Basics of Probability

*spelling errors may occur because No red squiggly line came up.

1. Probability

a. probability = chance = likelihood

b. probability = chance that something will occur in future

or

A Numerical measure of the likelihood that an event will occur

If E_i = i^{th} experimental outcome (sample point)
$P(E_i)$ = probability of Event i (like roll a 6) $P(6)=\frac{1}{6}$

c. $0 \le P(E_i) <= 1$ for all i

d. $P(E_i)$ is never know with certainty (only estimate)

e. $P(E_i)$ is an estimate of an event that may occur in future

2. Experiment

a. A process that generates well-defined outcomes. On any single repetition of an experiment, one and only one of the possible experimental outcomes can occur.

or

Any Activity that has 2 or more possible "outcomes" → results and it is uncertain which will occur.

b. Examples:

Experiment		Experimental outcomes (sample points)
1. Toss a coin	⇒	Head, Tail
2. Drive on Bridge	⇒	stuck in Traffic, Not stuck
3. Roll Die	⇒	1, 2, 3, 4, 5, 6
4. Make product	⇒	Defect, Not Defect

3 Sample point (Experimental outcome)

ⓐ one of the experimental outcomes

ⓑ Example:

Experiment: Flip coin 1 time

sample Point = Head

sample Point = Tail

ⓒ Element of sample space

4 Sample space

ⓐ List of all possible experimental outcomes (sample points)

ⓑ Example:

{Sample space for Flip coin 1 time} = S = {Head, Tail}

{sample space for Flip coin 2 times} = S = {(H,H), (H,T), (T,T), (T,H)}

(each is sample point or 1 experimental outcome)

5 Multi-Step Experiment

ⓐ Experiment with more than 1 step.

ⓑ Example:

① Not a Multi-step Experiment: Flip coin 1 time

② Multi-step Experiments:
- Flip Coin 3 times
- Drive across bridge 7 times

6 Tree Diagram

a) Graphical representation that helps to visualize mult-step experiments

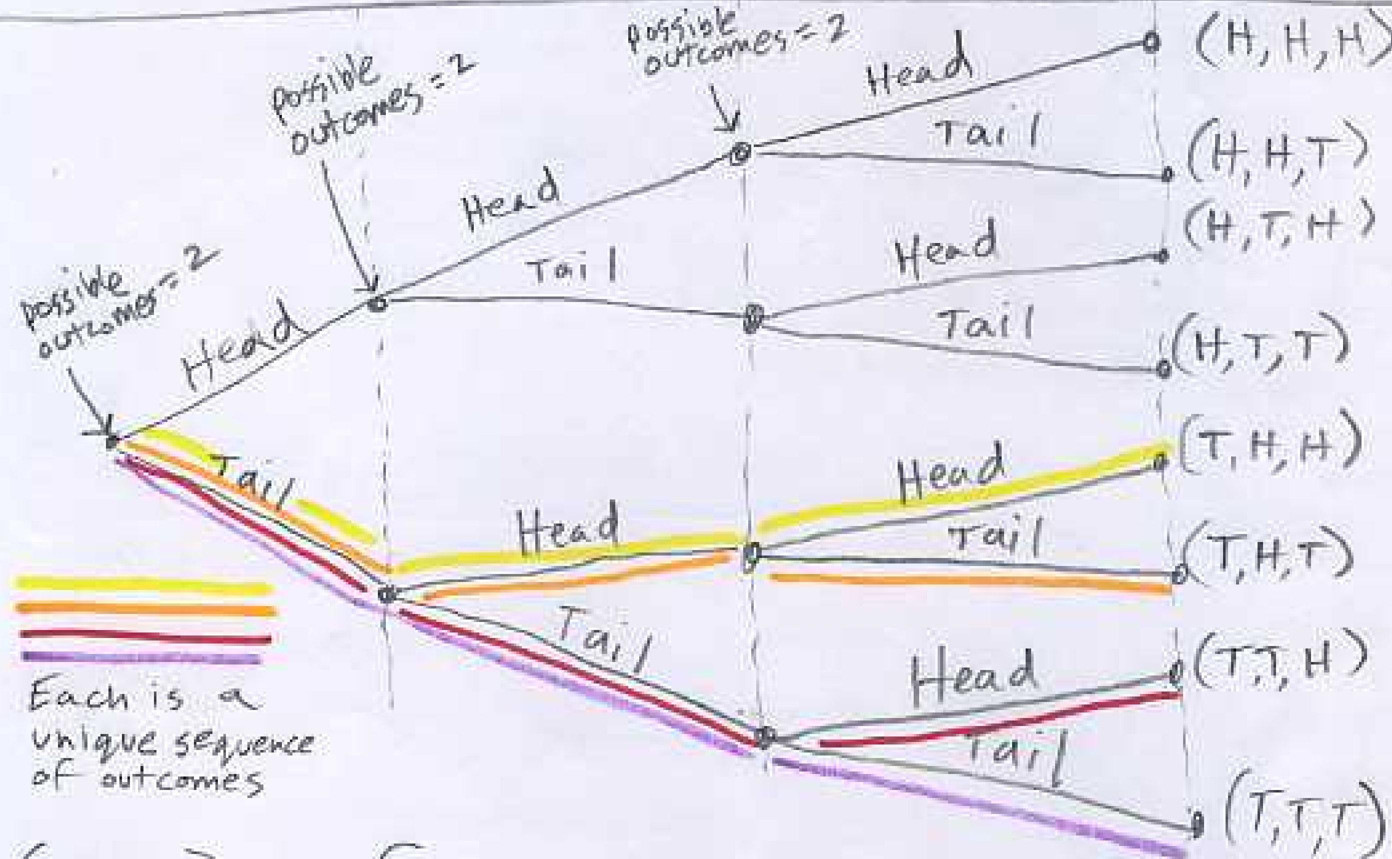
b) Without Tree Diagrams, it is hard to count all experimental outcomes (sample points)

