probability of -1.825 or less

$\mu_0 = \$1056$ = Ave Tax Refund

critical value = -1.645

Test statistic = z = -1.825

p-value = =NORM.S.DIST(-1.825)

= 0.034

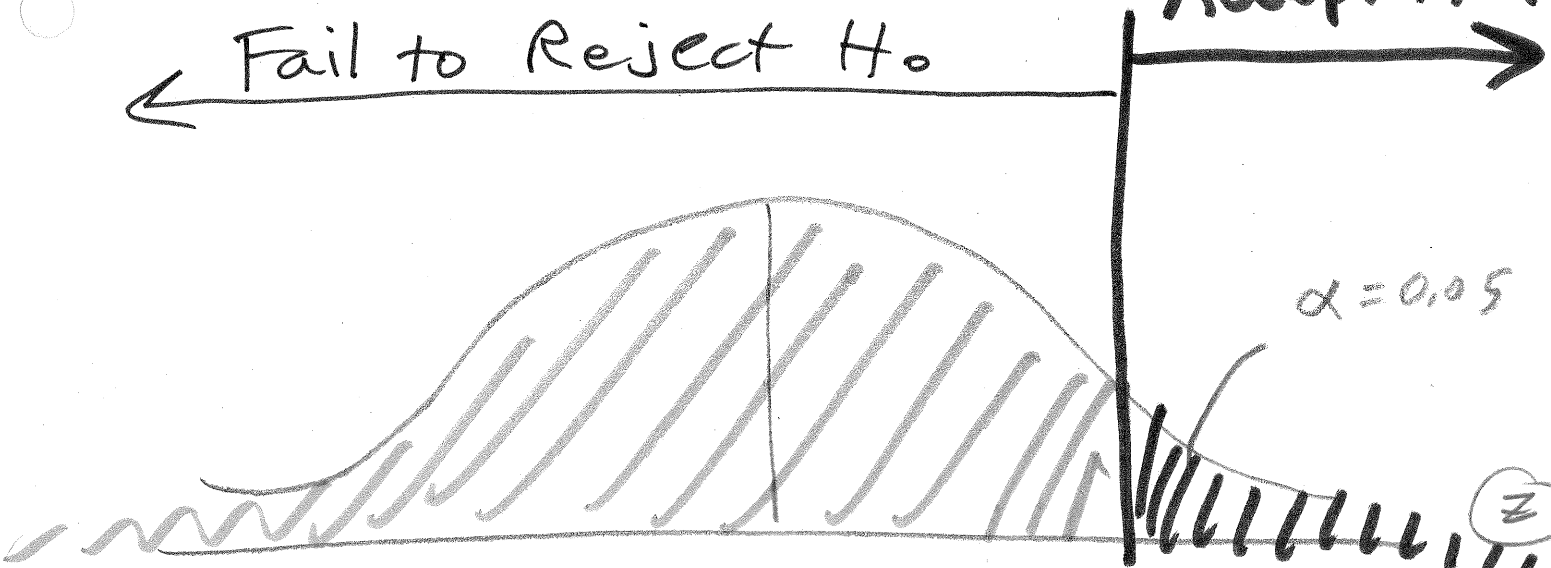
Probability of -1.825 or less

step 3

$H_0: \mu \leq 3173$

$H_a: \mu > 3173$

Reject H_0
Accept H_a

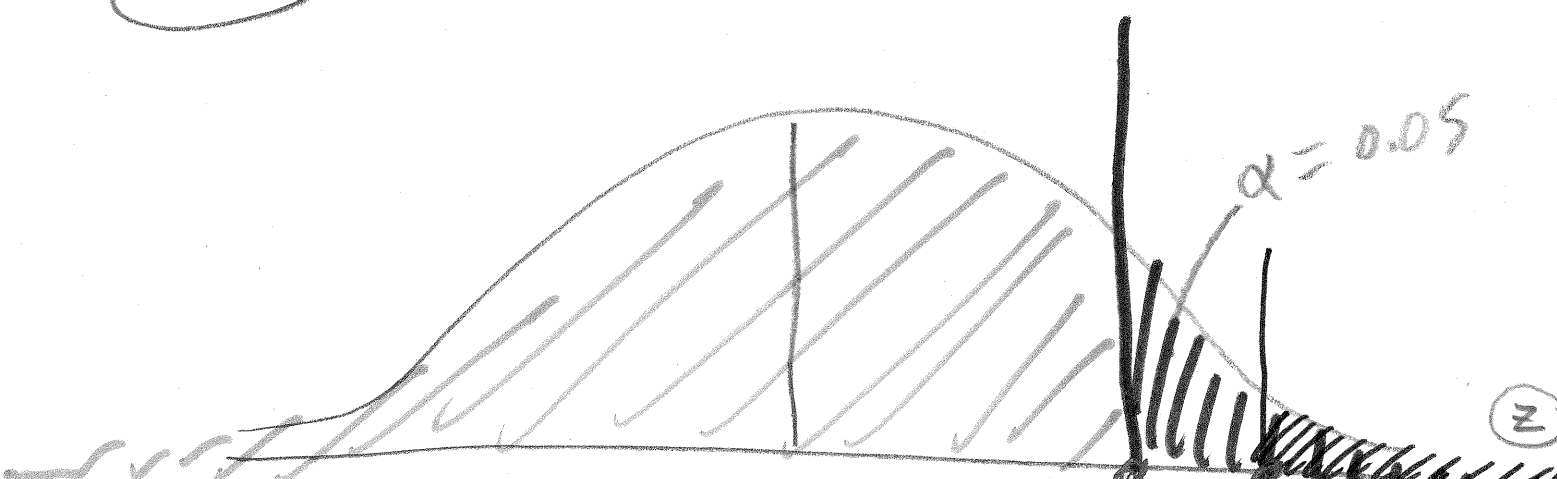


$\mu_0 = 3173$

credit card Balance
