Future value

Example I:

If you deposit \$125,000 in an account for 35 years that pays an Annual Rate of 5%, compounded quarterly, what is Future value?

variables:

$PV = 125{,}000$ $\qquad$ $FV = ?$

$x = 35$ $\qquad$ $\frac{i}{n} = \frac{5\%}{4} = 1.25\%$

$i = 5\%$ $\qquad$ $x * n = 35 * 4 = 140$

$n = 4$

math:

$$FV = PV * \left(1 + \frac{i}{n}\right)^{\wedge(x*n)}$$

$$FV = 125000 * (1 + .0125)^{\wedge 140}$$

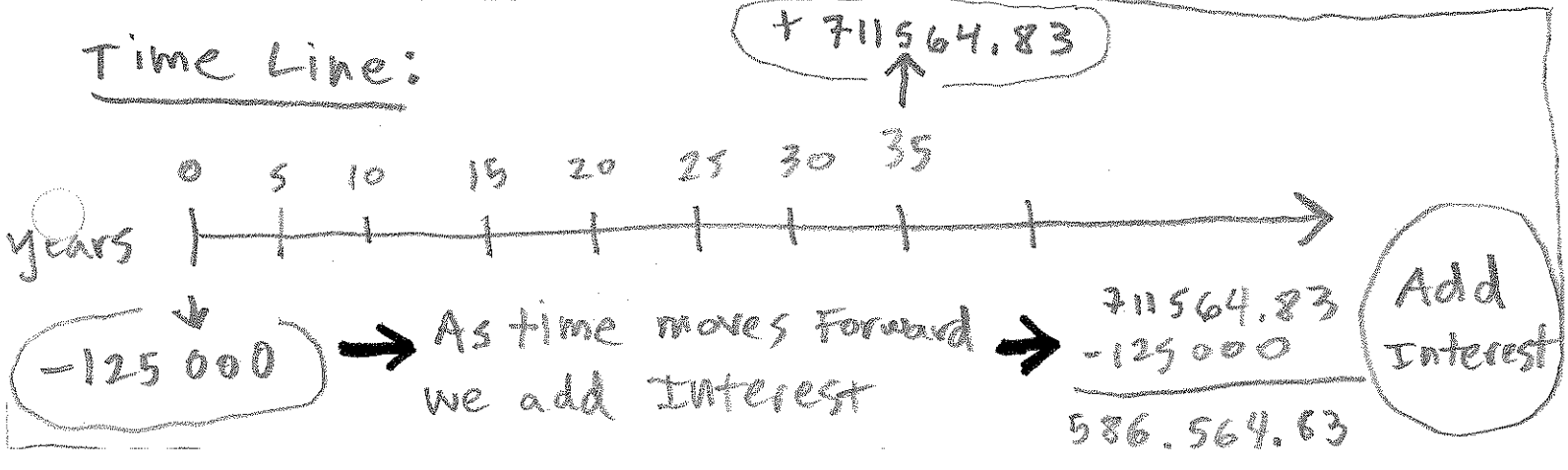
$$FV = \$711{,}564.83 \quad ✓$$

Excel:

$$FV = FV(0.0125, 140, , -125000)$$

$$= \$711{,}564.83 \quad ✓$$

Time Line:



But...

* what if we wanted to do the Reverse?

* we knew the Future Value that we wanted, and now we want to solve for the Present Value!!

FV Formula we have been using:

$$FV = PV * \left(1 + \frac{i}{n}\right)^{\wedge(x * n)}$$

* If we knew we wanted \$711,564.83 and we knew the bank offered 5%, compounded quarterly for 35 years.