Tend to under count

c) Example:

Multi-step Experiment: Toss coin 3 times - to see if we can see any patterns for getting a Head or Tail

Step 1	Step 2	Step 3	Sample space List of all Sample Points List of all Experimental outcomes



Each single one is sample point
whole List = sample space

$\begin{Bmatrix}\text{Sample}\\ \text{Space}\end{Bmatrix} = S = \{(H,H,H), (H,H,T), (H,T,H), (H,T,T), (T,H,H), (T,H,T), (T,T,H), (T,T,T)\}$

How many total outcomes in Experiment? Count = 8

(d) Alternative for visualizing Multi-step Experiment:

(Not as easy)

Experiment = Flip coin 3 times
H = Head T = Tail

	1st Toss	2nd Toss	3rd Toss	# Heads	Probability of Ex. outcome sample point
1	H	H	H	3	1/8 = .125
2	H	H	T	2	1/8 = .125
3	H	T	H	2	1/8 = .125
4	T	H	H	2	1/8 = .125
5	T	T	H	1	1/8 = .125
6	T	H	T	1	1/8 = .125
7	H	T	T	1	1/8 = .125
8	T	T	T	0	1/8 = .125

A Probabilities are: $0 \le P(E_i) \le 1$

$\left\{ \text{Add All Experimental outcome probabilities} \right\} = 1$

Each Probability is a number between 0 & 1

or

# of Heads in 3 Tosses	$P(E_i)$	$P(E_i)$
0	P(0 Heads)	1/8 = .125
1	P(1 Heads)	3/8 = .375
2	P(2 Heads)	3/8 = .375
3	P(3 Heads	1/8 = .125

all are between 0 & 1

$\sum P(E_i) = 1$

7 Counting Rule for Multi-step Experiment

(a) $\left\{\begin{matrix}\text{total Number} \\ \text{of Experimental} \\ \text{outcomes} \\ \text{(sample points)}\end{matrix}\right\} = n_1 * n_2 * \ldots * n_k = \left\{\begin{matrix}\text{size} \\ \text{of} \\ \text{sample} \\ \text{space}\end{matrix}\right\}$

If n is same each step $\{\text{total \#}\} = n^k$

k = number of steps in experiment

n_1 = number of possible outcomes step 1

n_2 = number of possible outcomes step 2

(b) Example:

Experiment: Flip coin 3 times

$n_1 = 2$ (Heads or Tails)

$n_2 = 2$ (Heads or Tails)

$n_3 = 2$ (Heads or Tails)

$K = 3$

$\left\{\begin{matrix}\text{Total number of} \\ \text{sample points} \\ \text{(Experimental} \\ \text{outcomes)}\end{matrix}\right\} = 2 * 2 * 2 = 8$

Because n is same each time:

$\left\{\begin{matrix}\text{total \#} \\ \text{outcomes}\end{matrix}\right\} = n^k = 2^3 = 8$

7) ⓒ Example:

Experiment: Roll 2 dice

$n_1 = 6$

$n_2 = 6$

$K = 2$

{Total # of sample points} $6 * 6 = 6^2 = 36$

sample space (List of sample points)

	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

↳ sometimes you want to find total # of sample points (Experimental outcomes) in a slightly different situation.

$n! =$ n factorial

example: 1 $5! = 5*4*3*2*1$

$5! = 120$

2

$$\frac{10!}{7!} = \frac{10*9*8*\cancel{7*6*5*4*3*2*1}}{\cancel{7*6*5*4*3*2*1}}$$

$= 10*9*8$

$= 90*8$

$= 720$

DON'T FORGET:

$0! = 1$

example $\frac{4!}{(4-4)!} = \frac{4!}{0!} = \frac{4!}{1} = 24$

Use Excel

=FACT(10)/FACT(7)

= 720

Remember:

$$\bar{x} = \frac{\Sigma x}{n}$$

$$\mu = \frac{\Sigma x}{N}$$

N = # of items in population
n = # of items in sample

we would like to be able to do this:

Find all possible combinations of sample size n, from a population with size N.

8 Count total number of experimental outcomes (sample points) when selecting n objects from a set of N objects (order does not matter $(1,2)=(2,1)$)

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a) Counting Rule for Combinations (order does not matter)

$$_{N}C_{n} = C_{n}^{N} = \binom{N}{n} = \frac{N!}{n!\,(N-n)!}$$

N = count of all objects (pop. size)
n = objects taken
items selected (sample size)
subset

9 Counting Rule for Permutations $\left(\begin{matrix}\text{order matters}\\ (2,1) \neq (1,2)\end{matrix}\right)$

$$ {}_N P_n = P^N_n = \binom{N}{n} = \frac{N!}{(N-n)!} $$

p. 9

$$_NC_n = \frac{N!}{n!\,(N-n)!}$$

Example ① The Dawson's Basketball Team has 12 players. How many 5 person teams can there be?

