BUSN
Math
chapter 2
Fractions

# Fractions



$\frac{1}{2} = 1 \div 2 = 2\overline{)1}$ = Division

$$\frac{1}{2}$$

1 ← Numerator, Dividend

2 ← Denominator, Divisor

$\frac{1}{2} = .5$ { Answer, Quotient

Fraction represents parts of a whole (total)

$$\frac{\text{2 pizza slice you eat}}{\text{6 total pizza slices}}$$




$\frac{2}{6} = \frac{\cancel{2}*1}{\cancel{2}*3} = \frac{1}{3}$

| Proper Fraction | Improper Fraction | Mixed Number | Like Fraction | Unlike Fractions |
|---|---|---|---|---|
| $\frac{1}{2}$ | $\frac{2}{1}$ | $1\frac{1}{2} = 1 + \frac{1}{2}$ | $\frac{1}{2}$ and $\frac{3}{2}$ | $\frac{1}{2}$ and $\frac{1}{4}$ |
| Numerator is smaller than Denominator | Numerator is bigger than Denominator | Whole number and Fraction shown together | Denominator the same in two or more fractions | Denominators not the same in 2 or more fractions |

# Prime Numbers



★ A prime number is a whole number that has exactly two factors: 1 and itself

★ 1 & itself are 2 different numbers

★ 1 is not a prime number because 1 and itself are not different numbers they are the same.

## List of Primes

2 = 2 * 1
3 = 3 * 1
5 = 5 * 1
7 ⋮
11
13
17
19
23
29
31
37
41
43
47
53
⋮

prime factor

Not prime factor

## Not Primes:

4 = 2 * 2

12 = 2 * 2 * 3

26 = 2 * 13

⋮

prime factor

prime factor

## Find All Prime Factors

① start with 2, then move to 3, then 5, etc.

NO 2

3 | 27
3 | 9
3

prime factors are: 3 * 3 * 3

② stop when last number is prime

2 | 56
2 | 28
2 | 14
7

prime factors are: 2 * 2 * 2 * 7

# Reduce Fractions

1. Break apart Numerator & denominator into prime factors
2. cancel when a prime factor is in both numerator & denominator

$$\frac{16}{26} = \frac{\cancel{2} * 2 * 2 * 2}{\cancel{2} * 13} = \frac{2 * 2 * 2}{13} = \frac{8}{13}$$

$$\begin{array}{r|l} 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline & 2 \end{array} \qquad \begin{array}{r|l} 2 & 26 \\ \hline & 13 \end{array}$$

## change Improper Fraction to Mixed Number

$$\frac{8}{3}$$

1. Do Division
2. put remainder over denominator

$$\begin{array}{r|l} & 2 \text{ R}2 \\ \hline 3 & 8 \\ & -6 \\ \hline & 2 \end{array} \Rightarrow 2\frac{2}{3}$$

OR:

$$\frac{8}{3} = \frac{6+2}{3} = \frac{3+3+2}{3}$$

$$= \frac{3}{3} + \frac{3}{3} + \frac{2}{3}$$

$$= 1 + 1 + \frac{2}{3}$$

$$= 2\frac{2}{3}$$

# Convert Mixed Numbers to Improper Fractions

$2\frac{2}{3}$

1. multiply denominator & whole number then add num to get new numerator
2. place new numerator over denominator

$$2\frac{2}{3} = \frac{3*2+2}{3} = \frac{8}{3}$$

## multiply Fractions

1. multiply numerators
2. multiply denominators
3. Reduce Fraction

$$\frac{1}{2}*\frac{1}{4} = \frac{1*1}{2*4} = \frac{1}{8}$$

$$\frac{6}{8}*\frac{2}{12} = \frac{6*2}{8*12} = \frac{12}{96} = \frac{\cancel{2}*\cancel{2}*\cancel{3}}{\cancel{2}*\cancel{2}*2*2*2*\cancel{3}} =$$

$$= \frac{1}{2*2*2} = \frac{1}{8}$$

# Divide Fractions

1. Invert (Flip) second fraction
2. multiply
3. Reduce

$$\frac{6}{12} \div \frac{1}{2} = \frac{6}{12} * \frac{2}{1} = \frac{6*2}{12} = \frac{12}{12} = 1$$

