Math 151: Week 1 Self-Assessment: You should be able to complete this within 55 minutes with only a calculator, pencil, and eraser.

1. Consider the graph of the function $y = f(x)$ given below. Draw the tangent line to the curve at each of the indicated points. At what point is the slope of the tangent the largest? At what point is the slope of the tangent line the smallest?

2. The graphs in the figures below represent the positions of moving particles as functions of time.

![Graph A](image1.png)  
![Graph B](image2.png)

a.) Do the instantaneous velocities at times $t_1, t_2, t_3$ in (A) form an increasing or decreasing sequence?

**decreasing sequence**

b.) Is the particle speeding up or slowing down in (A)?

**Slowing down**

c.) Is the particle speeding up or slowing down in (B)?

**Speeding up**
3. A stone is tossed in the air from ground level with an initial velocity of 15 meters/sec. Its height at time $t$ is $h(t) = 15t - 4.9t^2$ meters.

a.) Compute the stone's average velocity over the time intervals:

i.) $[1, 1.01]$
\[
\frac{h(1.01) - h(1)}{1.01 - 1} = 5.151 \text{ m/s}
\]

ii.) $[1, 1.001]$
\[
\frac{h(1.001) - h(1)}{1.001 - 1} = 5.1951 \text{ m/s}
\]

iii.) $[1, 1.0001]$
\[
\frac{h(1.0001) - h(1)}{1.0001 - 1} = 5.19951 \text{ m/s}
\]

iv.) $[.99, 1]$
\[
\frac{h(1) - h(.99)}{1 - .99} = 5.249 \text{ m/s}
\]

v.) $[.999, 1]$
\[
\frac{h(1) - h(.999)}{1 - .999} = 5.2049 \text{ m/s}
\]

vi.) $[.9999, 1]$
\[
\frac{h(1) - h(.9999)}{1 - .9999} = 5.20049 \text{ m/s}
\]

b.) What would be a good estimation, based on the above results, for the instantaneous velocity at $t = 1$?

Roughly 5.2 m/s
4. Fill in the tables and guess the value of the limit if they exist. If the limit does not exist explain why:

a.) \[ \lim_{{x \to 0}} \left( \frac{e^x - x - 1}{x^2} \right) \approx 0.5 \]

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x))</th>
<th>(x)</th>
<th>(f(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.5</td>
<td>0.426123</td>
<td>.5</td>
<td>5.94888</td>
</tr>
<tr>
<td>-0.1</td>
<td>0.483472</td>
<td>1</td>
<td>5.17092</td>
</tr>
<tr>
<td>-0.05</td>
<td>0.49177</td>
<td>.05</td>
<td>5.08439</td>
</tr>
<tr>
<td>-0.01</td>
<td>0.498337</td>
<td>.01</td>
<td>5.01671</td>
</tr>
</tbody>
</table>

b.) \[ \lim_{{x \to 0^+}} \ln x \text{ does not exist but approaches } -\infty \text{ (slowly).} \]

<table>
<thead>
<tr>
<th>(x)</th>
<th>.1</th>
<th>.01</th>
<th>.001</th>
<th>.0001</th>
<th>.00001</th>
</tr>
</thead>
<tbody>
<tr>
<td>(g(x))</td>
<td>-2.3</td>
<td>-4.6</td>
<td>-6.9</td>
<td>-9.2</td>
<td>-11.5</td>
</tr>
</tbody>
</table>

5. Determine the following limits numerically:

a.) \[ \lim_{{x \to 0^+}} \left( \frac{\sin x}{x} \right) = \sin x \quad x > 0 \]

b.) \[ \lim_{{x \to 0^-}} \left( \frac{\sin x}{x} \right) = \sin x \quad x < 0 \]

\[ \lim_{{x \to 0^+}} \left( \frac{\sin x}{x} \right) \text{ approaches } 1 \]

\[ \lim_{{x \to 0^-}} \left( \frac{\sin x}{x} \right) \text{ approaches } -1 \]

c.) What do the above results tell you about \( \lim_{{x \to 0}} \left( \frac{\sin x}{x} \right) \)?

This limit does not exist because

\[ \sin \frac{\sin x}{1 \times 1} \neq \lim_{{x \to 0^-}} \frac{\sin x}{|x|} \]
6. Given the graph of the \( y = f(x) \):

\[
\begin{array}{c}
\text{y = f(x)} \\
\end{array}
\]

Which of the following statements about the function \( y = f(x) \) are true and which are false?

a.) \( \lim_{x \to 1^-} f(x) = 1 \) \text{ True} \\
b.) \( \lim_{x \to 2} f(x) \) does not exist \text{ False} \\
c.) \( \lim_{x \to 2} f(x) = 2 \) \text{ False} \\
d.) \( \lim_{x \to 1^+} f(x) = 2 \) \text{ True} \\
e.) \( \lim_{x \to -1} f(x) \) does not exist \text{ True} \\
f.) \( \lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x) \) \text{ True} \\
g.) \( \lim_{x \to 2} f(x) \) exists at every \( c \) in \((-1, 1)\) \text{ True} \\
h.) \( \lim_{x \to 3^+} f(x) \) exists at every \( c \) in \((1, 3)\) \text{ True} \\

7. Sketch a function that satisfies the following conditions:

\[
\begin{align*}
\lim_{x \to 1^+} g(x) &= 2, \\
\lim_{x \to 3^-} g(x) &= 0, \\
\lim_{x \to 3^+} g(x) &= 4, \\
\lim_{x \to -2^+} g(x) &\text{ approaches } \infty \text{ and } g(5) = -2
\end{align*}
\]
8. Use the graph of \( f(x) \) below to help you find the requested limits. If a limit does not exist explain why and if it approaches \( \pm \infty \) state which it is.

\[
\begin{align*}
\text{a)} \quad \lim_{x \to -2^+} f(x) &\quad \text{approaches} \quad -\infty \\
\text{b)} \quad \lim_{x \to 3^-} f(x) &\quad \text{approaches} \quad +\infty \\
\text{c)} \quad \lim_{x \to 6} f(x) &\quad = \quad -2 \\
\text{d)} \quad \lim_{x \to 4} f(x) &\quad \text{does not exist} \\
\text{e)} \quad \lim_{x \to -2^-} f(x) &\quad \text{approaches} \quad +\infty 
\end{align*}
\]