N = 12
n = 5

$$_{12}C_5 = \frac{12!}{5!\,(12-5)!} = \frac{12\cdot11\cdot10\cdot9\cdot8\cdot\cancel{7}\cdot\cancel{6}\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}{5\cdot4\cdot3\cdot2\cdot1 * \cancel{7}\cdot\cancel{6}\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}$$

$$= \frac{12\cdot11\cdot10\cdot9\cdot8}{5\cdot4\cdot3\cdot2\cdot1} = \frac{\cancel{12}\cdot11\cdot\cancel{10}\cdot9\cdot8}{\cancel{12}\cdot\cancel{10}\cdot1} = 11*9*8$$

$$= 99*8 = 792$$

$\boxed{\text{use Excel}}$

= COMBIN(12,5) = 792

$${}_{n}P_{r} = \frac{N!}{(N-n)!}$$

Example ① Che wants to find out how many teams he can create from 12 players with each team having 5 players & he wants to rank each team (J, S, T, C, Z is different than S, J, T, C, Z)

$${}_{12}P_{5} = \frac{12!}{(12-5)!} = 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8$$

$$= 95040$$

Use Excel

= PERMUT(12, 5) = 95040

Boomerang B
Boomerang C
Boomerang S
Boomerang D

{ we would like to Find all possible comb. of sample size 2, from population size = 4 (used later w/ central Limit Theorem) }

Randomly select 2 of 4 to test. order Does not matter

$N = 4$
$n = 2$

$${}_4C_2 = \frac{4!}{2!(4-2)!} = \frac{\cancel{1} * \cancel{2} * 3 * 4}{1 * 2(1 * 2)} = \frac{12}{2} = 6$$

Excel = COMBIN(number, number_chosen)
= COMBIN(N, n)
= COMBIN(4,2) = 6

Experimental outcomes = BC, BS, BD, CS, CD, DS

order Does matter

$${}_4P_2 = \frac{4!}{(4-2)!} = \frac{1 * 2 * 3 * 4}{1 * 2} = 12$$

Experimental outcomes = BC, BS, BD, CB, CS, CD, SB, SC, SD, DB, DC, DS

10 Methods of Assigning Probability

(a) Classical Method

All experimental outcomes are equally likely.

$$P(E_i) = \frac{\text{\# of favorable outcomes}}{\text{total \# of Possible outcomes}}$$

Example: [1] Roll die, 6 equally likely outcomes

$P(\text{Roll a } 6) = \frac{1}{6}$

[2] Get Audited by district office, 3000 out of 3,000,000 Tax Returns

$P(\text{Audit}) = \frac{3000}{3000000} = \frac{1}{1000}$

(b) Relative Frequency Method

Use past Data to predict Future

or

Use past Data Available to estimate the proportion of the time the experimental outcome will occur if the Experiment is repeated a large number of times.

Example: instructor gave 10 A's out of 100 in past class. $P(A) = \frac{10}{100} = \frac{1}{10}$

Law of Large Numbers; over a large number of Trials, Relative Frequency will approach True probability.

(c) Subjective Method

[1] can't realistically assume outcomes equally likely

[2] Little Past Data exists

[3] You will use the best information you have, but it will be your personal belief.

Examples: who will win Super Bowl, merger between Google & Microsoft

11 Requirements for Assigning Probabilities (All 3)

Prob. between 0 & 1 $\boxed{1}$ $0 \leq P(E_i) \leq 1$ for all i

E_i = i^{th} Experimental outcome

$P(E_i)$ = probability

Sum of All = 1 $\boxed{2}$ $P(E_1) + P(E_2) + \ldots + P(E_n) = 1$

sum of the probabilities for <u>ALL</u> experimental outcomes must be one!

Example: Look back to page 4

12 **Event** = An event is a collection of sample points (1 or more)

*Note: sample points & events provide the foundation for the study of probability

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Experiment: Toss coin 3 times

Success = H

sample point = Experiments outcome

Total sample points	Toss 1	Toss 2	Toss 3	List of sample points	# of H	Probability for each is equal
1	H	H	H	H, H, H	3	1/8 = .125
2	H	H	T	H, H, T	2	1/8 = .125
3	H	T	H	H, T, H	2	1/8 = .125
4	T	H	H	T, H, H	2	1/8 = .125
5	H	T	T	H, T, T	1	1/8 = .125
6	T	H	T	T, H, T	1	1/8 = .125
7	T	T	H	T, T, H	1	1/8 = .125
8	T	T	T	T, T, T	0	1/8 = .125
						$\Sigma = 1$

$$\begin{array}{r} .125 \\ .125 \\ +\ .125 \\ \hline = .375 \end{array}$$

or

$3 * .125 = .375$

or

$\frac{3}{8} = .375$

Event = get 2 heads in 3 Flips = { 3 sample points match this requirement }

13 **Probability of an Event**

The probability of any event is equal to the sum of the probabilities of the sample points in the event. (Not always possible to list all sample points)

$S = \{(H,H,T),(H,T,H),(T,H,H)\}$

$$P\left(\text{get 2 Heads in 3 Tries}\right) = \begin{array}{r} .125\ P(HHT) \\ +\ .125\ P(HTH) \\ +\ .125\ P(THH) \\ \hline .375 \end{array}$$

(14) Because S, sample space, is an event, and it contains all sample points:

$$P(S) = 1$$

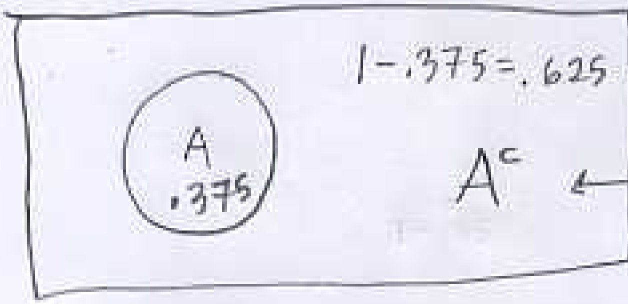
(15) when the classical method is used to assign probability (all sample points are equally likely) $\Rightarrow$

$$P(E_i) = \frac{\text{\# of favorable Exp. outcomes}}{\text{total \# of All Exp. outcomes}}$$

16 Venn Diagram

Event A = Get 2 Heads in 3 Flips.

A^c = Not Event A (0, 1 or 3 rolls)



S = sample space

complement of Event A

$P(S) = 1$

$P(A) = .375$

$P(A^c) = 1 - .375 = .625$

17 Complement Rule

$P(A^c) = 1 - P(A)$

or

$P(A) = 1 - P(A^c)$

"Dating only one person"

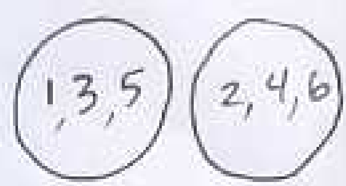
18 Mutually Exclusive

Two events are said to be mutually exclusive if the events have no sample points in common.