undergraduate students

#16
Ch. 9

step 4



$\mu_0 = 3173$

Critical value = 1.645

Test statistic = 2.039

p-value = 0.0207

probability of 2.039 or more

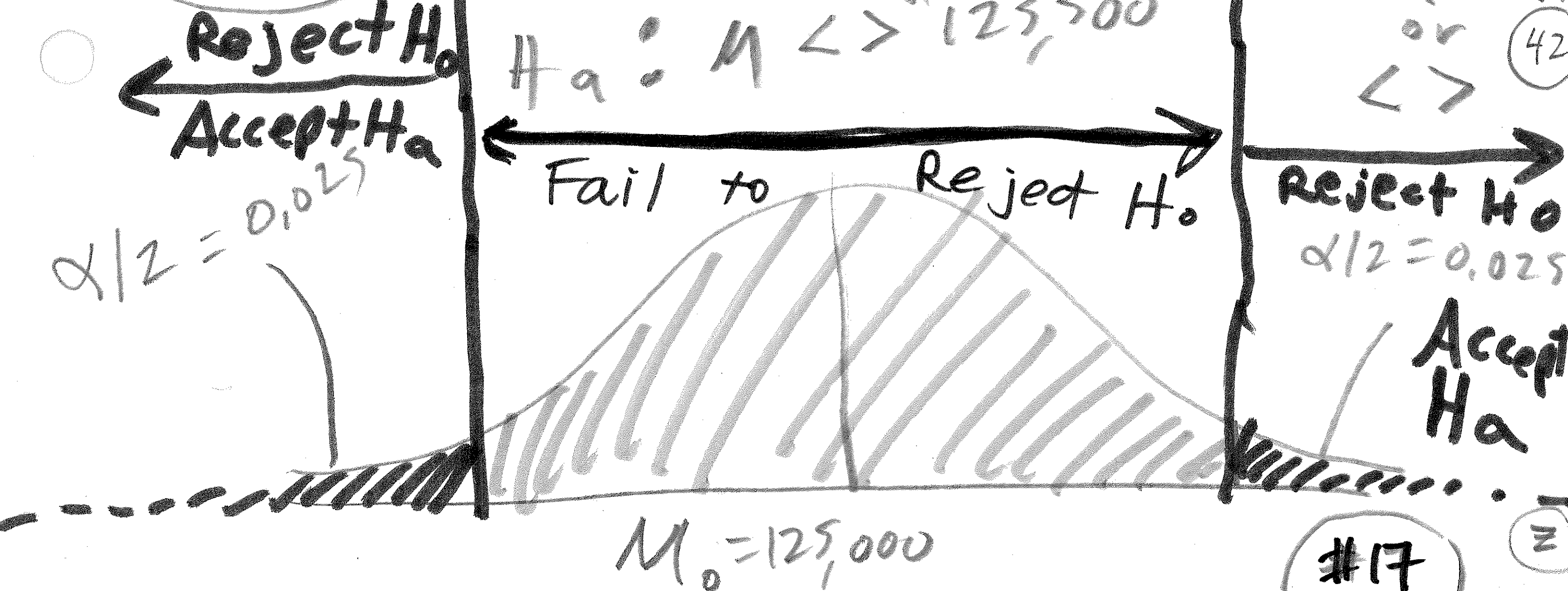
Step 3

$H_0: \mu = \$125,500$

$H_a: \mu <> \$125,500$

Not equal $\neq$ or <>

p. 42



#17 Ch. 9

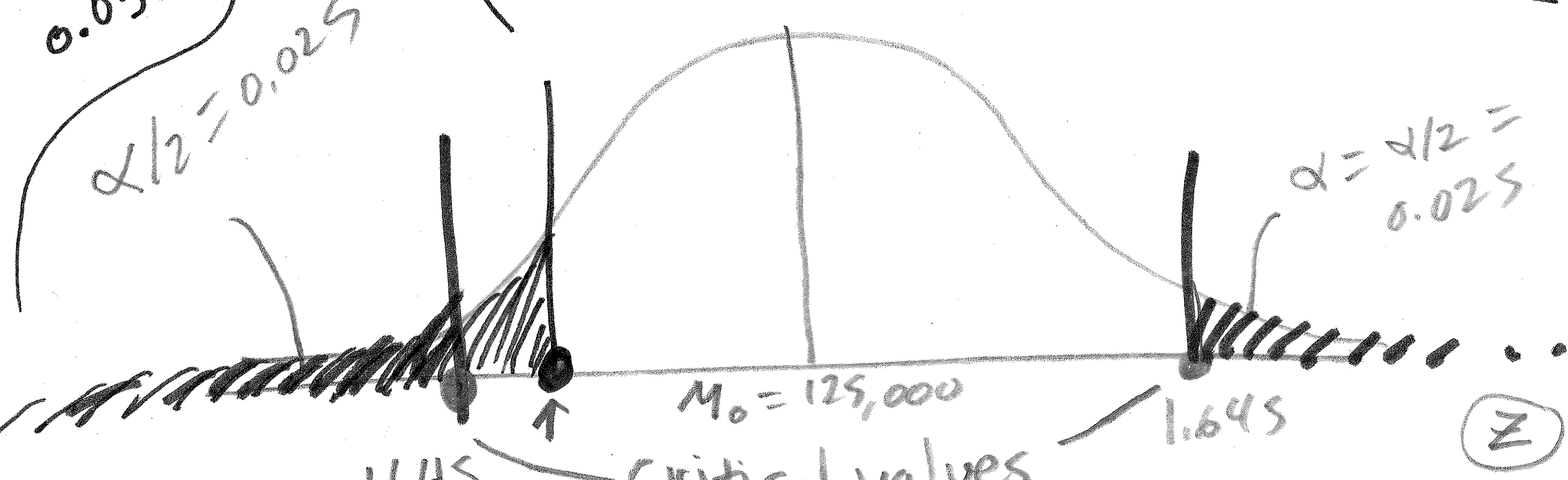
(p-value from low end) * 2 = 0.0569 * 2
= 0.1138