plug numbers in:

$$711{,}564.83 = PV * \left(1 + \frac{0.05}{4}\right)^{\wedge(35 * 4)}$$

solve for this

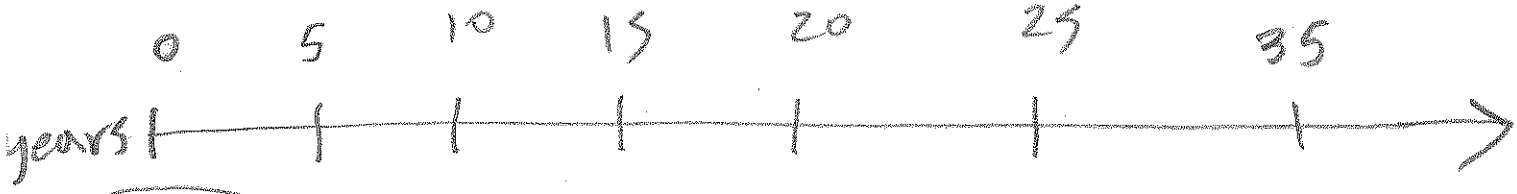
PV = Amount Deposit in Bank today to get Future value Amount.

We need to do this:

we know what amount we want in future

711,564.83

Time Line:



PV = ?

we need to go backwards & take all assumed interest out of the Future value Amount to Calculate the Present value (Amount to Deposit Today to get FV amount).

711,564.83
− 586,564.83

125,000

take interest out to get to Present value Amount

Present value

Define:

* Present value of lump sum Future Value Amount is the amount need to deposit today to get to that Future Value Amount

* Present value of lump sum Future Value Amount is the math calculation process of removing all of the assumed interest from the Future Value Amount (that you will earn) to calculate Amount to deposit today.

* Present Value is the current worth of future cash Flows discounted back at an assumed Annual Rate, Rate of Return or Discount Rate (all are synonyms)

* Think of it as "Interest Going Backwards"!

Present Value Formulas

For Lump Sum

Math:

$$PV = \frac{FV}{\left(1+\frac{i}{n}\right)^{\wedge(x*n)}}$$

$$= FV/\left(1+\frac{i}{n}\right)^{\wedge(x*n)}$$

Excel:

$$PV = PV(\text{rate}, \text{nper}, , FV)$$

skip PMT for Lump sum

Variables:

	Math	Excel
present value	PV	PV
Future Value	FV	FV
Annual Rate Discount Rate	i	
# compounding periods per Year	n	
years	x	
period Rate	i/n	rate
Total # compound periods	$x*n$	nper

PV example 2:

How much to deposit today if we want 711,564.83 in 35 years?