Example:

$Event_1$ = Roll odd number w/ 1 die

$Event_2$ = Roll even number w/ 1 die



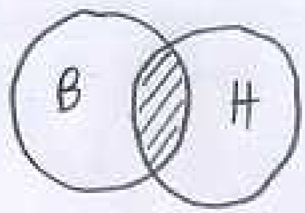
Sample Points Event 1 = 1, 3, 5

Sample Points Event 2 = 2, 4, 6

None in common. Therefore the events are mutually exclusive

$Event_B$ = have brown hair = B

$Event_H$ = have hazel eyes = H



Events are not mutually exclusive

(19) **Union**

The union of B and H is the event containing all sample points belonging to B or H or Both

symbol

union = B or H or Both = $\cup$ = OR = "at least 1" = "1 or more"

(20) **Intersection**

Given two events B & H, the intersection of B and H is the event containing the sample points belonging to both B and H.

symbol

Intersection = A and B = $\cap$ = AND = Both = Joint = concurrent = Both must be True

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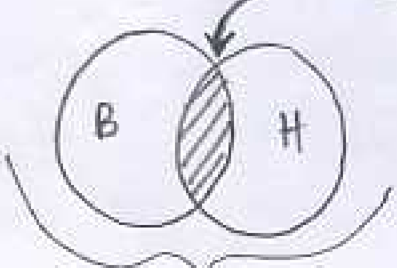
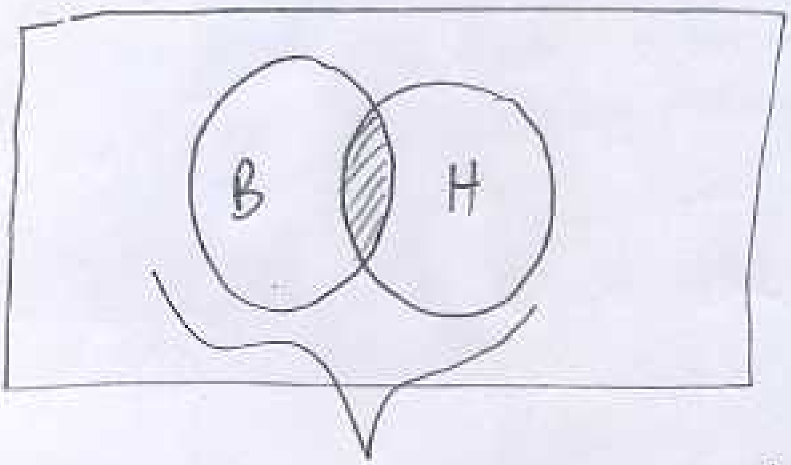
$P(B \cap H) = P(B \text{ AND } H)$ = Intersection



union = $P(B \cup H) = P(B \text{ OR } H)$ = Have Brown Hair or Hazel eyes or Both

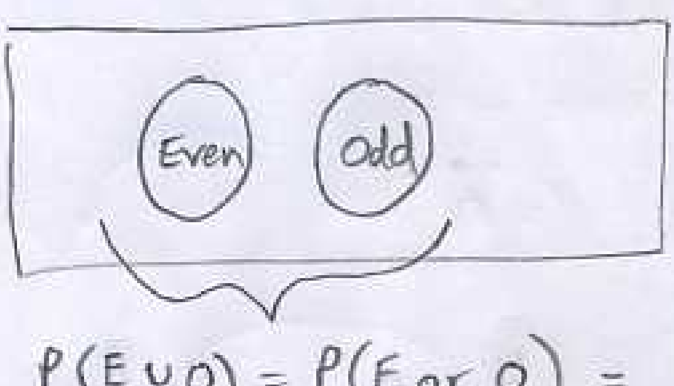
21 Addition Law for Probability



Not mutually Exclusive

$$P(B \cup H) = P(B \text{ or } H) = P(B) + P(H) - P(B \text{ and } H)$$

most subtract so we don't Double count!!!



Mutually Exclusive

$$P(E \cup O) = P(E \text{ or } O) = P(E) + P(O)$$

Example 1:

Flight Arrival		Frequency
Early	(E)	100
on time	(OT)	800
Late	(L)	75
Canceled	(C)	25
		1000

Mutually Exclusive Events

$P(E) = \frac{100}{1000} = .1$

$P(E \text{ or } OT) = P(E \cup OT) = \frac{(800+100)}{1000} = .9$

Example 2:

Travel survey of people who visit Seattle

visit Space Needle = SN
Visit Pike's Place Market = PP

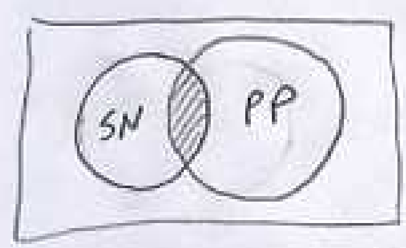
Not M.E. events

$P(SN) = .45$

$P(PP) = .65$

$P(SN \text{ and } PP) = .25 = P(SN \cap PP)$

$P(SN \text{ or } PP) = .45 + .65 - .25$



must subtract so we don't count twice!!

* Note problem may say: P(at least one site)
P(Either site)

(22) Conditional Probability $P(A \mid B)$

"given that"

① The probability of an event "given that" another event already occurred.

② sample space has changed.

Example: Probability of pulling a Queen from a deck of cards (without replacement) affects the prob. of pulling the next card.