Step 4

p-value = 0.1138

p-value = probability of -1.581 or less or 1.581 or more

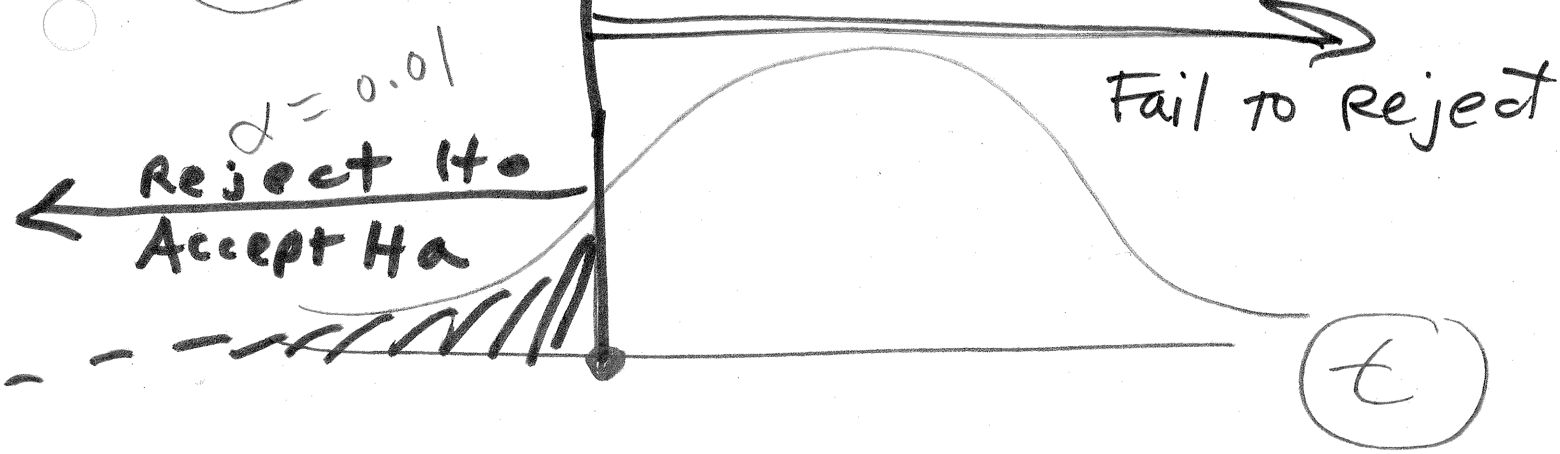
This probability 0.0569



Test statistic = -1.581

Step 3

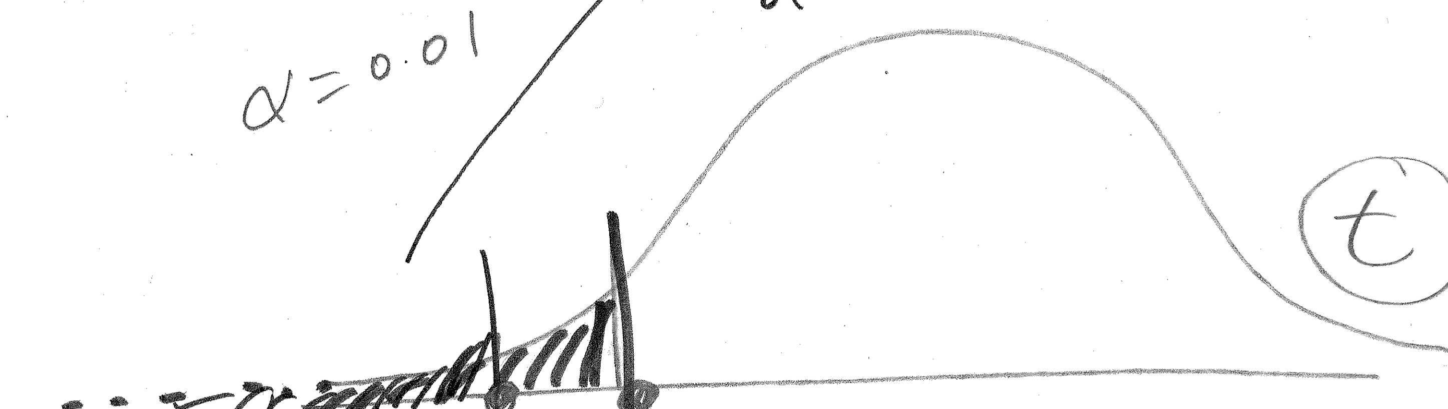
Ho : $\mu \geq 9$ years
Ha : $\mu < 9$ years



