Continuous Probability Distributions

Discrete Random Variable	Continuous Random Variable
① There are GAPs between numbers	① No GAPs between numbers
② can only assume clearly separated values	② can assume any value in interval or collection of intervals
③ usually from counting successes	③ depends on measuring instrument
④ Examples: - Number of bedrooms in house. - number of times late - number of times interrupted - number of Heads in 3 Flips	④ Examples: - weight of cereal box - score on statistics Excel Test - Salary Earned (Dollars) - Miles Auto Tires last
⑤ Discrete Probability Distribution	⑤ Continuous Probability Distribution

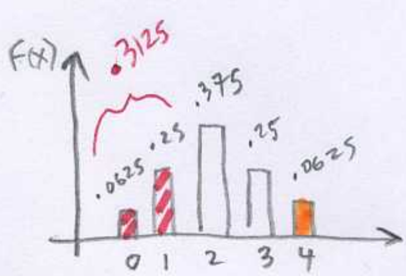
Both:

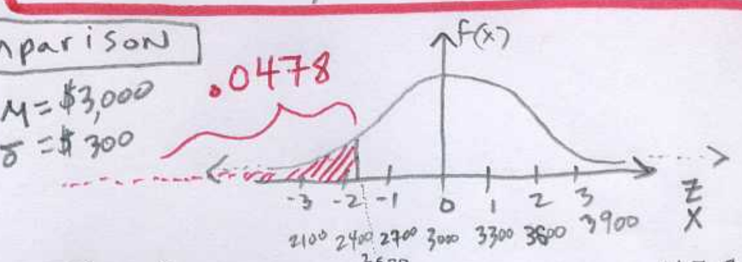
A listing of all the Random Variable outcomes of an Experiment & the probability associated with each outcome

Discrete Random Variable	Continuous Random Variable
⑥ Types: ① Binomial Distribution ② Poisson Distribution ③ Hypergeometric	⑥ Tyes we will study in this chapter: ① uniform Probability Distribution ② Normal (Bell) Prob. Distribution ③ Exponential Prob. Distribution
⑦ Questions we are allowed to ask: ① Probability of exactly X ② Probability between 2 X values	⑦ Questions we are allowed to ask: ① Probability between 2 X values we cannot calculate Probability for exactly X

Visual comparison

p = .5
n = 4





① $P(x=4) = .0625$

② $P(x \le 1) = .25 + .0625 = .3125$

① $P(X \le 2500) = P(X < 2500) = .0478$ *

⑧ How we calculate:
- Add individual Probabilities

⑧ How we calculate:
- Find area under curve between 2 values
- Area = Probability

* Because a line has No area, we cannot calculate $P(x=3)$ or $P(x=.12)$

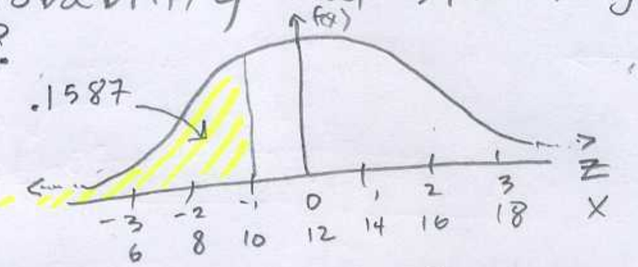
Continuous Probability Distributions:

we ask questions like:

Q: what is probability that student gets less than 20 on quiz?

All area = 1

$P(X \leq 10) = .1587$

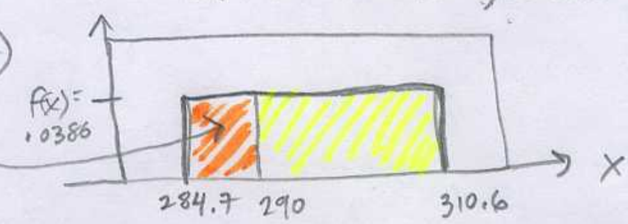


A: The probability will come from the area under the curve from $-\infty$ to 10 (x) or $-\infty$ to -1 (Z)

Q: what is probability that a top ten PGA golfer will drive between 284.7 & 290 yards?