Event Q_1 = Pull Queen from 52 card deck
Event Q_2 = Pull 2nd Queen after 1st Queen Pulled

$$P(Q_1) = \frac{4}{52}$$

(51 cards left - sample space has changed)

$$P(Q_2 \mid Q_1) = \frac{3}{51}$$

"given that"

① conditional probability
② Events are dependent
③ Events are not independent
④ sample space has changed
⑤ Q_1 is alread know to exist before we calculate $P(Q_2 \mid Q_1)$

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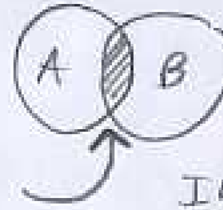
Joint Probability



$P(A \text{ and } B) = P(A \cap B) =$

Intersection
And
Joint
concurrent

Example:

	(M) Male	(F) Female	total
(P) promoted	288	36	324
Not Promoted (NP)	672	204	876
	960	240	1200

24

Joint Probability Table

Marginal Prob.
Joint Probabilities

	M	F	total
P	$\frac{288}{1200} = .24$	$\frac{36}{1200} = .03$	$.24 + .03 = .27$
NP	$\frac{672}{1200} = .56$	$\frac{204}{1200} = .17$	$.56 + .17 = .73$
total	$.24 + .56 = .8$	$.03 + .17 = .2$	$.27 + .73 = 1$ or $.8 + .2 = 1$

Joint

$P(M \text{ and } P) = .24$

$P(F \text{ and } P) = .03$

$P(M \text{ and } NP) = .56$

$P(F \text{ and } NP) = .17$

$P(M) = \frac{960}{1200} = .8$

$P(F) = \frac{240}{1200} = .2$

Marginal Prob.

conditional Probabilities

$P(P \mid M) = \frac{288}{960} = .3$

$P(P \mid F) = \frac{36}{204} = .15$

does not prove discrimination But does support the argument presented by Females that there is discrimination

Probability Rules of Multiplication

Independent

25 ① Events are independent if the occurrence of one event does NOT affect the occurrence of another event

② sample space is not changed.

③ Rule of Independence $P(B) = P(B/A)$ or $P(A) = P(A|B)$ {A unaffected by occurrence of B

Example: ① Rolling a 6, does not affect the next roll

② A traffic jam today, generally does not affect whether you are in traffic jam tomorrow

③ whether or not Whole Foods Market stock price goes up does not affect whether or not Google's goes up.

Dependent

① occurrence of one event affects the occurrence of another event

② sample space is changed

Example: ① Probability of pulling Queen from a deck of cards (without replacement) affects the probability of pulling the next card.

1st $P(Q_1) = \frac{4}{52}$

2nd $P(Q_2) = \frac{4}{51}$ or $\frac{3}{51}$ Depending on what you pulled on 1st

26 Multiplication Law for Probability

$$P(A \text{ and } B) = P(A \cap B) = P(B) * P(A|B)$$

or

$$P(A \text{ and } B) = P(A \cap B) = P(A) * P(B|A)$$

If events are independent:

since $P(B) = P(B/A)$

$P(A) = P(A|B)$

then

27 multiplication Law (Independence)

$$P(A \text{ and } B) = P(A \cap B) = P(A) * P(B)$$

Hint: And $= \cap = *$

OR $= \cup = +$

Note: Two events with non zero probabilities cannot be both M.E. → mutually exclusive & independent. Mutually exclusive means an event occurred & the other did not. Independent means the two exist but they are not related. M.E. event and other zero prob. event are dependent.

$$P(A \text{ and } B) = P(A) * P(B \mid A)$$

What if we want to solve for this?

Divide Both sides by $P(A)$

we get....

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{P(A \cap B)}{P(A)}$$

28 Calculate Conditional Probabilities

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{P(A \cap B)}{P(B)}$$

or

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{P(A \cap B)}{P(\ \ )}$$

A

Multiplication

$P(W \text{ and } G) = P(W) * P(G)$ *Events must be independent

example ①

$P\left(\begin{matrix}\text{WFMI stock}\\ \text{goes up}\\ \text{in next year}\end{matrix}\right) = .35$

$P\left(\begin{matrix}\text{GOOG stock}\\ \text{go up in}\\ \text{next year}\end{matrix}\right) = .20$

$P(W \text{ and } G) = P(W) * P(G) = .35 * .20 = .07$

②

$P\left(\begin{matrix}\text{defective product}\\ \text{coming of assembly}\\ \text{line}\end{matrix}\right) = .02$

$P\left(\begin{matrix}\text{Randomly selecting}\\ \text{3 Products and}\\ \text{all 3 are defective}\end{matrix}\right) = .02 * .02 * .02 =$

$= .000008$

---

multiplication

$= P(Q_1 \text{ and } Q_2) = P(Q_1) * P(Q_2 | Q_1)$

"given that"

① Events are not independent
② Events are dependent
③ conditional probability
④ sample space has changed
⑤ "Probability that a second Queen will be selected <u>given that</u> we already selected one card"

Example: ①

$P\left(\begin{matrix}\text{select 2}\\ \text{Straight}\\ \text{Queens}\end{matrix}\right) = P(Q_1) * P(Q_2 | Q_1) = \frac{4}{52} * \frac{3}{51} \approx .004525$

example 2

18 women in stats class
13 men in stats class
31 total

$$P\left(\begin{array}{l}\text{selecting 5 straight women}\\ \text{names from hat of all}\\ \text{names}\end{array}\right) = \frac{18}{31} * \frac{17}{30} * \frac{16}{29} * \frac{15}{28} * \frac{14}{27} =$$

Joint
"concurrent"
"and"

conditional
"given that"

$= P$(5 straight W. w/out replacement)

$= .0504264$

example 3

$$P\left(\begin{array}{l}\text{10 boomerangs}\\ \text{on table, 2 with}\\ \text{Blemish, probability}\\ \text{of pull 2 straight}\\ \text{w/ Blemish}\end{array}\right) = \frac{2}{10} * \frac{1}{9} = \frac{2}{90} = \frac{1}{45} = .0\overline{2}$$

---

success = pull woman name.