#28
Ch. 9

Step 4

p-value = probability of getting test statistic of −2.499 or less



critical value = −2.37

Test statistic = −2.49997

p-value = probability of −2.49997 or less

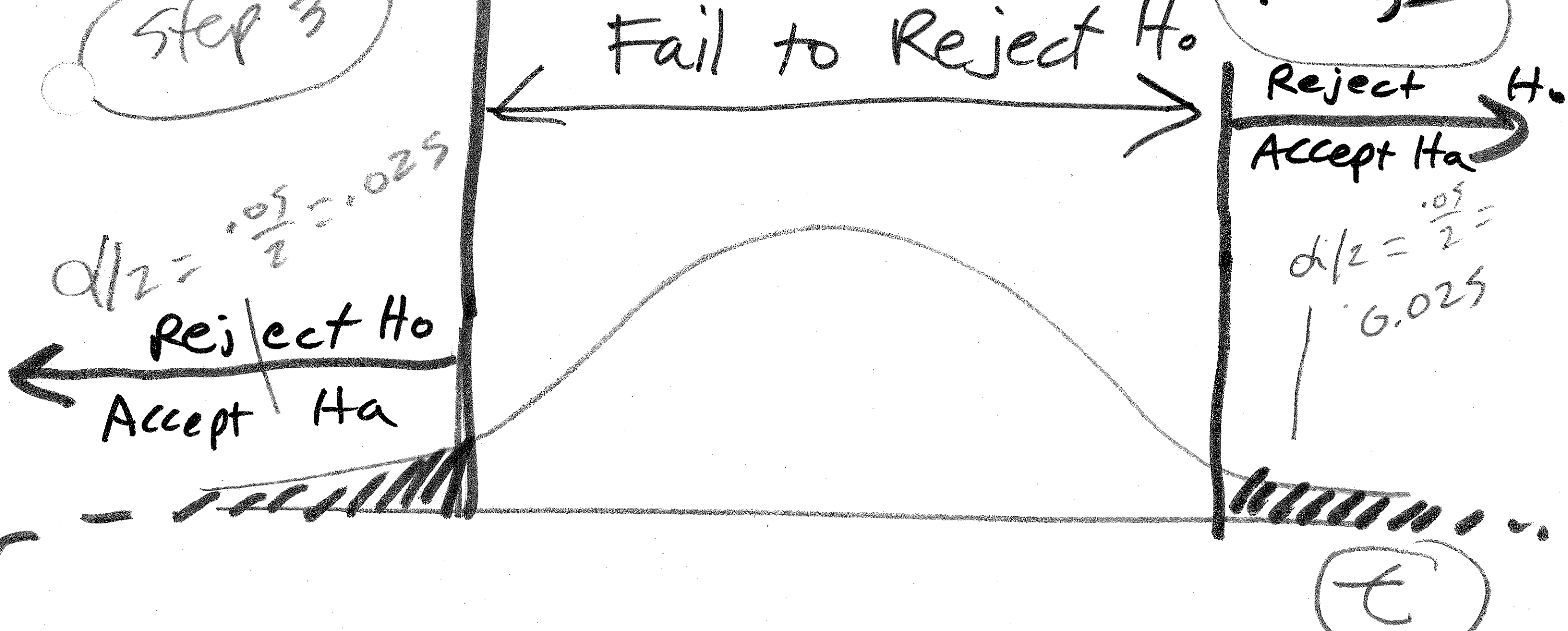
p-value = 0.0072

Ho: M = 10192
Ha: M <> 10192

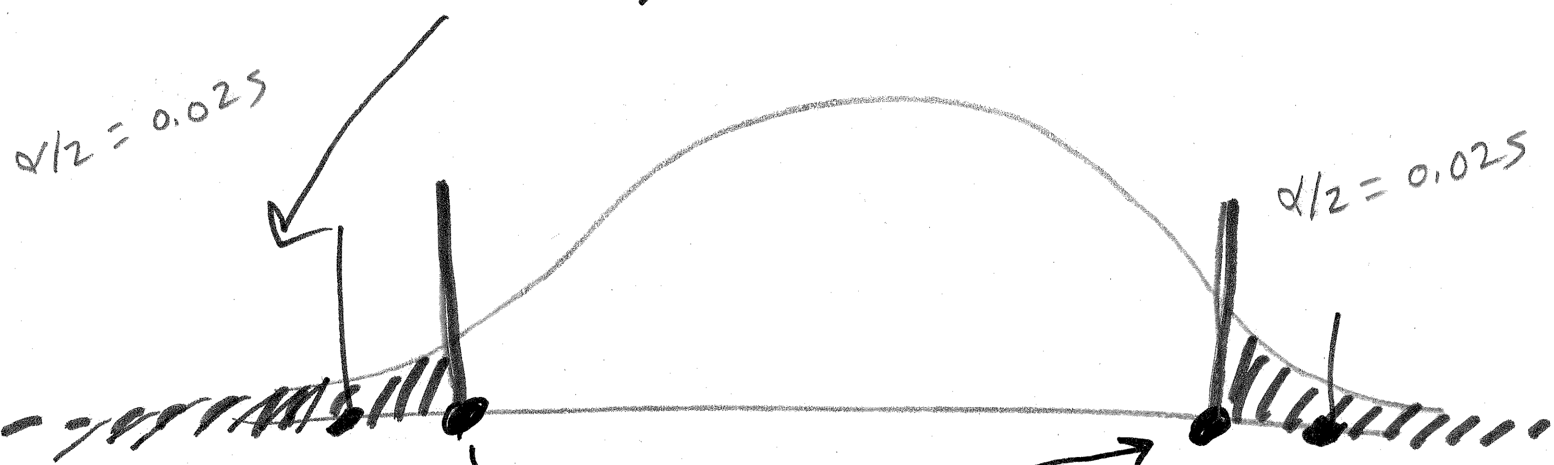
