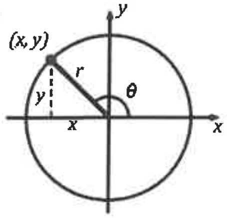
Definitions of the 6 Trigonometric Functions:

Circular function definitions, where θ is any angle:



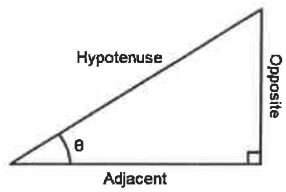
$r = \sqrt{x^2 + y^2}$ (unit circle r = 1)

$\sin\theta = \frac{y}{r}$ $\quad \csc\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r}$ $\quad \sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x}$ $\quad \cot\theta = \frac{x}{y}$

Right triangle definitions, where $0 < \theta < \frac{\pi}{2}$.



$\sin\theta = \frac{opp.}{hyp.}$ $\quad \csc\theta = \frac{hyp.}{opp.}$

$\cos\theta = \frac{adj.}{hyp.}$ $\quad \sec\theta = \frac{hyp.}{adj.}$

$\tan\theta = \frac{opp.}{adj.}$ $\quad \cot\theta = \frac{adj.}{opp.}$

Identities:

Reciprocal

$\sin\theta = \dfrac{1}{\csc\theta}$ $\quad \cos\theta = \dfrac{1}{\sec\theta}$ $\quad \tan\theta = \dfrac{1}{\cot\theta}$

$\csc\theta = \dfrac{1}{\sin\theta}$ $\quad \sec\theta = \dfrac{1}{\cos\theta}$ $\quad \cot\theta = \dfrac{1}{\tan\theta}$

Tangent and Cotangent

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\quad \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Pythagorean

$\sin^2\theta + \cos^2\theta = 1$ $\quad 1 + \tan^2\theta = \sec^2\theta$ $\quad 1 + \cot^2\theta = \csc^2\theta$

Co-function

$\sin(\frac{\pi}{2} - \theta) = \cos\theta$ $\quad \cos(\frac{\pi}{2} - \theta) = \sin\theta$ $\quad \tan(\frac{\pi}{2} - \theta) = \cot\theta$

$\csc(\frac{\pi}{2} - \theta) = \sec\theta$ $\quad \sec(\frac{\pi}{2} - \theta) = \csc\theta$ $\quad \cot(\frac{\pi}{2} - \theta) = \tan\theta$

Negative Angle (Even-Odd Properties)

Odd:		Even:
$\sin(-\theta) = -\sin\theta$	$\tan(-\theta) = -\tan\theta$	$\cos(-\theta) = \cos\theta$
$\csc(-\theta) = -\csc\theta$	$\cot(-\theta) = -\cot\theta$	$\sec(-\theta) = \sec\theta$

Formulas:

Sum and Difference

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

Sum-to-Product

$\sin\alpha \pm \sin\beta = 2\sin\left(\dfrac{\alpha \pm \beta}{2}\right)\cos\left(\dfrac{\alpha \mp \beta}{2}\right)$

$\cos\alpha + \cos\beta = 2\cos\left(\dfrac{\alpha + \beta}{2}\right)\cos\left(\dfrac{\alpha - \beta}{2}\right)$

$\cos\alpha - \cos\beta = -2\sin\left(\dfrac{\alpha + \beta}{2}\right)\sin\left(\dfrac{\alpha - \beta}{2}\right)$

Power-Reducing

$\sin^2\theta = \dfrac{1 - \cos 2\theta}{2}$ $\quad \cos^2\theta = \dfrac{1 + \cos 2\theta}{2}$ $\quad \tan^2\theta = \dfrac{1 - \cos 2\theta}{1 + \cos 2\theta}$

Product-to-Sum

$\sin\alpha\sin\beta = \dfrac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$

$\cos\alpha\cos\beta = \dfrac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$

$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha + \beta) - \sin(\alpha - \beta)]$

Double Angle

$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$ $\quad \tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$

Half Angle

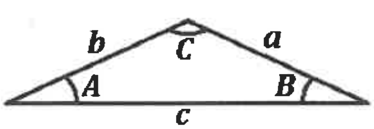
$\sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1 - \cos\theta}{2}}$ $\quad \cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1 + \cos\theta}{2}}$

$\tan\dfrac{\theta}{2} = \dfrac{1 - \cos\theta}{\sin\theta} = \dfrac{\sin\theta}{1 + \cos\theta}$

Laws:

Law of Sines

$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$ *and* $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$



Law of Cosines

$a^2 = b^2 + c^2 - 2bc\cos A$ $\quad A = \cos^{-1}(\frac{b^2 + c^2 - a^2}{2bc})$

$b^2 = a^2 + c^2 - 2ac\cos B$ $\quad B = \cos^{-1}(\dfrac{a^2 + c^2 - b^2}{2ac})$

