Hypothesis

- A statement about a population parameter subject to verification.

Example:
An official report claims:
"The yearly salary of full-time realtor is $85,000."

Hypothesis Testing

1. A statistical procedure that uses sample evidence & probability theory to determine whether a statement about the value of a population parameter:
   "Should be rejected" = "Reject"
   or
   "Should Not be rejected" = "Fail to Reject"

2. Make a concluding statement about the population parameter based on sample evidence.
Example 1

Statement from official Report:
"The yearly salary earned by full-time realtors is $85,000"

Researcher believes:
Realtors make more than $85,000

1. If we take a random sample & get $\bar{X} = 88,595$
2. we must decide if sampling Error of $88,595 - 85,000 = 3,595$ is acceptable.

Is the difference $3,595$

"Statistically significant" or

"Statistically insignificant"
Chapter 7

Sampling Distribution of $\bar{X}$

If our sample evidence provided $\bar{X} = 86,000$
Then original claim of $85,000$:
"Should not be rejected"
"Original claim seems reasonable"

If sample evidence provide $\bar{X} = 88,595$
Then original claim of $85,000$:
"Should be rejected"
"Original claim seems unreasonable"

We will use something called a "Test statistic" ($Z$ or $t$)
Compare it to our Hurdle Line.
"Test statistic" = # Standard Deviations above or below
Chapter 8

Sampling distribution of $X$

$Z = 0$

$M_X = 85,000$

If our sample evidence provided $\bar{X} = 86,000$

Then because interval contains 85,000, original claim:

"Should not be rejected"

or

"Original claim seems reasonable."

If our sample evidence provided $\bar{X} = 88,595$

Then because interval does not contain 85,000,

"Original claim should be rejected"

"Original claim seems unreasonable."

88,595

86,000
other examples of "statements about a value of a population parameter" that we can test:

Is the new contribution solicitation letter more effective than the old letter, which got 15% contributions?

Is the manufacturer's claim that 16 oz. of catsup is in each bottle?

Is the average wait time in line at McBurger's Restaurant less than 3 minutes?

Is the new machine faster than the old one?
Steps of Hypothesis Testing

1. Develop Null Hypothesis (H₀) & Alternative Hypothesis (Hₐ) or (Hₐ₁) or (Hₐ₂)

2. Specify the level of significance (α)

3. Collect sample data & compute value of test statistic (Z or t), Draw Picture.

P-value Approach

4. Use value of test statistic to compute p-value

5. Reject H₀ if p-value ≤ α

Critical Value Approach

4. Use level of significance to determine the critical value and

5. Use the value of the test statistic and the rejection rule to determine whether to reject H₀

Notes:
1. If population data is normally distributed, these methods are exact (.99 = cI, α = .01, then 99 intervals contain μ₀, I does not)
2. If population data is not normal, the bigger the n, the more exact.
   - Pop Normal = any n can be used
   - Approx. Normal n ≥ 15
   - Not Normal n ≥ 30
   - Outliers n ≥ 50
**Step 1**

Develop Null Hypothesis ($H_0$) & Alternative $H_0$ ($H_a$)

**Null Hypothesis** = $H_0$

The hypothesis tentatively assumed true in the hypothesis testing procedure. Based on sample evidence we either "Reject $H_0$" or "Fail to Reject $H_0$".

**Alternative Hypothesis** = $H_a$

Based on sample evidence, the hypothesis concluded to be true if the null hypothesis is rejected. We either "Fail to Reject $H_0$" or "Reject $H_0$, accept $H_a$".
Research Hypothesis

Start with alternative hypothesis and make it the conclusion the researcher hopes to support.

Example:
Realtors make more than $85,000

Validity of a Claim

Assumption that population parameter is true

Example:
Is catsup bottle filled with 16oz?

Decision Making

Choose between 2 things.

Example:
Should we accept box of shipped products, yes or no.
Notes about Step I

- Developing \( H_0 \) & \( H_a \) can be difficult & takes practice to learn how to do.