All area = 1 (colored area)

$P(284.7 \leq X \leq 290) = .2046$



A: The probability will come from the area under the curve from 284.7 to 290

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* Area = Probability

* Lines Don't have area, thus:

① we can't calculate $P(X=3)$

② $P(X \leq 3) = P(X < 3)$

Discrete Functions	Continuous Functions
Binomial $F(x) = \frac{n!}{x!(n-x)!} * p^n * (1-p)^{(n-x)}$	Normal $F(x) = \frac{1}{\sigma\sqrt{2\pi}} * e^{-(x-\mu)^2/2\sigma^2}$
① can calculate probability	① cannot calculate Probability Directly
	② continuous Probability functions, f(x) (called Density functions) can calculate the height of curve & can help to Plot the curve & area, but we must use Integral Calculus to Find Area. Luckily →

→ Instead of Integral calculus we can use Excel functions that will calculate the area under the curve.

Note about chapter 6

In chapter 6 we are dealing with population data that make up Distribution.

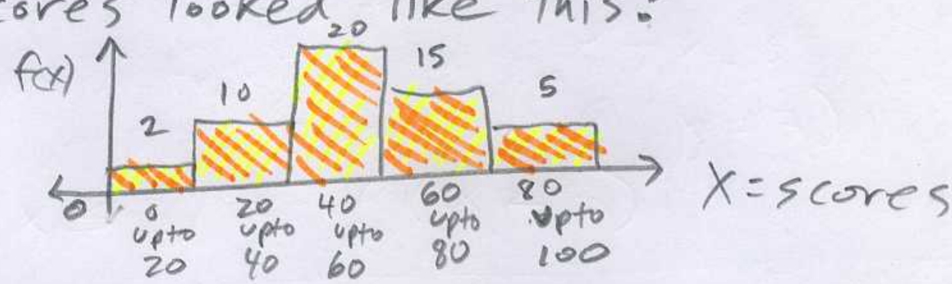
μ = mean of population
σ = Standard Deviation of population

Later we will deal with sample Data (after we learn "Central Limit Theorem"):

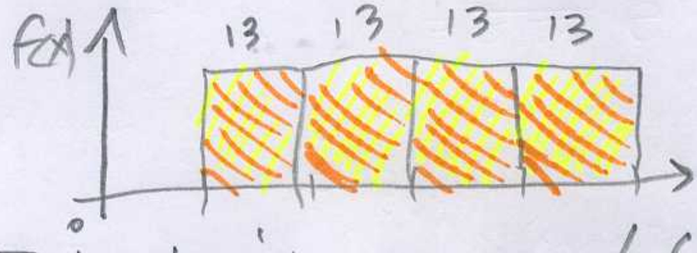
$\bar{X}$ = mean of sample
σ = SD of sample

Visual Introduction to Uniform Probability Distr.

Imagine the distribution of statistics Quiz scores looked like this:



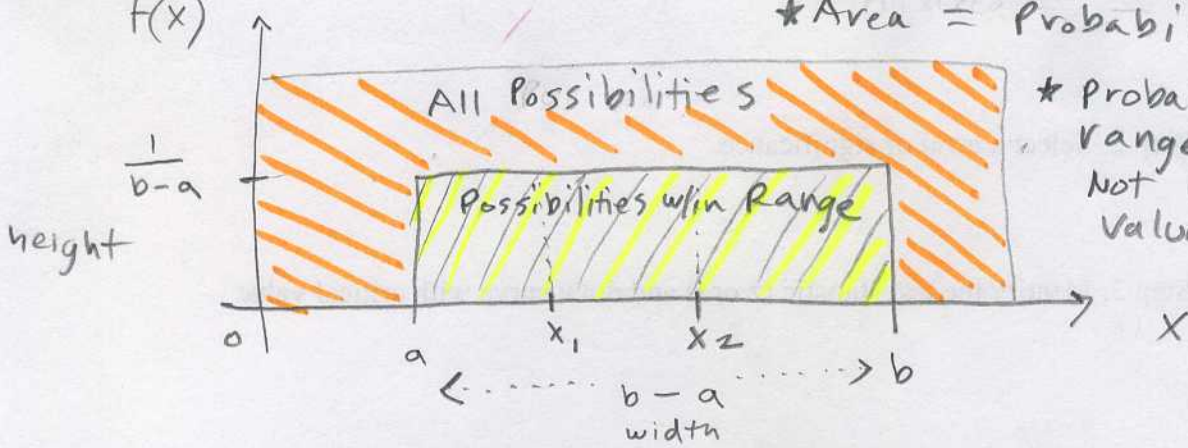
Now imagine if it looked like this:



Uniform Distribution!!

1 uniform Probability Distribution

* All area within Range = 1
* Area = Probability



* Probability over a range of values, Not for a particular value.

$$\text{Area} = \underbrace{1 = \text{height} * \text{width} = \text{height} * (b-a)}$$

$$1 = \text{height} * (b-a)$$

$$\frac{1}{(b-a)} = \text{height}$$ (height is not a probability)

2 UNIFORM PROBABILITY DENSITY FUNCTION

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \le X \le b \\ 0 & \text{elsewhere} \end{cases}$$ also $f(x) \ge 0$ for all X

a = Minimum value

b = Maximum value

X_1 = particular value #1

X_2 = particular value #2

Area = height * width = Probability = $P(X_1 \le X \le X_2) = \frac{1}{b-a} * (X_2 - X_1)$

Area = Probability = Area under graph of $f(x)$ over the interval X_1 to X_2.

$P(X = X_1)$ = Probability for particular value = 0. Lines have NO Area

(Pop Data)

μ = mean = $E(x) = \frac{a+b}{2}$

σ = standard Deviation = $\sqrt{\frac{(b-a)^2}{12}}$

Note: Because Lines have NO Area

$P(1 \le X \le 2) = P(1 < X < 2)$

endpoints included or Not!!

*height of probability Density function is not a probability. Lines have no area.

Example of Uniform Probability Distribution:

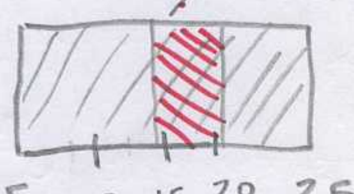
Suppose the time that you wait on the telephone for a live representative of your phone company to discuss your problem with you is uniformly distributed between 5 & 25 minutes.

1. What is the mean wait time?
2. What is standard deviation?
3. What is probability that you will wait between 15 & 20 minutes

Variables:

$X_1 = 15$ minutes.
$X_2 = 20$ minutes.
min $= a = 5$ minutes
max $= b = 25$ minutes

$$\frac{1}{20} = \frac{1}{25-5}$$



$$\mu = \frac{a+b}{2} = \frac{5+25}{2} = 15 \text{ minutes}$$

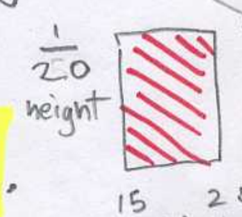
$$\sigma = \sqrt{\frac{(b-a)^2}{12}} = \sqrt{\frac{(25-5)^2}{12}} = \sqrt{\frac{400}{12}} = 5.77 \text{ minutes}$$

$$P(15 < X < 20) = \frac{1}{25-5} * (20-15) = .05 * 5 = .25$$

Finding Probability for Uniform Distribution is a simple Geometry problem of

width * height = $\boxed{\text{Area} = \text{Probability}}$.

$$\underset{\downarrow}{(20-15)} * \underset{\downarrow}{\frac{1}{20}} = \frac{5}{20} = .25$$