Flip

Also:

$$\frac{\frac{6}{12}}{\frac{1}{2}} = \frac{6}{12} \div \frac{1}{2} = \frac{6}{12} * \frac{2}{1} = \frac{12}{12} = 1$$

15 Express .875 as a common fraction

$$.875 = \frac{.875}{1} = \frac{0.875}{1.000} = \frac{0.875}{1.000}$$

Any number divided by 1 is that number

slide the decimal in both the numerator & denominator and the number remains the same

$$= \frac{875}{1000} = \frac{\cancel{5}*\cancel{5}*\cancel{5}*7}{\cancel{5}*\cancel{5}*\cancel{5}*2*2*2} = \frac{7}{8}$$

$$\begin{array}{r|l} 5 & 875 \\ \hline 5 & 175 \\ \hline 5 & 35 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 2 & 1000 \\ \hline 2 & 500 \\ \hline 2 & 250 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

List all prime factors

OR:

$$.875 = \frac{.875}{1} = \frac{.875}{1} * 1 =$$

$$\frac{.875}{1} * \frac{1000}{1000} = \frac{875}{1000} = \frac{7}{8}$$

Any number times 1 is that number

Answer $\frac{7}{8}$

(16) Express $\frac{3}{4}$ as a decimal (7)

$$
\begin{array}{r}
.75 \\
4\overline{)3.0} \\
-28 \\
\hline
20 \\
-20 \\
\hline
0
\end{array}
$$

Answer: $\frac{3}{4} = .75$

# Add Fractions (or subtract) 7.5

① if denominators are same:

Ⓐ Add numerators

Ⓑ Keep same denominator

Ex:

$$\frac{2}{5} + \frac{1}{5} = \frac{2+1}{5} = \frac{3}{5}$$

Ex:

$$\frac{2}{5} - \frac{1}{5} = \frac{2-1}{5} = \frac{1}{5}$$

② if denominators are different:

Ⓐ Find "Least common Denominator" LCD
also called
"Lowest common Denominator" LCD
also called
"Least common multiple" LCM

Ⓑ Add Numerators & use LCD as denominator

Ex: $\frac{5}{8} + \frac{3}{12}$

Steps to Find LCD

| ① List all Prime Factors | Needs |
|---|---|
| $8 = 2*2*2$ | 3 |
| $12 = 2*2*3$ | 2 |

LCD $= 2*2*2*3 = 24$

② For each distinct prime factor select the one that is listed the most times

③ Multiply each fraction by the number 1 with the goal of ending up with like denominators. Ask: what does each denominator need to get to be LCD

$$\frac{5}{8}*1 + \frac{3}{12}*1 = \frac{5}{8}*\frac{3}{3} + \frac{3}{12}*\frac{2}{2} = \frac{15}{24} + \frac{6}{24} = \frac{15+6}{24} = \frac{21}{24} =$$

$$= \frac{\cancel{3}*7}{\cancel{3}*8} = \frac{7}{8}$$

# Add Fractions

$$\frac{2}{12} + \frac{1}{8} + \frac{3}{4} =$$

## Lowest Common Denominator (LCD)

| | | | |
|---|---|---|---|
| Prime factors | 12 = | 2 * 2 * 3 | |
| | 8 = | 2 * 2 * 2 | |
| | 4 = | 2 * 2 | |

only 1 2 because LCD already uses 2 2s.

LCD = 2 * 2 * 3 * 2

Don't need any of these 2s because LCD already has 3 2s.

LCD = 24

needs 2 needs 3 needs 2*2

$$\frac{2}{12} * \frac{2}{2} + \frac{1}{8} * \frac{3}{3} + \frac{3}{4} * \frac{2*3}{2*3}$$

☆ Key concept

multiplying by 1 keeps number the same.

$$\frac{4}{24} + \frac{3}{24} + \frac{18}{24} = \frac{25}{24} = \frac{24}{24} + \frac{1}{24} = 1\frac{1}{24}$$

# Adding Fractions

$$\frac{5}{8} + \frac{3}{4} + \frac{1}{2}$$

{Least common Denominator} = LCD =

① List all prime factors

| | Need |
|---|---|
| $8 = 2 * 2 * 2$ | |
| $4 = 2 * 2$ | 2 |
| $2 = 2$ | $2 * 2$ |

② for each distinct prime factor, select the one that is listed the most times.