$c^2 = a^2 + b^2 - 2ab\cos C$ $\quad C = \cos^{-1}(\dfrac{a^2 + b^2 - c^2}{2ab})$

Domain and Range:

Trig Functions

$\sin\theta$ *Domain*: $\mathbb{R}$ *Range*: $[-1,1]$

$\cos\theta$ *Domain*: $\mathbb{R}$ *Range*: $[-1,1]$

$\tan\theta$ *Domain*: $\mathbb{R} \to \theta \neq \frac{\pi}{2} + n\pi$ *Range*: $\mathbb{R}$

$\sec\theta$ *Domain*: $\mathbb{R} \to \theta \neq \frac{\pi}{2} + n\pi$ *Range*: $(-\infty,-1] \cup [1,\infty)$

$\csc\theta$ *Domain*: $\mathbb{R} \to \theta \neq n\pi$ *Range*: $(-\infty,-1] \cup [1,\infty)$

$\cot\theta$ *Domain*: $\mathbb{R} \to \theta \neq n\pi$ *Range*: $\mathbb{R}$

Inverse Trig Functions

$\sin^{-1}x$ *Domain*: $[-1,1]$ *Range*: $[-\frac{\pi}{2},\frac{\pi}{2}]$

$\cos^{-1}x$ *Domain*: $[-1,1]$ *Range*: $[0,\pi]$

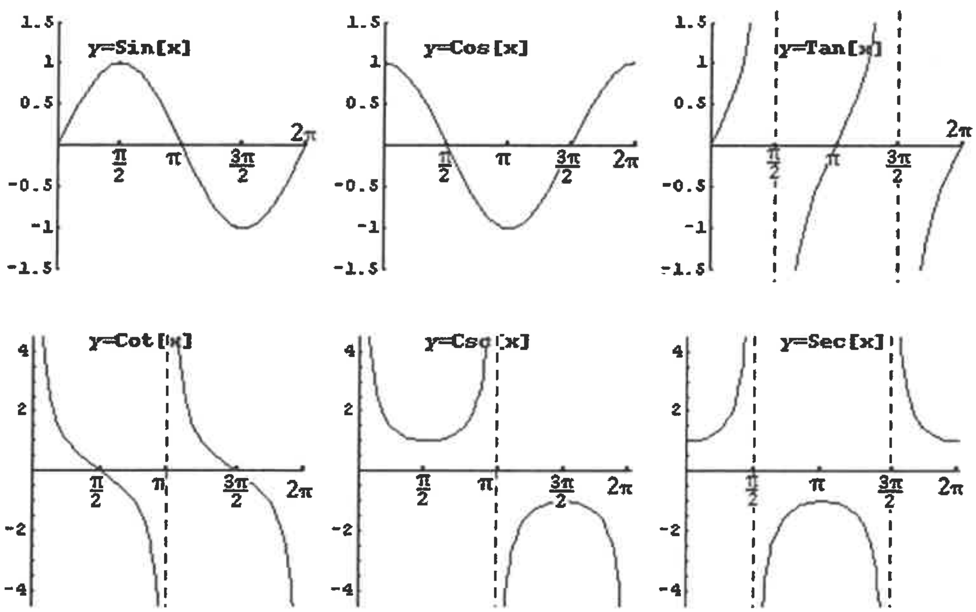
$\tan^{-1}x$ *Domain*: $\mathbb{R}$ *Range*: $(-\frac{\pi}{2},\frac{\pi}{2})$

$\sec^{-1}x$ *Domain*: $(-\infty,-1] \cup [1,\infty)$ *Range*: $\left[0,\frac{\pi}{2}\right) \cup \left[\pi,\frac{3\pi}{2}\right)$

$\csc^{-1}x$ *Domain*: $(-\infty,-1] \cup [1,\infty)$ *Range*: $\left(0,\frac{\pi}{2}\right] \cup \left(\pi,\frac{3\pi}{2}\right]$

$\cot^{-1}x$ *Domain*: $\mathbb{R}$ *Range*: $(0,\pi)$

Graphs:



Period

$\sin\theta$, $\cos\theta$, $\sec\theta$, *and* $\csc\theta$: 2π

$\tan\theta$, $\cot\theta$: π

Unit Circle

Θ°	0°	30°	45°	60°	90°	180°	270°
Θ ^{rad}	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	-	0	-
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\frac{2}{\sqrt{2}}$	2	-	-1	-
$\csc\theta$	-	2	$\frac{2}{\sqrt{2}}$	$\frac{2}{\sqrt{3}}$	1	-	-1
$\cot\theta$	-	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	-	0

(-) = undefined