Ch 9
32

P. 44

Step 3



single low p-value = 0.015094
DOUBLE IT!!!



critical value = +/- 2.009575

test statistic = +/- 2.2324

P-value = 2 * 0.015094 = 0.030189 =

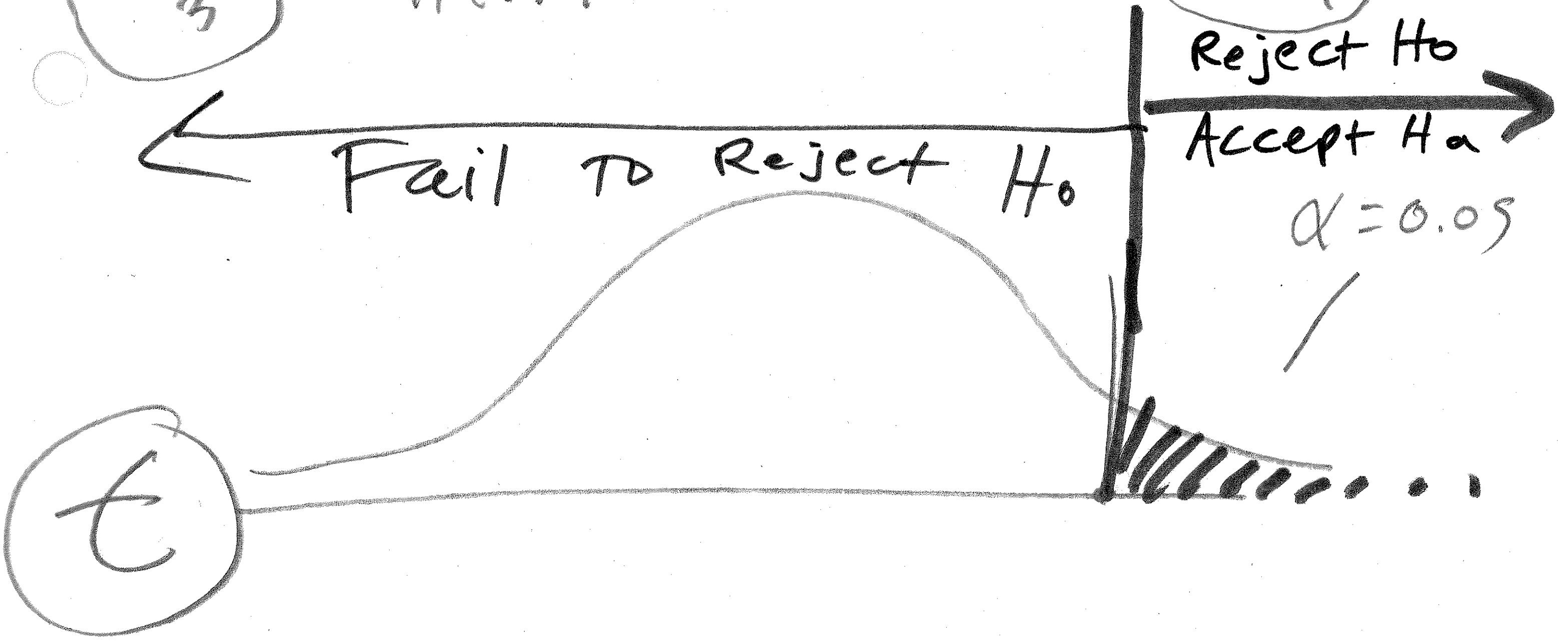
P-Vale means probability of less than or equal to -2.23 [redacted] or greater than or equal to 2.23