In our case 2 is listed 3 times.

$2 * 2 * 2 = 8 = LCD$

order of operations says do * before +

③ multiply each fraction by 1 with the goal of ending up with like denominators

$$\frac{5}{8} * 1 + \frac{3}{4} * 1 + \frac{1}{2} * 1 = \frac{5}{8} + \frac{3}{4} * \frac{2}{2} + \frac{1}{2} * \frac{2*2}{2*2}$$

$$= \frac{5}{8} + \frac{3*2}{4*2} + \frac{1*2*2}{2*2*2} = \frac{5}{8} + \frac{6}{8} + \frac{4}{8}$$

$$= \frac{5+6+4}{8} = \frac{15}{8} = \frac{8+7}{8} = \frac{8}{8} + \frac{7}{8} = 1 + \frac{7}{8} = 1\frac{7}{8}$$

answer

# Add Mixed Numbers

$$\begin{array}{r} 7\ \tfrac{1}{2} \\ 2\ \tfrac{1}{4} \\ +\ 10\ \tfrac{2}{3} \\ \hline \end{array}$$

| LCD | | Prime factors | Need |
|---|---|---|---|
| 2 | = | 2 | 2 * 3 |
| 4 | = | 2 * 2 | 3 |
| 3 | = | 3 | 2 * 2 |

LCD = 3 * 2 * 2 = 12

$$\begin{array}{r} 7 + \tfrac{1}{2} * \tfrac{2*3}{2*3} \\ 2 + \tfrac{1}{4} * \tfrac{3}{3} \\ +\ 10 + \tfrac{2}{3} * \tfrac{2*2}{2*2} \\ \hline \end{array} \Rightarrow \begin{array}{r} 7 + \tfrac{1}{2} * \tfrac{6}{6} \\ 2 + \tfrac{1}{4} * \tfrac{3}{3} \\ +\ 10 + \tfrac{2}{3} * \tfrac{4}{4} \\ \hline \end{array} \Rightarrow \begin{array}{r} 7 + \tfrac{6}{12} \\ 2 + \tfrac{3}{12} \\ +\ 10 + \tfrac{8}{12} \\ \hline \end{array}$$

one times any number is that number $\underbrace{2 * 1 = 2 * \tfrac{6}{6} = \tfrac{12}{6}}_{\text{example}}$

order of operations says do * before +

$$\begin{array}{r} 7\ \tfrac{6}{12} \\ 2\ \tfrac{3}{12} \\ +\ 10\ \tfrac{8}{12} \\ \hline 19\ \tfrac{17}{12} \end{array} = 19 + \tfrac{12}{12} + \tfrac{5}{12} = 19 + 1 + \tfrac{5}{12} = 20\tfrac{5}{12}$$

Answer $20\tfrac{5}{12}$

# Subtract Fractions

$$\frac{5}{8} - \frac{5}{24}$$

| LCD | prime factors | Need |
|---|---|---|
| 8 = | 2 * 2 * 2 | 3 |
| 24 = | 2 * 2 * 2 * 3 | |

LCD = 2 * 2 * 2 * 3

one times any number is that number

$$= \frac{5}{8} * 1 - \frac{5}{24} * 1 = \frac{5}{8} * \frac{3}{3} - \frac{5}{24} = \frac{5*3}{8*3} - \frac{5}{24} =$$

$$= \frac{15}{24} - \frac{5}{24} = \frac{15-5}{24} = \frac{10}{24} =$$

Reduce to lowest terms:
List all prime factors in numerator and denominator, then cancel when factor is in both numerator (you are canceling a one!)

$$\frac{10}{24} = \frac{2*5}{2*2*2*3} = \frac{2}{2} * \frac{5}{2*2*3}$$

$$= 1 * \frac{5}{2*2*3} = 1 * \frac{5}{12} = \frac{5}{12}$$

or

$$\frac{10}{24} = \frac{\cancel{2}*5}{\cancel{2}*2*2*3} = \frac{5}{12}$$

Answer = $\frac{5}{12}$

subtract Mixed Num.