$FV = 711,564.83$

Annual Rate $= i = 5\%$

compound periods $= n = 4$

years $= X = 35$

period rate $= \frac{i}{n} = \frac{5\%}{4} = 1.25\% = 0.0125$

total periods $= X * n = 35 * 4 = 140$

Math

$PV = FV/(1+\frac{i}{n})^{\wedge}(X*n)$

$PV = 711,564.83/(1+0.0125)^{\wedge}140$

$PV = 125,000$

Excel

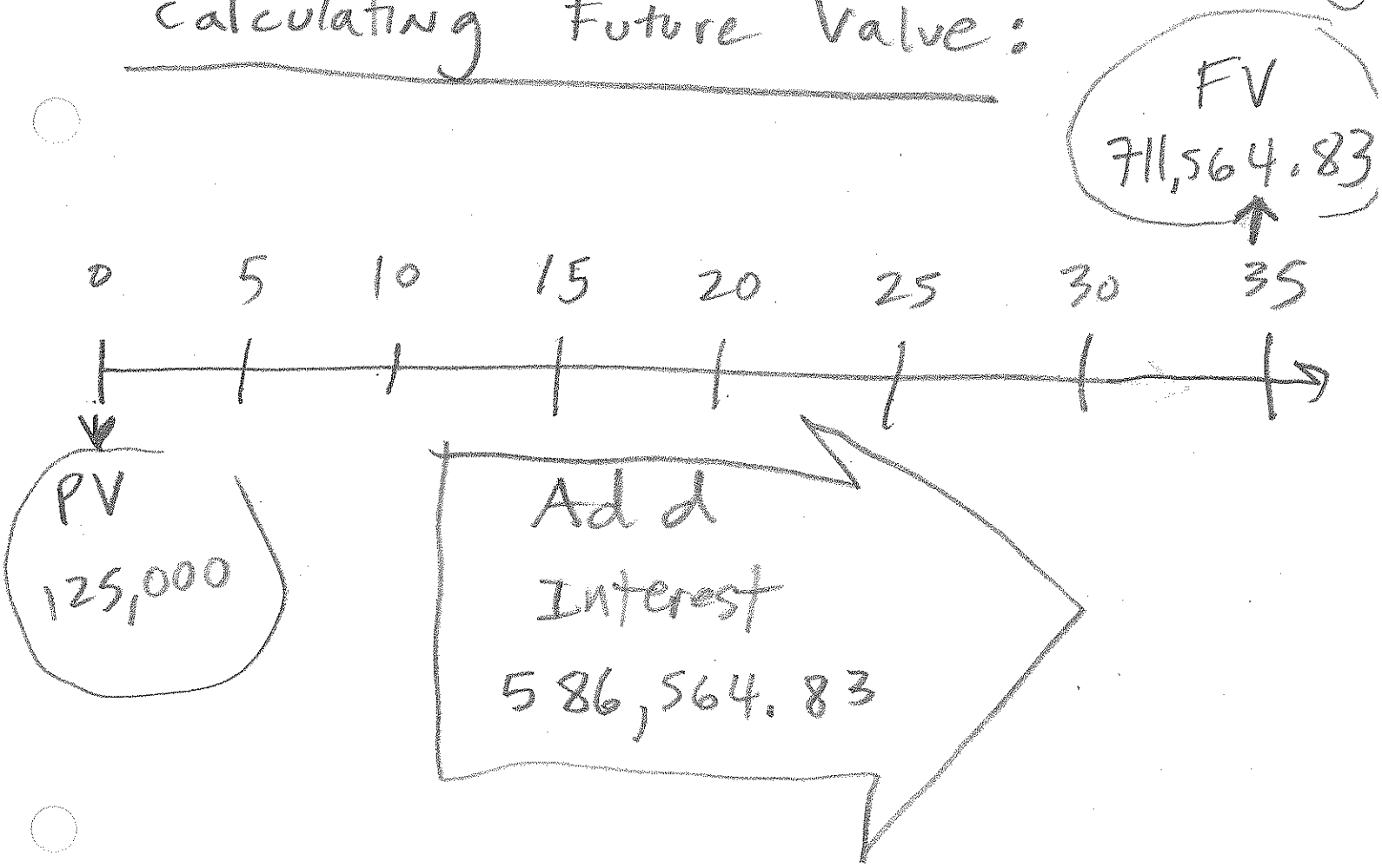
$= PV(\text{rate}, \text{nper}, , FV)$

$= PV(0.0125, 140, , 711564.83)$

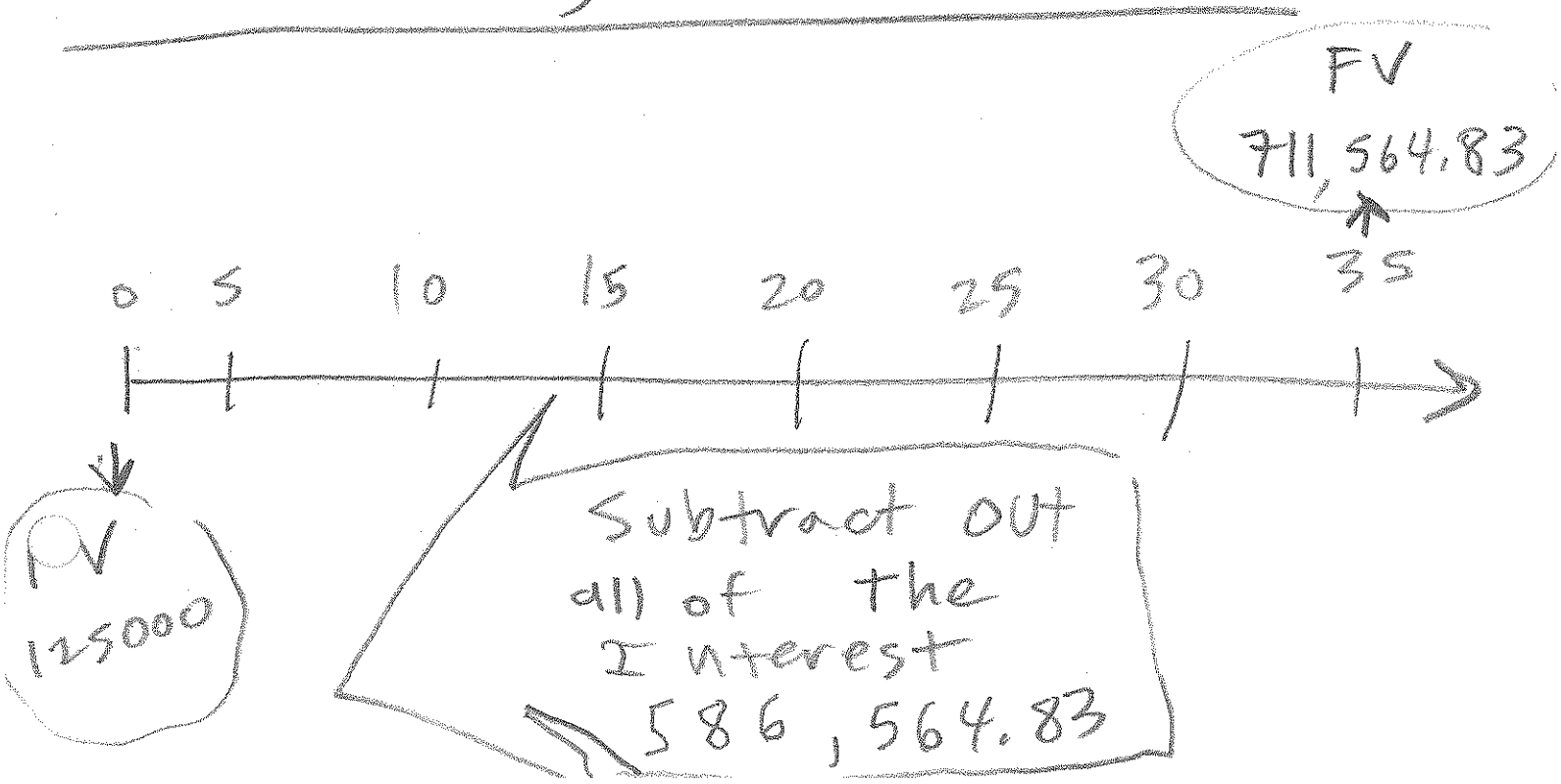
$= -125,000$

Excel knows cash flow (when you deposit \$, it is a negative out of your wallet)

Calculating Future Value:



Calculating Present Value:



PV example 3:

If you wanted to be a millionare in 40 years how much would you have to put in bank today if you could earn 10%, compounded monthly?

$FV = 1{,}000{,}000$

$i = 10\% \Rightarrow 0.10$

$n = 12$

$\frac{i}{n} = \frac{0.10}{12} = .00833$

$x = 40$

$x * n = 40 * 12 = 480$

Math

$PV = FV / (1 + \frac{i}{n})^{\wedge}(x * n)$

$= 1000000 / (1.00833\overline{3})^{\wedge}480$

$= 18{,}621.74$

Excel

$= PV(rate, nper, , FV)$

$= PV(.1/12, 480, , 1000000) = -18{,}621.74$

Example : How much do we have to put in the bank today ($i = .1$, $n = 12$) to be a millionare in 40 years?