step 3

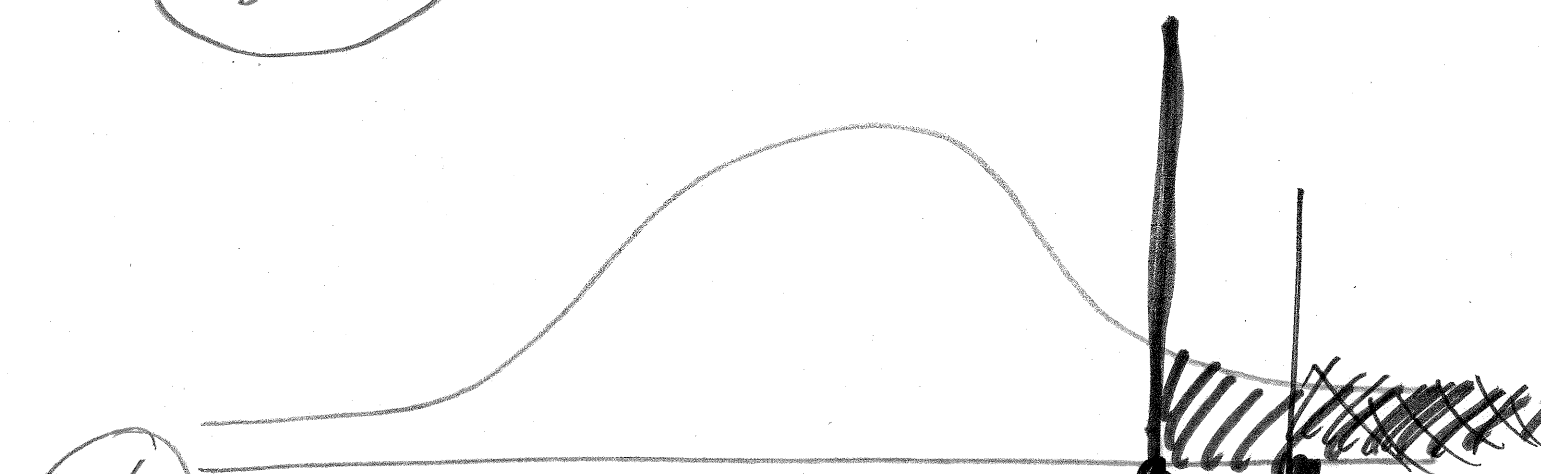
$H_0: \mu \leq 21.6$
$H_a: \mu > 21.6$

#33 ch. 9

P. 45



step 4



critical value = 1.753

test statistic = 2.05

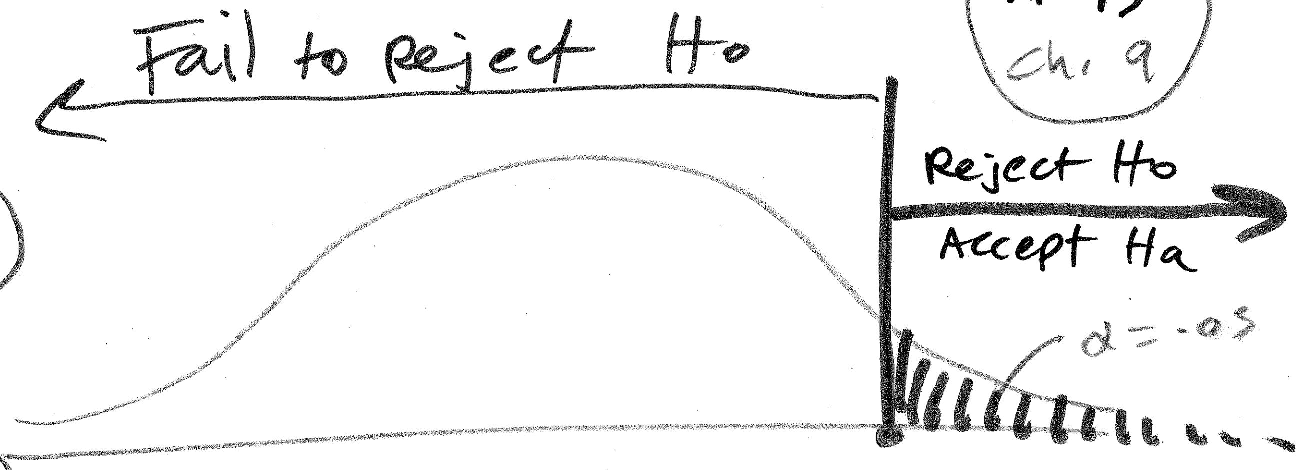
p-value = 0.02895 = Probability of 2.05 or more