$$\begin{array}{r} 83\ \frac{3}{4} \\ -\ 21\ \frac{2}{5} \\ \hline \end{array}$$

| LCD | Prime factors | Need |
|---|---|---|
| 4 = | 2 * 2 | 5 |
| 5 = | 5 | 2 * 2 |
| LCD = 5 * 2 * 2 | | |

$$\begin{array}{r} 83 + \frac{3}{4} * \frac{5}{5} \\ -\ 21 + \frac{2}{5} * \frac{2*2}{2*2} \\ \hline \end{array} \Rightarrow \begin{array}{r} 83 + \frac{3*5}{4*5} \\ -\ 21 + \frac{2*2*2}{5*2*2} \\ \hline \end{array} \Rightarrow \begin{array}{r} 83 + \frac{15}{20} \\ -\ 21 + \frac{8}{20} \\ \hline \end{array}$$

order of operations says do * before − or +

$$\begin{array}{r} 83\ \frac{15}{20} \\ -\ 21\ \frac{8}{20} \\ \hline \end{array}$$

$$83 - 62\ \frac{15-8}{20} = 62\ \frac{7}{20}$$

Answer $62\ \frac{7}{20}$

$$\frac{3}{8} * \frac{3}{5} = \frac{3*3}{8*5} = \frac{9}{40}$$

Multiply Fractions

Answer $\frac{9}{40}$

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Divide Fractions

$$15\frac{1}{4} \div 5\frac{1}{8} = \frac{15*4+1}{4} \div \frac{8*5+1}{8} = \frac{61}{4} \div \frac{41}{8}$$

$$\frac{61}{4} \div \frac{41}{8} = \frac{61}{4} * \frac{8}{41} = \frac{61}{\cancel{4}_{1}} * \frac{\cancel{8}^{2}}{41} = \frac{61}{1} * \frac{2}{41} =$$

Invert and multiply

$$= \frac{61*2}{1*41} = \frac{122}{41} = 2\frac{40}{41}$$

$$\begin{array}{r} 2\frac{40}{41} \\ 41\overline{)122} \\ -82 \\ \hline 40 \end{array}$$

check: $\frac{82+40}{41} = \frac{122}{41}$ ✓

Answer $\frac{122}{41} = 2\frac{40}{41}$

7 change $\frac{43}{32}$ to a mixed number

$$\begin{array}{r|l} & 1 \quad \frac{11}{32} \\ 32 \,\big) & 43 \\ - & 32 \\ \hline & 11 \end{array}$$

Answer $1\frac{11}{32}$

8 write $\frac{30}{42}$ in lowest terms

$$\frac{3*10}{2*21} = \frac{\cancel{3}*\cancel{2}*5}{\cancel{2}*\cancel{3}*7} = \frac{5}{7}$$

Answer $\frac{5}{7}$

List all prime factors in numerator & denominator & cancel factors that appear in both numerator and denominator

Dividing

| | | | | | |
|---|---|---|---|---|---|
| | | 3 | 1 | 5 | $\frac{7}{35}$ |
| 35 | 1 | 1 | 0 | 3 | 2 |
| | - | 1 | 0 | 5 | |
| | | | 5 | $\cancel{3}^{2}$ | $2^{12}$ |
| | | - | 5 | 2 | 5 |
| | | | | | 7 |

Remainder / Denominator

Must Reduce to lowest terms

Prime factors

$$\begin{array}{r} 1 \\ 35 \\ *\ 3 \\ \hline 105 \end{array} \qquad \begin{array}{r} 2 \\ 35 \\ *\ 15 \\ \hline 175 \\ 350 \\ \hline 525 \end{array}$$

$$5 \overline{)1.0} \quad .2$$

Also

$= 315 + \frac{1}{5}$

$315 + 1 \div 5$

$= 315 + .2$

$= 315.2$

$$= 315 + \frac{7}{35} = 315 + \frac{\cancel{7}}{\cancel{7} * 5}$$

$$= 315 + \frac{1}{5} = 315\frac{1}{5}$$

Answer $= 315\frac{1}{5} = 315.2$

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(6) change $6\frac{3}{8}$ to an improper fraction

$$= 6\frac{3}{8} = 6 + \frac{3}{8} = 6 * 1 + \frac{3}{8} = 6 * \frac{8}{8} + \frac{3}{8}$$

$$= \frac{6}{1} * \frac{8}{8} + \frac{3}{8} = \frac{48}{8} + \frac{3}{8} = \frac{48+3}{8} = \frac{51}{8}$$

Answer $\frac{51}{8}$

Any number times one is that number