= HYPGEOMDIST(5, 5, 18, 31) = .050426

\# of successes

\# of sample

\# successes in population

population size

seattle survey

Example: - visit space needle = SN
- visit Pike's Place Market = PP

$P(SN) = .45$

$P(PP) = .65$

$P(SN \& PP) = .25$

Find $P(SN \mid PP) = \dfrac{P(SN \text{ and } PP)}{P(PP)}$

$P(SN \mid PP) = \dfrac{.25}{.65} = \dfrac{5}{13} = .3846$

Executives were asked whether or not they would remain with the company if they received a better offer from a different company

Loyalty	Length of Service: Less than 1 year	1-5 years	6-10 years	More than 10 years	Total
Would remain	10	30	5	75	120
Would not remain	25	15	10	30	80
Total	35	45	15	105	200

Joint Probability — Root Prob. — Conditional Prob.

And $*$ P(Randomly selecting executive who Would remain and who has More than 10 years) $= P(WR) * P(>10 \mid WR) = \frac{120}{200} * \frac{75}{120} = \frac{75}{200} = \frac{5 \cdot 5 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 5 \cdot 5} = \frac{3}{8} = .375$

And $*$ P(Randomly selecting executive who Would not remain and who has More than 10 years) $= P(WNR) * P(>10 \mid WNR) = \frac{80}{200} * \frac{30}{80} = \frac{30}{200} = \frac{3}{20} = .15$

OR $+$ P(Randomly selecting executive who Would not remain or Less than 1 year) $= P(WNR) + P(<1) - P(WNR \text{ and } <1) = \frac{80}{200} + \frac{35}{200} - \frac{25}{200} = \frac{90}{200} = .45$

| P(Randomly selecting executive who Would not remain given that 1-5 years) $= P(WNR \mid 1\text{-}5 \text{ years}) = \frac{15}{45} = \frac{3}{9} = \frac{1}{3} = .\overline{3}$

$P\left(\text{randomly select executive with 1-5 years service}\right) = \frac{45}{200} = .225$

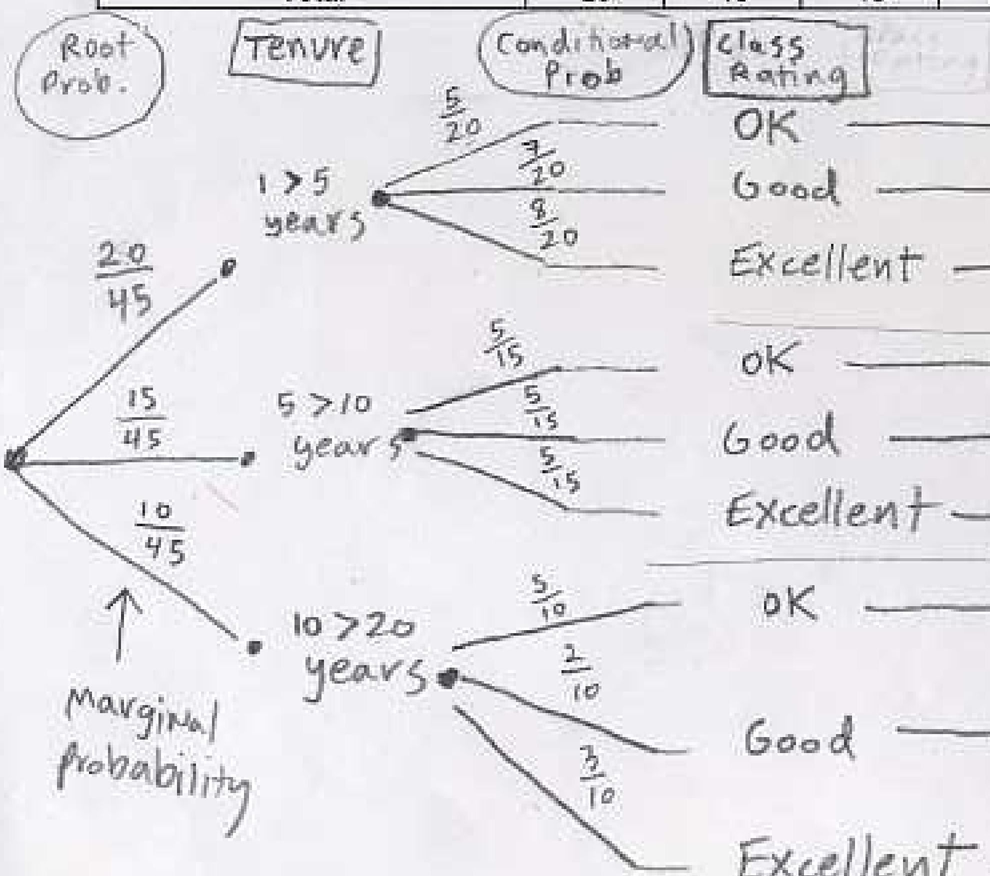
Tree Diagram

shows all conditional and joint probabilities for a contingency table

1. Heavy dot represents root
2. Draw branches with Root Prob.
3. Draw branches with conditional Prob.
4. Write out Joint Probabilities

Tenure and Class ratings

Class rating	Had Tenure for # of years 1 > 5 years	5 > 10 years	10 > 20 years	Total
OK	5	5	5	15
Good	7	5	2	14
Excellent	8	5	3	16
Total	20	15	10	45



Joint probabilities

OK → $\frac{20}{45} * \frac{5}{20} = \frac{5}{45} = \frac{1}{9}$

Good → $\frac{20}{45} * \frac{7}{20} = \frac{7}{45} = \frac{7}{45}$

Excellent → $\frac{20}{45} * \frac{8}{20} = \frac{8}{45} = \frac{8}{45}$

OK → $\frac{15}{45} * \frac{5}{15} = \frac{5}{45} = \frac{1}{9}$

Good → $\frac{15}{45} * \frac{5}{15} = \frac{5}{45} = \frac{1}{9}$

Excellent → $\frac{15}{45} * \frac{5}{15} = \frac{5}{45} = \frac{1}{9}$

OK → $\frac{10}{45} * \frac{5}{10} = \frac{5}{45} = \frac{1}{9}$

Good → $\frac{10}{45} * \frac{2}{10} = \frac{2}{45} = \frac{2}{45}$

Excellent → $\frac{10}{45} * \frac{3}{10} = \frac{3}{45} = \frac{1}{15}$

Problem 7

A deli bar offers a special sandwich for which there is a choice of five different cheeses, four different meat selections, and three different rolls. How many different sandwich combinations are possible?