$FV = 1{,}000{,}000$
$i = .1$
$n = 12$
$X = 40$

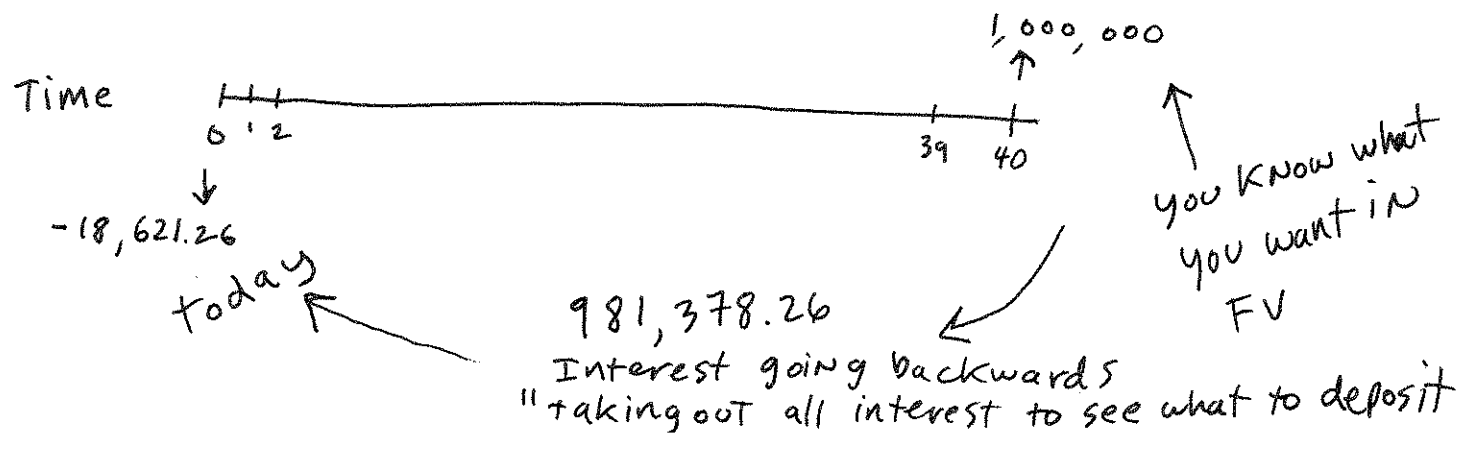
$$PV = \frac{1{,}000{,}000}{\left(1+\frac{.1}{12}\right)^{12*40}} = 18\,621.74$$

$$= PV\left(\tfrac{.1}{12}, 12*40, 1000000\right) = -18{,}621.74$$

Notice that Excel knows that the money going into the investment has a negative cash flow

Answer: If we want to be a millionare in 40 years & we could earn 10% compounded monthly, we would need to invest \$18,621.74 today.

$$\begin{array}{r} 1\,000\,000 \\ -\quad 18\,621.74 \\ \hline = 981{,}378.26 \end{array}$$ Interest earned



Example 4:

How much would we have to invest today, if we wanted to have \$150,000 for our daughter's college tuition in 18 years and we could earn an annual interest rate (discount rate) of 6.95% compounded daily (365 times a year)?

$PV = ?$

$FV = \$150{,}000$

$X = 18$

$i = .0695 \Rightarrow 6.95\%$

$n = 365$

$$PV = \frac{FV}{\left(1+\frac{i}{n}\right)^{n*x}}$$

$$PV = \frac{150\,000}{\left(1+\frac{.0695}{365}\right)^{(365*18)}} = \frac{150\,000}{(1.0001904109589041)^{6570}}$$