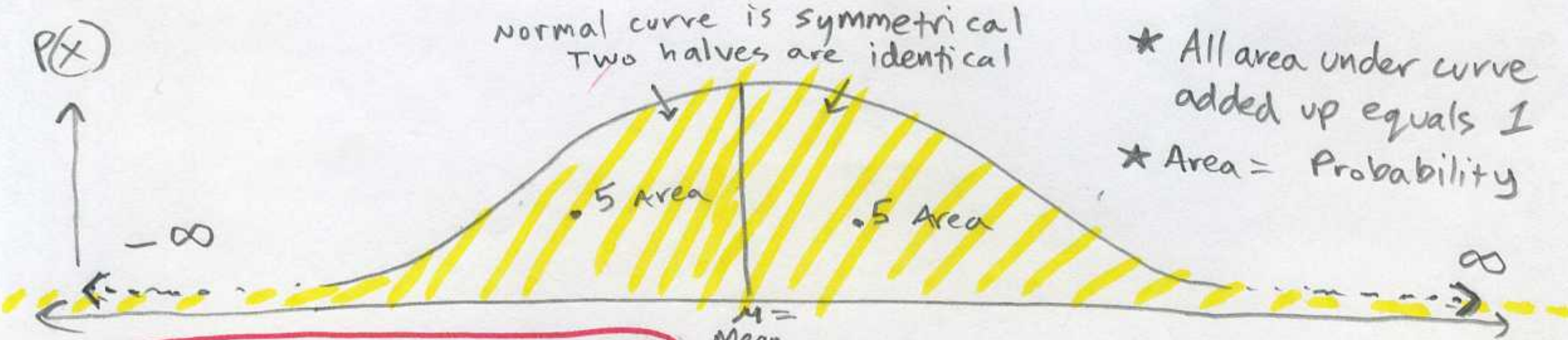


* Because Lines have No Area $P(15 \le X \le 20) = P(15 < X < 20)$

A: The average wait time is 15 minutes & the probability of waiting between 15 & 20 minutes is .25

③

Normal (Bell) Probability Distribution



★ All area under curve added up equals 1

★ Area = Probability

μ = Mean = median = mode

X

This is a Frequency Polygon (FP) that extends infinitely in both directions - the area under the FP line is our Probability.

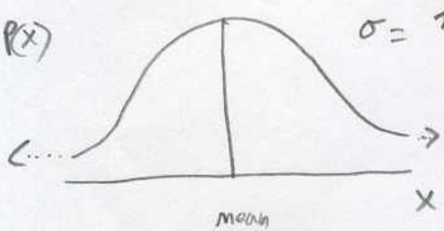
$-\infty \leftarrow$ Asymptotic in Both Directions $\rightarrow \infty$

Family of Normal Probability Distributions

As long as the Frequency Polygon looks like a bell shape when you graph it, then you have a Normal Probability Distribution. There are many Distributions that tend to follow the Normal (Bell) Distribution

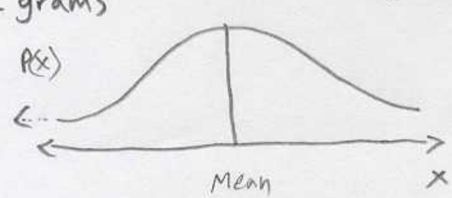
weight of cerial Box

μ = 250 grams
σ = 2 grams



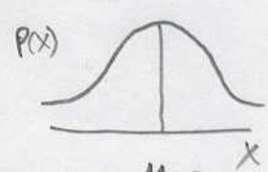
mean
median
mode

μ = 300 grams
σ = 3 grams

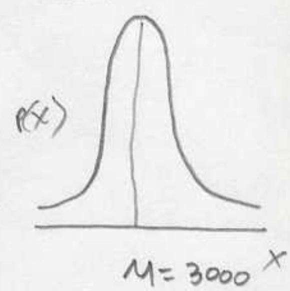


Mean
median
mode

Salaries



μ = 3000
σ = 400



μ = 3000
σ = 100

Excel:

For a normal probability distribution:

= NORMDIST(X, μ, σ, 1) ← cumulative from $-\infty$ to X