$H_o: p \le .1$
$H_a: p > .1$

#43
Ch. 9

P. 46

Step 3

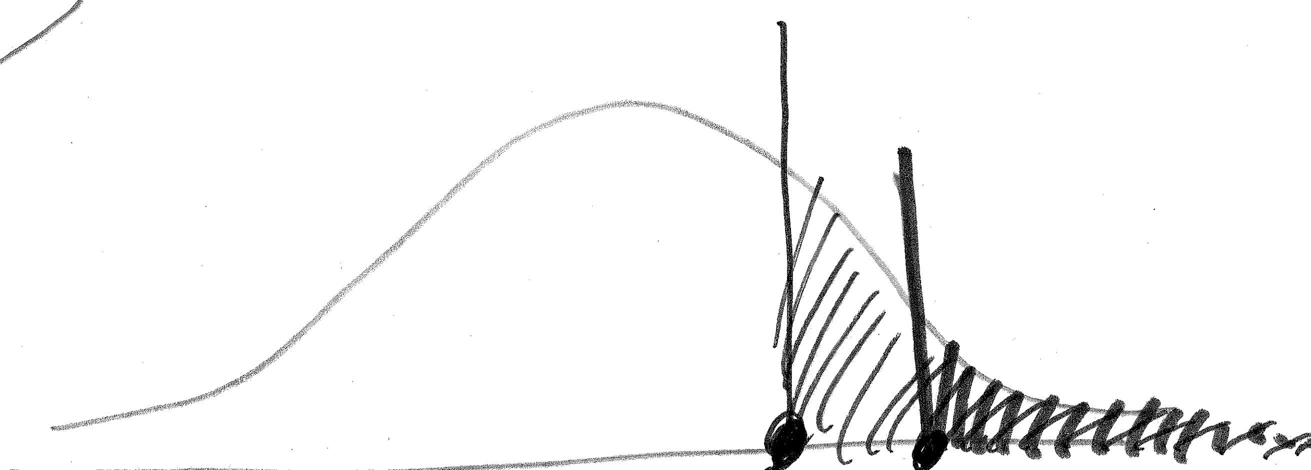


Z

Step 4

p-value = probability of getting Test statistic of 1 or greater = 0.1587

Z



critical value = 1.645

Test statistic = 1

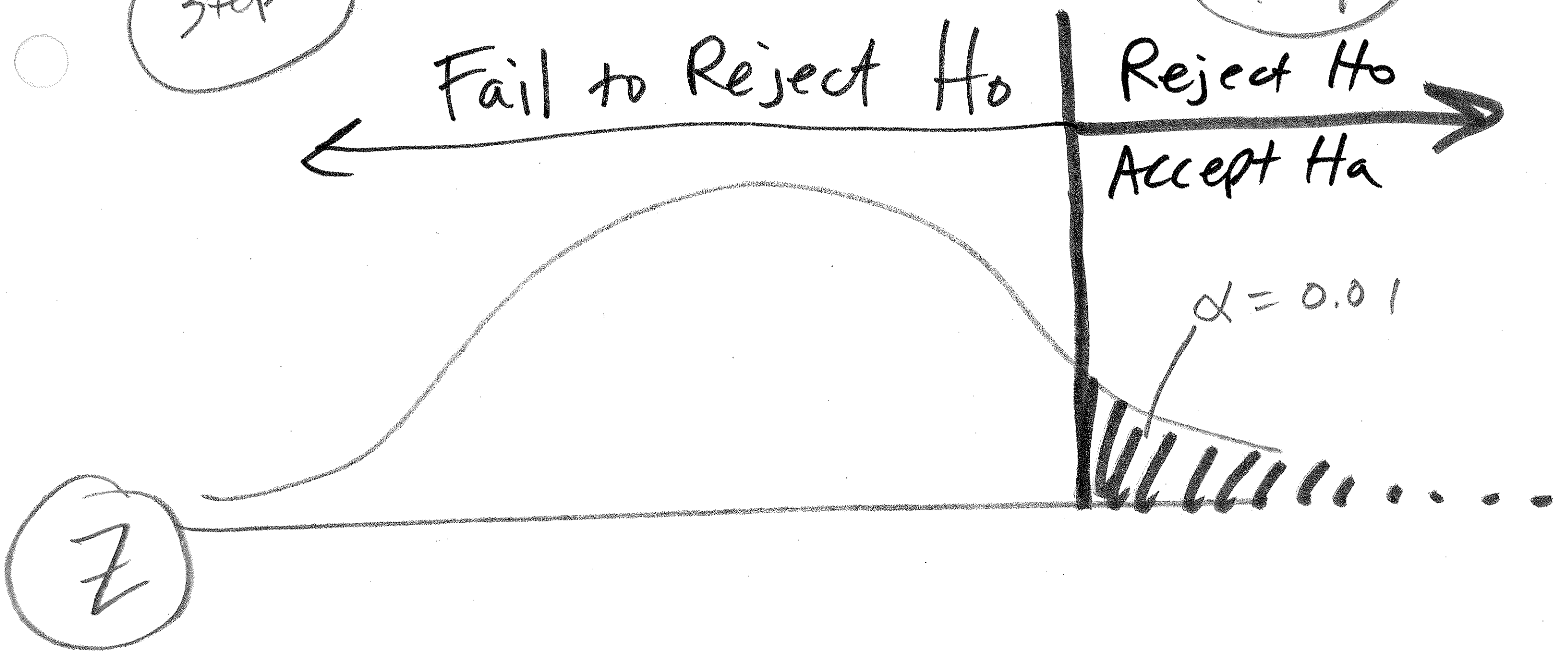
