Chapter 10.1, 10.2, 10.3

Do not read textbook – they use "Financial Tables" which are no longer used in the world

Topics:

1. New Terms
2. Compare Simple Interest & Compound Interest
3. FV Lump Sum Formula (section 10.1, 10.2)
4. PV Lump Sum Formula (section 10.3)

Define Finance:

$$\boxed{\text{How to allocate scarce resources across assets over time in order to earn a return.}}$$

What Should we invest in to earn a return?

Should we use cash or incur debt when we invest?

Chapters 10 & 11 are about Investing to get a return

New Terms:

PV = Present Value = Amount Invested or Borrowed today

Example: if you put \$5000 in Bank today this is called "Present value"

i = Annual Compound Interest Rate

n = Number of compounding periods in one year

Example: If you are paid interest daily, $n = 365$

If you are paid interest Quarterly, $n = 4$

Note: the more compounding periods per year, the more interest you are paid on savings Account.

x = years for investment or years amount is borrowed for.

FV = Future Value of investment after "x" number of years, or amount in savings account on Maturity Date. (Book calls this Compounded Amount)

Interest

1. "Rent on money"
2. when you borrow money, you pay interest.
3. when you lend money or make an investment, you earn interest.

Annual Interest Rate

Most Interest Rates are given as annual rates, and are usually given as a percentage.

Period Rate

$$\text{Period Rate} = \frac{i}{n} = \frac{\text{Annual Rate}}{\text{\# compounding perids per year}}$$

Rate earned per compounding period.

Example:

CD Bank account pays 8%, compounded Quarterly

Annual Rate $= i = 8\%$

Number compound periods per year $= n = 4$

$$\left\{\begin{matrix}\text{Period} \\ \text{Rate}\end{matrix}\right\} = \frac{\text{Annual Rate}}{\text{Number compounding periods pear year}} = \frac{i}{n} = \frac{8\%}{4} = 2\%$$

Simple Interest

Interest earned on the original Investment only (or paid on original loan)

Example: Invest \$5000 in bank that pays simple interest rate of 8%, how much in Bank at end of year?

$$\text{Interest} = 5000 * .08 * 1$$
$$= 400$$

$$\left\{\begin{array}{c}\text{Future} \\ \text{Value}\end{array}\right\} = \begin{array}{c}\text{original} \\ \text{Amount}\end{array} + \text{Interest}$$

↓

of investment $\quad 5000 + 400$

$$FV = \$5400$$

Compound Interest

Interest earned on both original investment and interest reinvested from prior periods.

Example: Invest \$5000 in bank that pays 8%, compounded Quarterly.

PV = Present value = Amount put in Bank Today = \$5000

i = Annual Rate = 8%

n = {compounding periods per year} = 4

$\frac{i}{n}$ = period Rate = $\frac{8\%}{4}$ = 2%

Quarter	Amount in Account at start	period Rate	Interest	Amount in Account at END
Q1	5000	.02	5000 * .02 = 100	5000 + 100 = 5100
Q2	5100	.02	5100 * .02 = 102	5100 + 102 = 5202
Q3	5202	.02	5202 * .02 = 104.04	5202 + 104.04 = 5306.04
Q4	5306.04	.02	5306.04 * .02 = 106.12	5306.04 + 106.12 = 5412.16

Interest is included when calculating New interest

Interest Earned Formula

$$FV - PV$$

Amount Taken out at End

Amount Invested

Simple Interest

Interest = FV − PV

Interest = 5400 − 5000 = 400

Compound Interest

Interest = FV − PV

Interest = 5412.16 − 5000 = 412.16

FV for simple Interest	\$5400
FV for Compound Interest	− \$5412.16

{Interest Earned ON Interest} = \$12.16

OR

Difference between our simple Interest example & our compound Interest Example is:

Interest Earned ON Interest

Earned simple Interest	400
Earned compound Interest	− 412.16
Interest Earned on Interest →	12.16

Formula for Calculating Future Value

$$FV = PV * \left(1 + \frac{i}{n}\right)^{\wedge(x * n)}$$

FV = Future Value

PV = Present value

i = Annual Interest Rate (usually APR)

n = number of compounding periods per year

X = number of years for investment

$\frac{i}{n}$ = period Rate

$X * n$ = total number of periods

Example

$PV = \$5000$

$i = 8\%$

$n = 4$

$X = 1$

$\frac{i}{n} = \frac{8\%}{4} = 2\% \Rightarrow 0.02$

$X * n = 1 * 4 = 4$

$$FV = 5000 * \left(1 + \frac{.08}{4}\right)^{\wedge(4*1)}$$

$= 5000 * (1 + .02)^{\wedge 4}$

$= 5000 * 1.08243216$

$= 5412.1608$

Excel Function for Future Value

Variable Name	symbol we use on paper	symbol used in Excel: the argument
Future value	FV	FV
Present value	PV	PV
period Rate	$\frac{i}{n}$	rate
Total periods	$X*n$	nper

Excel Function & arguments

=FV(rate, nper, , -PV)

Skip PMT argument (Must type extra comma)

PV is negative because FV function understands cash flow. Putting $ in Bank is negative cash flow to you (out of wallet).

$=FV(\frac{i}{n}, X*n, , -PV)$

$= FV(0.02, 4, , -5000)$

$= 5412.1608$

FV example:

If you deposit \$125,000 for 35 years at 5%, compounded quarterly, what would the future value be?