- The context, situation, or point of view will help determine the correct \( H_0 \) & \( H_a \)

1. **Research Hypothesis** → usually start with \( \rightarrow H_a \)

   - *Example:* "Realtors make more than $85,000?"
     - \( H_0: M \leq 85,000 \)
     - \( H_a: M > 85,000 \)

2. **Validity of Claim** → usually start with \( \rightarrow H_0 \)

   - *Example:* "Is catsup bottle filled with 16oz?"
     - Consumer's point of view
     - Assume bottle is filled with 16oz. But if not, take action
     - \( H_0: M \geq 16 \text{ oz.} \)
     - \( H_a: M < 16 \text{ oz.} \)
     - {Consumer cares about this}

     - Manufacturer's point of view
     - Manufacture cares about filled too much or too little
     - \( H_0: M = 16 \text{ oz.} \)
     - \( H_a: M \neq 16 \text{ oz.} \)

3. **Decision Making** → (choose between) \( \rightarrow H_0 \) or \( H_a \)

   - 2 things
step 1: Develop H₀ and Hₐ

Original statement:
The yearly salary earned by full-time realtor is $85,000 (σ = $12,549).

Competing statement:
Researcher believes realtors make more than $85,000.

1st write this: color says "Here is Hypothesis"

H₀: M
Hₐ: M

2nd: Use "more than $85,000" to determine comparative operator for Hₐ

H₀: M
Hₐ: M > 85,000

3rd: Once you know comparative operator for Hₐ, put opposite comparative operator and equal sign for H₀.

H₀: ≤ 85,000
Hₐ: > 85,000

4th: H₀ ALWAYS get = sign 1-tail to Left.

5th: Hₐ comparative operator
Step 1 Continued...

H₀: \( M \leq 85,000 \)
Hₐ: \( M > 85,000 \)

If we get \( \bar{x} \) here we say:

"Based on the sample evidence, we fail to reject H₀. There is little statistical evidence that the mean salary is more than \$85,000." *Don't say H₀ is TRUE

If we get \( \bar{x} \) here we say:

"Based on the sample evidence, we reject H₀ and accept Hₐ. There is statistical evidence that the mean salary is more than \$85,000."

*careful in our language because we are taking samples.

*only two possible outcomes.
3 possible forms of $H_o$ & $H_a$

1. $H_o: M = M_0$
   $H_a: M \neq M_0$

2. $H_o: M \leq M_0$
   $H_a: M > M_0$

3. $H_o: M \geq M_0$
   $H_a: M < M_0$
Level of significance = Alpha = $\alpha$

1. $\alpha$ determines the cut off point, which is the threshold used to decide whether the test statistic is statistically significant.

$H_0: M \leq 85,000$
$H_a: M > 85,000$

If we get $\bar{x} = 88,595$ and it is out here, this is statistically significant and we reject $H_0$ and accept $H_a$. $M > 85,000$.

- If we choose $\alpha = 0.05$, we are taking a 5% risk of rejecting $H_0$ even though it was true.
- Because we choose $\alpha$, we can say we are doing a "Significance Test"
These picture examples show the 3 possibilities at 3 different alpha values.
Step 2 Specify Level of Significance (\( \alpha \))

- Because hypothesis testing is based on sample data, we must allow for the possibility of errors.
- Unless we test the whole population, you run the risk of error.

1. Notice:

   ![Sampling Distribution of \( \bar{X} \)]

   This is the entire distribution of possible \( \bar{X} \) values.

2. It is possible to take a sample and get an \( \bar{X} = 88,595 \) that is just sample error.

   ![Distribution with \( \mu = \mu_{\bar{X}} = 85,000 \) and \( \bar{X} = 88,595 \)]

   So we could get \( \bar{X} = 88,595 \) but \( \mu = 85,000 \) is still true.
It is very unlikely that we could get an $\bar{X}$ out here.

But...