$5 * 4 * 3 = 60$

Problem 8

Three scholarships are available for needy students. Their values are: $2,000, $2,400, and $3,000. Twelve students have applied and no student may receive more than one scholarship. Assuming all twelve students are in need of funds, how many different ways could the scholarships be awarded?

$n = 12$
$r = 3$

$${}_nP_r = \frac{n!}{(n-r)!}$$

$${}_nP_r = {}_{12}P_3 = \frac{12!}{(12-3)!} = 12 * 11 * 10 = 1320$$

The scholarships could be awarded 1320 different ways

Problem 9

The basketball coach of Dalton University is quite concerned about their 40 straight losses. The frustrated coach decided to select the starting lineup for the DU-UCLA game by drawing five names from the 12 available players at random. (Assume that a player can play any position.) How many different starting lineups are possible?

(No ranking)

$n = 12$
$r = 5$

$${}_nC_r = \frac{n!}{r!(n-r)!} = \frac{12!}{5!(12-5)!} = \frac{12 * 11 * 10 * 9 * 8}{5 * 4 * 3 * 2} =$$

$6 * 2 * 2 * 3 * 11$

72
11
72
720
792

792

Exercise 5.5

Check your answers against those in the ANSWER section.

Five hundred adults over 50 years of age were classified according to whether they smoked or not, and if they smoked, were they a moderate or heavy smoker. Also, each one was asked whether he or she had ever had a heart attack. The results are given.

	Heart Attack		
	Yes	No	Total
Do not smoke	30	220	250
Moderate smoker	60	65	125
Heavy smoker	90	35	125
Totals	180	320	500

+ OR a. What is the probability of selecting a person who either has had a heart attack, or who is a heavy smoker?

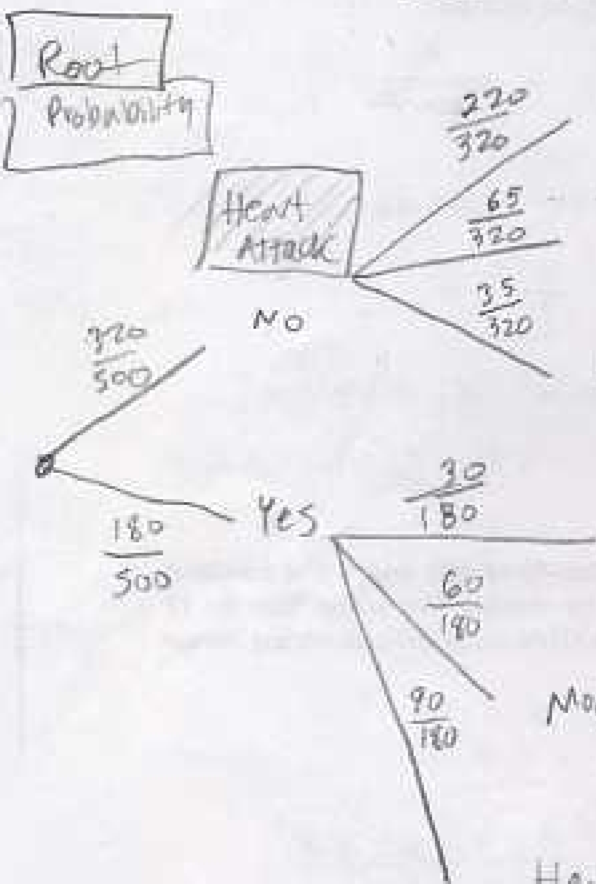
$P(\text{HA or HS}) = \frac{180}{500} + \frac{125}{500} - \frac{90}{500} = \frac{215}{500} = .43$

✱ AND b. What is the probability of selecting a heavy smoker who did not have a heart attack?

$\frac{125}{500} * \frac{35}{125} = \frac{35}{500} = \frac{7}{100} = .07$

Conditional probability "Given that"

Joint Probability "Concurrent"



NO $= \frac{320}{500} * \frac{220}{320} = \frac{22}{50} = \frac{11}{25} = .44$

moderate $= \frac{320}{500} * \frac{65}{320} = \frac{65}{500} = .13$

Heavy $= \frac{320}{500} * \frac{35}{320} = \frac{35}{500} = .07$

No $= \frac{180}{500} * \frac{30}{180} = \frac{30}{500} = .06$

Moderate $= \frac{180}{500} * \frac{60}{180} = \frac{6}{50} = .12$

Heavy $= \frac{180}{500} * \frac{90}{180} = \frac{9}{50} = .18$

$\Sigma = 1$

$\Sigma = 1$

$\Sigma = 1$? ✓

P.31

Problem 5

A large department store is analyzing the per-customer amount of purchase and the method of payment. For a sample of 140 customers, the following contingency table or cross-classified table presents the findings.

Payment Method	Amount of Purchase: B_1: Less than \$20	B_2:\$20 up to \$50	B_3: \$50 or more	Total
A_1: Cash	15	10	5	30
A_2: Check	10	30	20	60
A_3:Charge	10	20	20	50
Total	35	60	45	140

a. What is the probability of selecting someone who paid by cash or made a purchase of less than \$20?

b. What is the probability of selecting someone who paid by check and made a purchase of more than \$50?