Present value	$PV =$	\$125,000
years	$X =$	35
Annual Rate	$i =$	5%
compound periods per year	$n =$	4

FV = ?

$\frac{i}{n}$ period rate $= \frac{5\%}{4} = 1.25\%$

$x * n$ = total periods $= 35 * 4 = 140$

$$FV = PV * \left(1 + \frac{i}{n}\right)^{\wedge(x*n)}$$

$$FV = 125{,}000 * (1 + 0.0125)^{\wedge 140}$$

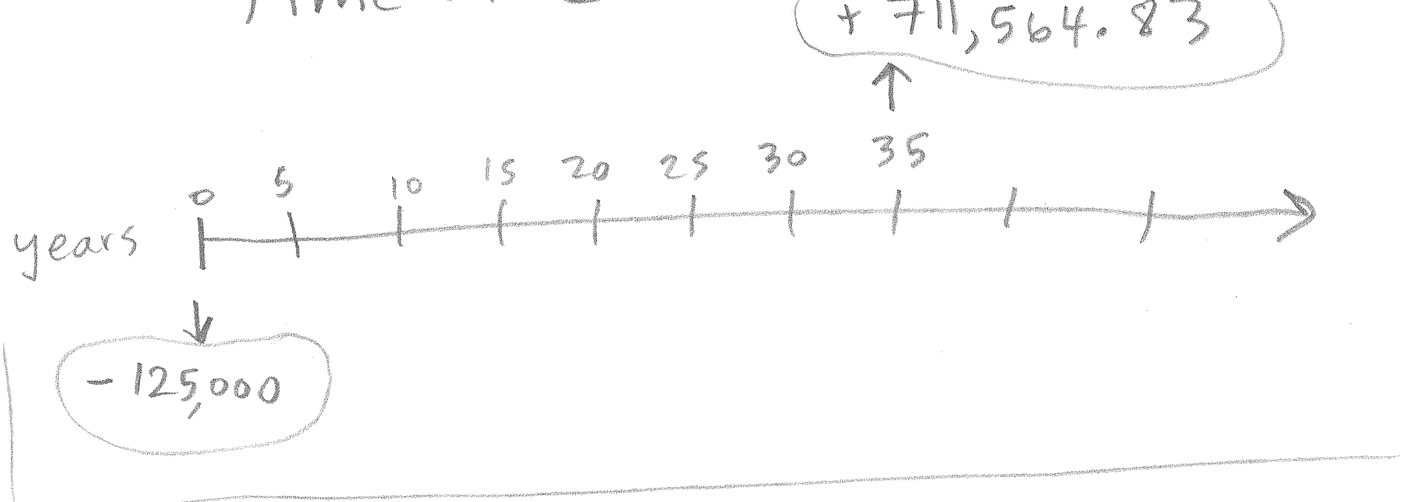
$$FV = \$711{,}564.83$$

Excel:

=FV(0.0125, 140,, -125000)

= \$711,564.83

Time Line



But if you wanted \$711,564.83 in 35 years? How much would you have to put in bank if you could earn 5%, compounded quarterly?

Formula we have been using:

$$FV = PV*\left(1+\frac{i}{n}\right)^{\wedge(x*n)}$$

plug numbers in

$$711\,564.83 = PV\,(1+0.0125)^{\wedge 140}$$

solve for this

$PV = \$125,000$

$\boxed{PV = \text{Present value} = \text{Amount to put in Bank today to get a certain amount in Future.}}$

Formula for calculating Present value

Present value = How much to put in Bank today to have a certain amount in Future

Math | Excel argument

FV = Future value → FV
PV = Present value → PV
i = Annual Interest Rate (APR)
n = number of compound periods per year
x = years
$\frac{i}{n}$ = Period Rate → rate
$x * n$ = Total number of periods → nper

Math Formula: → $PV = FV / (1 + \frac{i}{n})^\wedge(x * n)$

Excel Function → = PV (rate, nper, , FV)

skip PMT argument (Extra comma)

* PV function gives you negative amount because when you deposit it, it comes out of your walet.

PV example:

$FV = 711,564.83$

Annual Rate $= i = 5\%$

compound periods $= n = 4$

years $= X = 35$

period rate $= \frac{i}{n} = \frac{5\%}{4} = 1.25\% = 0.0125$

total periods $= X*n = 35*4 = 140$

Math

$PV = FV/(1+\frac{i}{n})^{\wedge(X*n)}$

$PV = 711,564.83/(1+0.0125)^{\wedge 140}$

$PV = 125,000$

Excel

= pv (rate, nper, , FV)

=PV(0.0125, 140, , 711564.83)

$= \underbrace{-125,000}$

Excel knows cash flow (when you deposit \$, it is a negative out of your wallet)

PV example 2:

If you wanted to be a millionare in 40 years how much would you have to put in bank today if you could earn 10%, compounded monthly?

$FV = 1,000,000$

$i = 10\% \Rightarrow 0.10$

$n = 12$

$\frac{i}{n} = \frac{0.10}{12} = .00833$

$x = 40$

$x * n = 40 * 12 = 480$

Math

$PV = FV / (1+\frac{i}{n})^\wedge(x*n)$

$= 1000000 / (1.00833\overline{3})^\wedge 480$

$= 18,621.74$

Excel

= PV(rate, nper,, FV)

= PV(.1/12, 480,, 1000000) = -18,621.74