It is a possibility that our particular sample just happened to have a lot of big numbers in it so we got:

$\bar{X} = \$88,595$

while the full population mean was still: $M = \$85,000$

So...

Because we can't usually test the whole population, we have to pick a cut off point and reject the original statement (Ho) if we go beyond that point.

This means we take a risk of an error...
Define: \( \alpha = \text{Level of Significance} = \text{Type I Error} \)

Probability (Risk) of rejecting \( H_0 \) even though it is true (as an equality).

- \( H_0: M \leq 85,000 \)
- \( H_a: M > 85,000 \)

Example: \( M = 85,000 \)

\[ M = \bar{X} = 85,000 \]

If we get \( \bar{X} = 88,595 \) & Reject \( H_0 \),

But \( H_0 (M = 85,000) \) is actually TRUE this is:

"Type I Error"

"Innocent but found Guilty"

Most of the time \( \bar{X} = 88,595 \) will lead to correct conclusion (about 95 out of 100).

"Innocent but found Guilty" Error 5 out of 100 times.
Designer of Hypothesis Test selects alpha \( \alpha \) and thereby controls the probability of a Type I Error.

\[
\begin{align*}
\text{Error 10 out of 100 times} & \quad \alpha = 0.10 \\
\text{Error 5 out of 100 times} & \quad \alpha = 0.05 \\
\text{Error 1 out of 100 times} & \quad \alpha = 0.01
\end{align*}
\]

\[\rightarrow \text{As you move } \rightarrow \text{this way} \rightarrow \text{you reduce } \alpha \rightarrow \]
\[\rightarrow \text{and Reduce } \rightarrow \text{the risk of Type I Error.}\]

Example:
1. Drug company may want to set \( \alpha \) very small, so they are sure new drug really works.
2. Quality Control may want to set \( \alpha \) low so they are more sure that the quality is high.

Selecting \( \alpha \):
- **If cost of making Type I Error is high**, choose small \( \alpha \).
- **If cost of making Type I Error is not high**, choose bigger \( \alpha \).
Errors & Correct Conclusions in Hypothesis Testing

<table>
<thead>
<tr>
<th>Actual Population Condition</th>
<th>H₀ TRUE (Hₐ FALSE)</th>
<th>H₀ FALSE (Hₐ TRUE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reject H₀, Accept Hₐ</td>
<td>Type I Error</td>
<td>Correct Conclusion</td>
</tr>
<tr>
<td>Alpha</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fail To Reject H₀</td>
<td>Correct Conclusion</td>
<td>Type II Error</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Beta</td>
</tr>
</tbody>
</table>

**Type I Error**

H₀ True, but we Reject H₀

Alpha = α

"I innocent but found guilty"

Because we control for α, we can say "Accept Hₐ" in our conclusion.

**Type II Error**

H₀ False, but we Fail to Reject H₀

Beta = β

"Guilty but found innocent"

Because we don't control for β (in this textbook) we can't say "Accept H₀".
Other Wording:

<table>
<thead>
<tr>
<th>Conclusion (based on Sample)</th>
<th>Actual Population Condition</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>H₀ TRUE (Hₐ FALSE)</td>
<td>H₀ FALSE (Hₐ TRUE)</td>
<td></td>
</tr>
<tr>
<td>Reject H₀</td>
<td>Type I Error</td>
<td>Correct Conclusion</td>
</tr>
<tr>
<td>Accept Hₐ</td>
<td>Alpha (Level of Significance)</td>
<td>&quot;True Positive&quot;</td>
</tr>
<tr>
<td></td>
<td>&quot;False Positive&quot;</td>
<td></td>
</tr>
<tr>
<td>Fail To Reject H₀</td>
<td>Correct Conclusion</td>
<td>Type II Error</td>
</tr>
<tr>
<td></td>
<td>&quot;True Negative&quot;</td>
<td>Beta</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&quot;False Negative&quot;</td>
</tr>
</tbody>
</table>