$H_o: p \leq .51$

$H_a: p > .51$

#44 ch. 9

p. 47

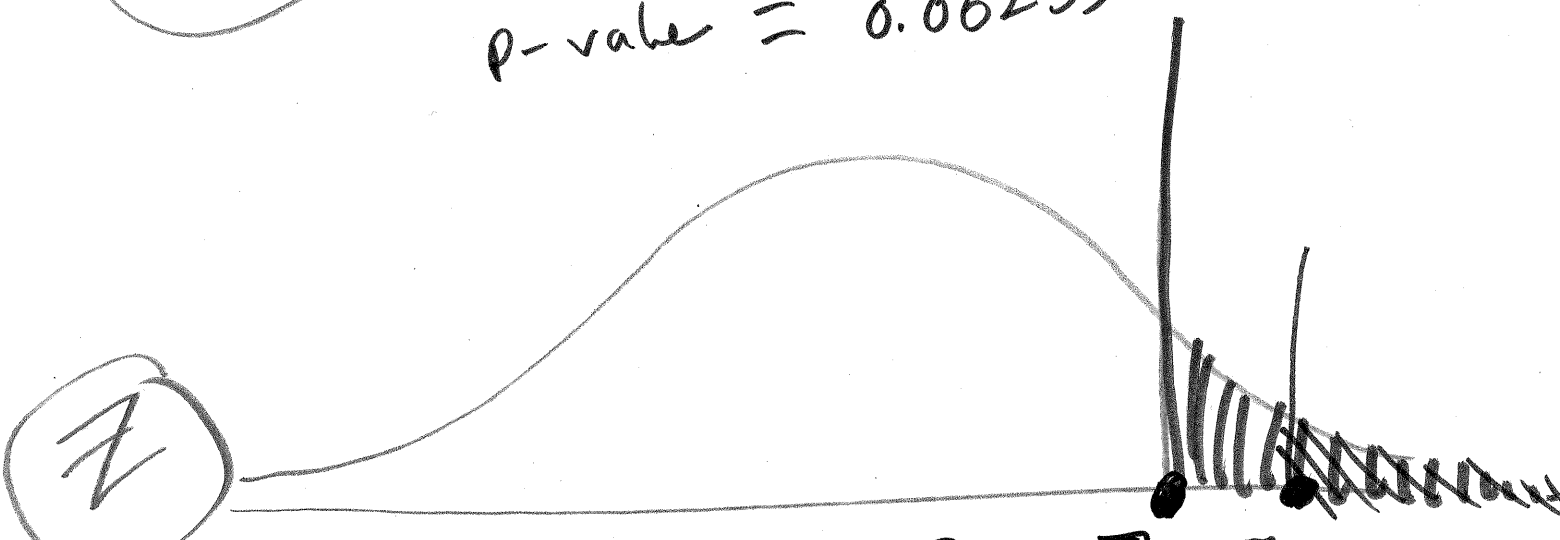
Step 3



Step 4

p-value = probability of getting a test statistic of 2.8 or more =

p-value = 0.00255



critical value = 2.33

Test statistic = 2.8