$$= \frac{150\,000}{3.493419000892} = \$42{,}937.88$$

Excel

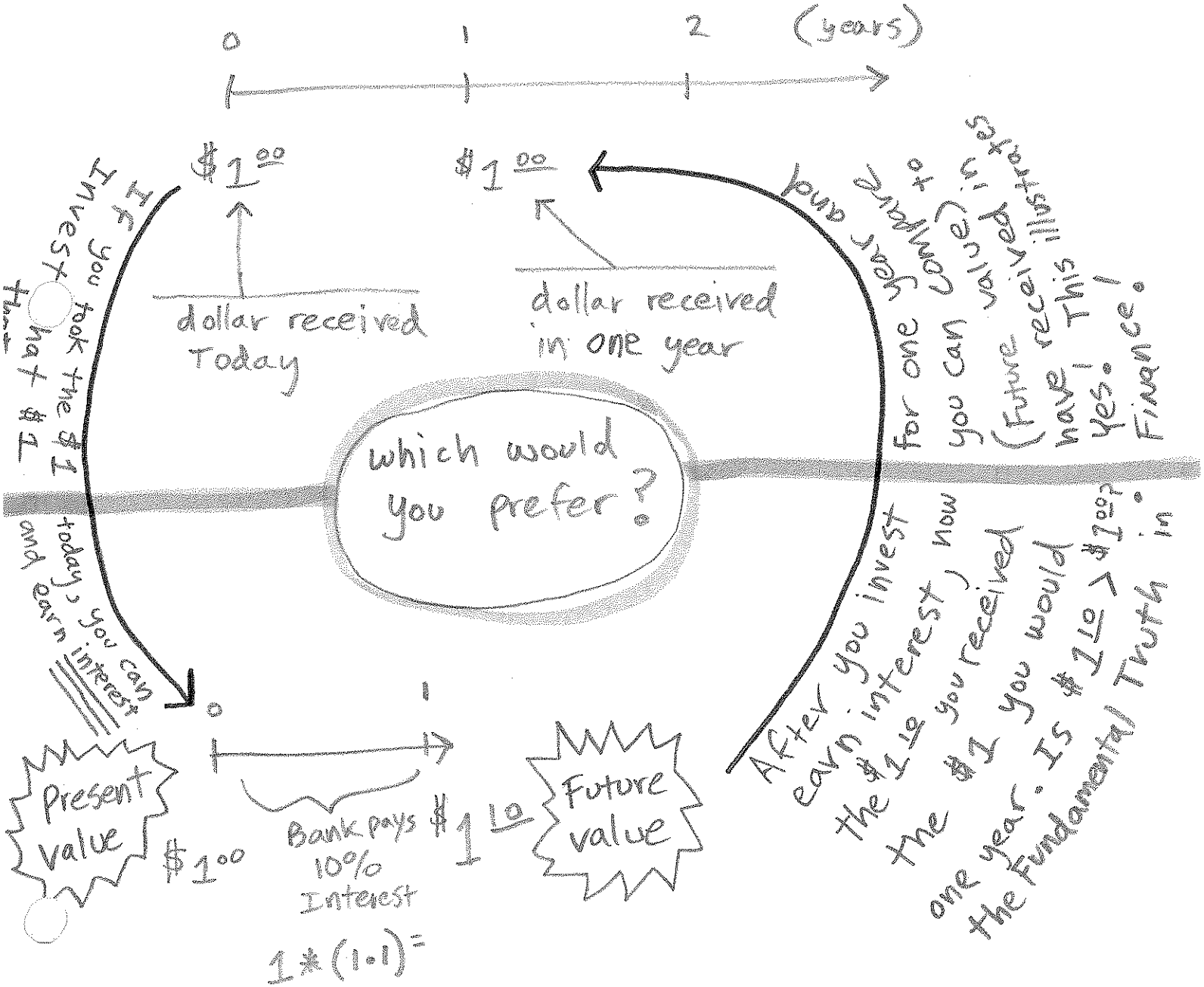
$$= PV\left(\frac{.0695}{365}, 365*18, , 150\,000\right) = \underbrace{-42{,}937.88}$$

Answer: If we want to have \$150,000 in 18 years to pay for our daughter's college tuition & we can earn 6.95% compounded daily, we would need to invest \$42,937.88 today.

Notice: Excel shows this as negative because this is a cash flow out of your wallet & into your bank

Fundamental Truth in Finance

A dollar received today is worth more than a dollar received later. (This is true because of interest).



Present value

0 → 1 year

Future value

$1.00 — Bank pays 10% Interest — $1.10

$1 * (1.1) = $1.10

$1.00 Present value	+	$0.10 Interest	=	$1.10 Future value

$1.00 — The Future value of $1.00 is $1.10 — $1.10

$1.00 — The present value of $1.10 is $1.00 — $1.10

$1.00 — This way you add interest — $1.10

$1.00 — This way you take out (subtract out) interest — $1.10