**False Positive**: Our Alternative (Hₐ) was selected even though the Null (H₀) was true.

**False Negative**: Our Alternative (Hₐ) was not selected even though the Null (H₀) was false.
Example 1:

**Step 1**
- $H_0: M \leq 85,000$  Annual Realtor Salary
- $H_a: M > 85,000$  Annual Realtor Salary

**Step 2**
- $\alpha = \text{Type I Error} = 0.05 = \left(\text{cost of error not too big}\right)$

**Step 3**
- We go out & get a sample
  - $\bar{X} = 88,595$
  - $\sigma$ known = 12,549
  - $n = 36$  (Big enough to accommodate some outlier salaries)

But we need test statistic
\( \alpha \) determines cut off point

test statistic beyond cut off point, then we reject \( H_0 \) and accept \( H_a \)

Test statistic here = Reject \( H_0 \), Accept \( H_a \)

Test statistic here = Fail to Reject \( H_0 \)

Test statistic here = Reject \( H_0 \), Accept \( H_a \)
2 Methods for determining whether test statistic is past cut off ("statistically significant")

1. p-value: \( p \leq \alpha \)
   - Reject \( H_0 \), Accept \( H_a \)

   or

2. Critical value:
   - If test statistic is past critical value
   - Reject \( H_0 \), Accept \( H_a \)
Critical value

Hurdle point that determines if the Null Hypothesis is Rejected & the Alternative Hypothesis is Accepted. Calculate critical value based on Alpha.

- 1-tail to Left Lower Tail
- 2-Tail Test
- 1-tail to Right Upper Tail

Reject Ho Accept Ha

Fail To Reject Ho

Critical value = \(-Z_{\alpha}\)

\(-Z_{\alpha/2}\) - critical values - \(Z_{\alpha/2}\)

Fail To Reject Ho

Reject Ho Accept Ha

\(Z_{\alpha} = \text{critical value}\)
**P-value**  "observed level of significance"

Probability of getting the test statistic value or worse (less or more).

1 tail to Left

\[ \alpha = 0.05 \]

- Test statistic
- Critical value

\[ p\text{-value} = \text{probability of getting test statistic or less} \]

\[ p\text{-value} \times 1 \]

1 tail to Right

\[ \alpha = 0.05 \quad \alpha \div 2 = 0.025 \]

- Critical values
- Test statistic

\[ p\text{-value} = \text{probability of getting test statistic or more} \]

\[ p\text{-value} \times 1 \]

Two tail

\[ \alpha = 0.05 \quad \alpha \div 2 = 0.025 \]

- Critical values
- Test statistics

\[ p\text{-value from 1 side} = \text{probability of getting test statistic or worse (less or more)} \]

\[ \text{then,} \]

\[ p\text{-value} \times 2 \]

**Rejection Rule:**

\[ p\text{-value} \leq \alpha \], Reject Ho, Accept Ha
### Interpreting P-value

<table>
<thead>
<tr>
<th>P-value</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 0.10</td>
<td>Insufficient evidence to say $H_a$ True</td>
</tr>
<tr>
<td>0.05 ≤ P-value ≤ 0.10</td>
<td>Weak evidence to say $H_a$ True</td>
</tr>
<tr>
<td>0.01 ≤ P-value ≤ 0.05</td>
<td>Strong evidence to say $H_a$ True</td>
</tr>
<tr>
<td>≤ 0.01</td>
<td>Overwhelming evidence to say $H_a$ True</td>
</tr>
</tbody>
</table>

Advantage of p-value (over critical value) is that it tells you how significant the results are:

1. What probability of getting a test statistic or worse (less or more)
2. What the Type I Error Rate is, like we got $\hat{88,595}$ for $x \bar{\Sigma}$ at that value would appear as a True sample mean w/ $M = 85,000$ 4 in 100 times.
Test statistic (z or t) for Hypothesis Testing About a Population Mean

<table>
<thead>
<tr>
<th>Known σ</th>
<th>Not Known σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$</td>
<td>$t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$</td>
</tr>
</tbody>
</table>

- $\mu_0 =$ hypothesized mean
- $z$ & $t =$ calculated test statistic, used to determine whether to reject the null hypothesis. Compare $z$ or $t$ to critical value to make decision, or used to calculate $p$-value. $z \pm t =$ number of standard errors above/below hypothesized mean.