OR a)

$P(\text{pay cash}) = \frac{30}{140}$

$P(\text{purchase} < 20) = \frac{35}{140}$

$P(\text{cash and} < 20) = \frac{15}{140}$

$P(\text{cash or pr} < 20) = \frac{30}{140} + \frac{35}{140} - \frac{15}{140} = \frac{50}{140} = \frac{5}{14} = .3571$

concurrent JOINT

AND b)

$P(\text{paid check}) = \frac{60}{140}$

$P(\text{Purchase} > 50) = \frac{20}{60}$

$P(\text{check and purchase} > 50) = \frac{60}{140} * \frac{20}{60} = \frac{20}{140} = \frac{2}{14} = \frac{1}{7} =$

.142857

Problem 4

Yesterday, the Bunte Auto Repair Shop received a shipment of four carburetors. One is known to be defective. If two are selected at random and tested:

a. What is the probability that neither one is defective?

b. What is the probability that the defective carburetor is located by testing two carburetors?

shipment of carburetor

multiplication

Independent? NO

Dependent

$P(A \& B) = P(A) * P(B|A)$

a) $P(\text{first two are defective}) = \frac{3}{4} * \frac{2}{3} = \frac{2}{4} = \frac{1}{2} = .5$

Each time select 2

b) $\underbrace{P(\text{select defective 1st time}) * P(\text{can't select defective 2nd time})}_{\boxed{\text{select def., then not}}} + \underbrace{P(\text{Don't select defect 1st Time}) * P(\text{select defective 2nd time})}_{\boxed{\text{Not, then select}}}$

$\frac{1}{4} * \frac{3}{3} + \frac{3}{4} * \frac{1}{3} = \frac{6}{12} = \frac{1}{2}$

$\frac{3}{12} + \frac{3}{12} = \frac{6}{12} = .5$

D = Defective

ND = Not Defective

b) $\underbrace{P(1^{st} \text{ is D}) * P(2nd \text{ is ND} \mid 1st \text{ was D})}_{\text{And}} \underset{\text{OR}}{+} \underbrace{P(1st \text{ ND}) * P(2nd \text{ is D} \mid 1st \text{ ND})}_{\text{And}}$

$= \frac{1}{4} * \frac{3}{3} + \frac{3}{4} * \frac{1}{3}$

$= \frac{3}{12} + \frac{3}{12}$

$= \frac{6}{12}$

$= .5$

Exercise 5.1

Check your answers against those in the ANSWER section.

A study was made to investigate the number of times adult males over 30 visit a physician each year. The results for a sample of 300 were:

Number of Visits	Number of Adult Males
0	30
1	60
2	90
3 or more	120
Total	300

a. What is the probability of selecting someone who visits a physician twice a year?

b. What is the probability of selecting someone who visits a physician?

Problem 2

A local community has two newspapers. The ***Morning Times*** is read by 45 percent of the households. The ***USA Today*** is read by 60 percent of the households. Twenty percent of the households read both papers. What is the probability that a particular household in the city reads at least one paper?

EX 5.1

$P(X=2) = \frac{90}{300} = \frac{9}{30} = \frac{3}{10} = .3$

$P(X>0) = P(X=1 \text{ or } X=2 \text{ or } X \geq 3) = 1 - \frac{30}{300} = 1 - \frac{3}{30} = 1 - \frac{1}{10}$

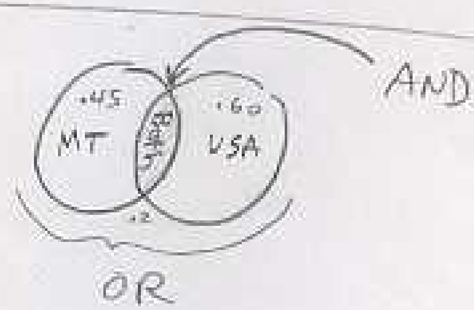
$.9$

Pr. 2

$P(\text{read MT}) = .45$

$P(\text{USA today}) = .60$

$P(\text{MT and USA}) = .2$



$P(\text{reads at least one}) = P(\text{MT or USA}) = P(\text{read MT or read USA or both})$

$= P(\text{read MT}) + P(\text{USA today}) - P(\text{MT and USA}) = .45 + .60 - .2 = .85$

Problem 1

Dunn Pontiac has compiled the following sales data regarding the number of cars sold over the past 60 selling days. Answer the following questions for the sales data shown.

Dunn Pontiac Sales Data	
Number of Cars Sold	Number of Days
0	5
1	5
2	10
3	20
4	15
5 or more	5
Total	60

a. What is the probability that two cars are sold during a particular day?

b. What is the probability of selling 3 or more cars during a particular day?

c. What is the probability of selling at least one car during a particular day?

a) empirical   relative frequency   mutually exclusive categories ~~collectively exhaustive~~

$P(x=2) = \frac{10}{60} = \frac{1}{6} = .1\bar{6}$

b) $P(x \geq 3) = \frac{20}{60} + \frac{15}{60} + \frac{5}{60} = \frac{40}{60} = \frac{4}{6} = \frac{2}{3}$

$P(x=3 \text{ or } x=4 \text{ or } x \geq 5) = \frac{40}{60} = \frac{4}{6} = \frac{2}{3} = .6\bar{6}$

c) $P(x \geq 1) = 1 - P(x=0) = 1 - \frac{5}{60} = \frac{55}{60} = \frac{11}{12} = .916\bar{6}$

$P(x=1 \text{ or } x=2 \text{ or } x=3 \text{ or } x=4 \text{ or } x \geq 5) = \frac{5+15+20+10+5}{60} = \frac{55}{60}$