- $\bar{x} =$ sample mean
- $\sigma =$ population standard deviation
- $s =$ sample standard deviation
- $n =$ sample size
Test statistic for Hypothesis Tests About A Population Proportion

\[ Z = \frac{\bar{p} - p_0}{\sqrt{p_0 \times (1-p_0)/n}} \]

\[ \bar{p} = \text{sample proportion} = \frac{\text{successes}}{n} \]

\[ p_0 = \text{hypothesized pop. proportion} \]

\[ n = \text{sample size} \]

\[ SE = \sigma_{\bar{p}} = \sqrt{\frac{p_0 \times (1-p_0)}{n}} \]

Must verify:

1. Are there fixed # Trials?
2. Are results Independent?
3. Does each Trial result in success or failure?
4. \( p \) stay same on each trial?
5. \( n \times p > 5 \)
   \( n \times (1-p) > 5 \)

- text book assumes true for all problems.

\[ \text{since exact sampling distribution of } \bar{p} \left( \text{Poisson} \right) \]

is Discrete, small samples require additional steps that we will not do in this textbook.
The histogram shows non-normal or outliers, increasing the sample size and improve the calculations.

Although histogram is not conclusive, sometimes it may be the best clue that you have.
If the population distribution is not known a histogram based on a sample may give you a clue.

**Notes:**

1. If pop distribution is not normal, n > 30 usually adequate
2. If pop distribution is normal or near normal, smaller than 30 sample size may be used
3. If pop distribution is skewed or has outliers, n > 50 should be used

A: When population distribution is normally distributed or near normal, or is sufficiently large enough
Q: When are we allowed to use t distribution?
P-value = A - T.DIST(T(\(\bar{x}, df, z\)), df)

\[ T \cdot \text{INV}(1 - \alpha, df) \]

Upper critical value

P-value = T.DIST(upper, \(\bar{x}, df, I\))

Z on Lowend

\[ -\text{NORM.S.INV}(1 - \alpha/2) \]

Lower critical value

P-value = T.DIST(lower, \(\bar{x}, df, I\))

Two-tail

\[ \text{NORM.S.INV}(\alpha/2) \]

When to use:

Tail to Right

A tail to left

Tail to Left

Z distribution

Excel Functions

\[ \Sigma \text{not known} \]

\[ \Sigma \text{known} \]

Populations, when 4 tests met.
<table>
<thead>
<tr>
<th>Test Type</th>
<th>Hypothesis Testing</th>
<th>Z Distribution (Sigma Known)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>α</td>
<td></td>
</tr>
<tr>
<td></td>
<td>critical value</td>
<td></td>
</tr>
</tbody>
</table>

**1 Tail Test to Left**

- Lower Tail
- \( H_0: M \geq M_0 \)
- \( H_a: M < M_0 \)

**2 Tail Test**

- \( H_0: M = M_0 \)
- \( H_a: M \neq M_0 \)

**Upper Tail**

- \( H_0: M \leq M_0 \)
- \( H_a: M > M_0 \)

**Test Statistic**

\[
Z = \frac{\bar{X} - M_0}{\sigma / \sqrt{n}}
\]

**P-Value Rejection Rule**

\[ \text{IF: } p\text{-value} \leq \alpha \]
\[ \text{THEN: Reject } H_0, \text{ Accept } H_a \]

**Excel p-Value**

- \( = \text{NORM.S.DIST}(Z, 1) \)
- \( = \text{NORM.S.DIST}(Z, 1) \times 2 \)
- \( = 1 - \text{NORM.S.DIST}(Z, 1) \)

**Critical Value Rejection Rule**

- \( Z \leq -Z_{\alpha} \)
- Then: Reject \( H_0 \), Accept \( H_a \)
- \( -Z_{\alpha} = \text{critical value (low end)} \)

- \( Z \geq Z_{\alpha} \)
- Then: Reject \( H_0 \), Accept \( H_a \)
- \( Z_{\alpha} = \text{Critical Value} = \text{NORM.S.INV}(1-\alpha) \)

**Excel Critical Value**

- \( -Z_{\alpha} = \text{NORM.S.INV}(\alpha) \)
- \( Z_{\alpha/2} = \text{Upper critical value} \)
- \( +/\!/- \text{critical value} = \) \( \text{NORM.S.INV}(\alpha/2) \)
# Hypothesis Testing

## Z Distribution (Proportions)

<table>
<thead>
<tr>
<th>Test Type</th>
<th>Hypothesis</th>
<th>Test Statistic</th>
<th>( p )-value Rejection Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Tail Test to Left</td>
<td>( H_0: \hat{p} \geq p_0 ) &lt;br&gt;( H_a: \hat{p} &lt; p_0 )</td>
<td>( { \text{Standard Error} } = SE = \sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} )</td>
<td>( IF: p\text{-value} \leq \alpha )&lt;br&gt;Then: Reject ( H_0 ), Accept ( H_a )</td>
</tr>
<tr>
<td>2 Tail Test</td>
<td>( H_0: \hat{p} = p_0 ) &lt;br&gt;( H_a: \hat{p} \neq p_0 )</td>
<td>( { \text{Standard Error} } = SE = \sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} )</td>
<td>( IF: \frac{\hat{p} - p_0}{SE} \leq -Z_{\alpha/2} )&lt;br&gt;( IF: \frac{\hat{p} - p_0}{SE} \geq Z_{\alpha/2} )&lt;br&gt;Then: Fail TO Reject ( H_0 )&lt;br&gt;( Z_{\alpha/2} = \text{low critical value (lower tail)} )&lt;br&gt;( Z_{\alpha/2} = \text{upper critical value} )</td>
</tr>
</tbody>
</table>
### Hypothesis Testing: t Distribution (Sigma Not Known)

<table>
<thead>
<tr>
<th>Test Type</th>
<th>Hypothesis</th>
<th>Test Statistic</th>
<th>( t = \frac{\bar{X} - M_0}{S/\sqrt{n}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>One Tail Test to Left</td>
<td>( H_0: M \geq M_0 ) ( H_a: M &lt; M_0 )</td>
<td>( \alpha )</td>
<td>( t = T \cdot \text{DIST}(t, df, 1) )</td>
</tr>
<tr>
<td>Two-Tail Test</td>
<td>( H_0: M = M_0 ) ( H_a: M \neq M_0 )</td>
<td>( \alpha/2 ) ( \alpha/2 )</td>
<td>( t = T \cdot \text{DIST}.2T(upper, df) )</td>
</tr>
<tr>
<td>One Tail Test to Right</td>
<td>( H_0: M \leq M_0 ) ( H_a: M &gt; M_0 )</td>
<td>( \alpha )</td>
<td>( t = T \cdot \text{DIST}.RT(t, df) )</td>
</tr>
</tbody>
</table>

#### P-value Rejection Rule

- If \( p \)-value \( \leq \alpha \), then: Reject \( H_0 \), Accept \( H_a \)

#### Excel P-value

- One Tail: \( = T \cdot \text{DIST}(t, df, 1) \)
- Two-Tail: \( = T \cdot \text{DIST}.2T(upper, df) \) \( = T \cdot \text{DIST}(lower, df, 1) \times 2 \)
- Right Tail: \( = T \cdot \text{DIST}.RT(t, df) \)

#### Critical Value

- One Tail: \( t = -t_\alpha \)
- Two-Tail: \( -t_{\alpha/2} < t < t_{\alpha/2} \)
- Right Tail: \( t = t_\alpha \)

- One Tail: \( -t_\alpha = T \cdot \text{INV}(\alpha, df) \)
- Two-Tail: \( t_{\alpha/2} = T \cdot \text{INV}(\alpha/2, df) \) \( t_{\alpha/2} = T \cdot \text{INV}(1 - \alpha/2, df) \)
- Right Tail: \( t_\alpha = T \cdot \text{INV}(1 - \alpha, df) \)
Example of step 3 (collect data, calculate test statistic, draw picture)

Because we know the population standard deviation \( \sigma = 12,549 \), we can use the test statistic, \( Z \).

\[
\begin{align*}
\bar{x} &= 88,595 \\
\sigma &= 12,549 \\
M &= M_{\bar{x}} = 85,000 \\
\sigma_{\bar{x}} &= \frac{12,549}{\sqrt{36}} = \frac{12,549}{6} = 2,091.50 \\
\alpha &= .05
\end{align*}
\]

\( H_0 : M \leq 85,000 \)

\( H_a : M > 85,000 \)

\[
\begin{align*}
\text{Critical value} &= \text{NORMSINV}(1-.05) = 1.6448 \\
M &= M_{\bar{x}} = 85,000 \\
\sigma_{\bar{x}} &= 2,091.50
\end{align*}
\]

Decision Rule:
If our test statistic is greater than or equal to 1.6448, we reject \( H_0 \) and accept \( H_a \).

\[
Z = \frac{\bar{x} - \theta}{\sigma_{\bar{x}}} = \frac{88,595 - 85,000}{2,091.50} = 1.72
\]
conclude with critical value & Rejection Rule

\[
\{ \text{Calculated test statistic} \} = \frac{88,595 - 85,000}{\frac{12549}{\sqrt{36}}} = 1.72
\]

**Make Decision:**

Because our calculated test statistic is greater than 1.645, we reject \( H_0 \) and accept \( H_1 \). It is reasonable to assume that the mean salary for real estate agents is greater than $85,000.

Based on the statistical evidence our \( \bar{X} \) of 88,595 is statistically significant & provides good evidence that the mean salary for real estate agents is greater than $85,000.
Because the p-value is less than alpha (0.04282 ≤ 0.05), we reject $H_0$ and accept $H_1$. It is reasonable to assume that the mean salary for real estate agents is greater than $85,000.
Summary for Real Estate Example:

- Population mean: \( \mu = M_{x} = M_{0} = 85,000 \)
- Sample mean: \( \bar{x} = 88,595 \)
- Size: \( n = 36 \)
- Population variance: \( \sigma^2 = 12,549 \)
- Standard Error: \( \sigma_{\bar{x}} = \frac{12,549}{\sqrt{36}} = 2,091.50 \)
- Significance level: \( \alpha = 0.05 \)

Test statistic:

\[
 t = \frac{88,595 - 85,000}{2,091.50} = 1.72
\]

Critical Value:

- Dividing point between the region where the Null Hypothesis is rejected and the region where it is not rejected.

\[
\alpha = 0.05
\]

p-value:

\[
p-value = 1 - \text{NORMSDIST}(1.72) = 0.04282
\]

Upper test:

\[
p-value = \text{NORMSDIST}(1 - \alpha) = \text{NORMSDIST}(1 - 0.05) = 0.6448
\]

Critical value = 1.6448
**Concluding:**

Use $z$ or $t$ to compare to Critical value

Use $p$-value to compare to alpha $\alpha$

---

**Critical Value**

- If $t$ or $Z \geq$ critical value, Reject $H_0$, Accept $H_a$
- If $t$ or $Z \leq$ critical value, Reject $H_0$, Accept $H_a$

---

$p$-value

- $p$-value $\leq \alpha$, Reject $H_0$
- $p$-value $> \alpha$, Accept $H_0$
Confidence Interval Hypothesis Testing

If:

\[ H_0: M = M_0 \]
\[ H_a: M \neq M_0 \]

Then:

\[ M_0 = \text{hypothesized population mean} \]

1. Select a simple random sample from the population and use the value of the sample mean \( \bar{X} \) to develop a confidence interval for the population mean \( M \).

\[ \bar{X} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \]

- \( \bar{X} \): sample mean
- \( Z_{\alpha/2} \): upper Z
- \( \sigma \): pop. S.D.
- \( n \): sample size

2. If the confidence interval contains the hypothesized value (\( M_0 \)) \( M_0 \), do not reject \( H_0 \), otherwise,

Reject \( H_0 \) (Reject \( H_0 \) if \( M_0 \) is one of the endpoints).

Example:

Accept \( H_0 \)
Reject $H_0$
Accept $H_a$

$H_0: M \geq 1056$
$H_a: M < 1056$

Fail to Reject $H_0$

$M_0 = 1056$
Ave. Tax Refund

Step 3

Step 4

$\alpha = 0.05$

Area = Probability of $-1.825$ or less

Test statistic $= z = -1.825$

$P$-value $= \text{NORM.S.DIST}(-1.825) = 0.034$

Probability of $-1.825$ or less
Step 3

\[ H_0 : \mu \leq 3173 \]
\[ H_a : \mu > 3173 \]

Fail to Reject \( H_0 \)

\[ M_0 = 3173 \]

credit card balance
undergraduate students

Step 4

\[ M_0 = 3173 \]

Critical value = 1.645

Test statistic = 2.039

Probability of 2.039 or more

\[ \alpha = 0.05 \]

\[ p-value = 0.0207 \]
Step 3

$H_0: \mu = \$125,000$

$H_a: \mu \neq \$125,000$

Fail to Reject $H_0$

$p$-value from low end $\times 2 = 0.0569 \times 2 = 0.1138$

$p$-value = 0.1138

$p$-value = probability of $-1.581$ or less or $1.581$ or more

$p$-value = 0.1138

Step 4

This probability $0.0569$

$\alpha = \frac{\alpha}{2} = 0.025$

Test statistic $= -1.581$
Step 3

Reject H₀
Accept H₁

\[ \alpha = 0.01 \]

Fail to Reject

\[ t \]

#28
ch. 9

Step 4

\[ \alpha = 0.01 \]

critical value = -2.37
Test statistic = -2.49997

\[ p-value = \text{probability of } -2.49997 \text{ or less} \]

\[ p-value = 0.0072 \]
**Step 3**

- Reject $H_0$ if $p-value < 0.025$

- Accept $H_a$

---

### Single low p-value = 0.015094

**DOUBLE IT!!!**

---

$\alpha/2 = 0.025$

**Critical Value**

$ t / - = 2.09575$

**Test Statistic**

$ t / - = 2.2324$

**P-value**

$2 \times 0.015094 = 0.030189$

**P-value** means probability of less than or equal to 2.23 or greater than or equal to 2.23
STEP 3

$H_0: M \leq 21.6$
$H_a: M > 21.6$

Fail To Reject $H_0$

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ch. 9

Reject $H_0$
Accept $H_a$

$\alpha = 0.05$

STEP 4

critical value = 1.753

test statistic = 2.05

$p\text{-value} = 0.02895 = \text{probability of 2.05 or more}$
$H_0: p \leq 0.1$
$H_a: p > 0.1$

Fail to Reject $H_0$

$p$-value = probability of getting test statistic of 1 or greater = 0.1587

critical value = 1.645

Test statistic = 1

Step 3

Step 4

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Ch. 9

p. 46

 Reject $H_0$
Accept $H_a$

$\alpha = 0.05$
Step 3: Fail to Reject $H_0$

Step 4: $p$-value = probability of getting a test statistic of $2.8$ or more = $p$-value = 0.00255

Critical value = 2.33

Test statistic = 2.8