100	90s	80s	70s	60s	<60
3	5	12	6	2	3

3:22

Math 220
Spring 2024
Assessment 6
Dusty Wilson

high = 100% 3x
$\bar{x}$ = 81%
med = 84.2%

Name: Key

Mathematics is the queen of the sciences and number theory is the queen of mathematics.
Carl Friedrich Gauss (1777-1855)
German mathematician

No work = no credit

1. Warm-ups

   (a) (1 point) If $A\vec{v} = \lambda\vec{v}$, what is $A^2\vec{v}$ $\lambda^2\vec{v}$

   (b) (1 point) Eigenvalue(s) of I_{2x2} 1.

   (c) (1 point) If A is singular, then an eigenvalue is: $\lambda = 0$.

2. (1 point) Based upon the quote (above), what branch of mathematics did Gauss most esteem? Answer using complete English sentences.

   Gauss thought number theory was the queen.

3. (4 points) Find the eigenvalue(s) of matrix $A = \begin{bmatrix} 7 & 3 & -2 \\ 0 & 1 & 0 \\ 2 & 4 & 3 \end{bmatrix}$. and multiplicities.

   solve $0 = \det(A - \lambda I)$

   $= (1-\lambda)[(7-\lambda)(3-\lambda)+4]$

   $\lambda^2 - 10\lambda + 21 + 4$

   $= (1-\lambda)(\lambda-5)^2$

   $\lambda = 1$ and $\lambda = 5$ (alg mult 2)

4. (4 points) Find the eigenspace of $A = \begin{bmatrix} -13 & -6 & -2 \\ 28 & 13 & 4 \\ 28 & 12 & 5 \end{bmatrix}$ with associated eigenvalue $\lambda = 1$. Is every vector in this eigenspace an eigenvector?

   $\text{rref}(A - 1I) = \begin{bmatrix} 1 & \frac{3}{7} & \frac{1}{7} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

   $E_{\lambda=1} = \text{span}\left\{ \begin{bmatrix} -3/7 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1/7 \\ 0 \\ 1 \end{bmatrix} \right\}$ OR $E_{\lambda=1} = \text{span}\left\{ \begin{bmatrix} -3 \\ 7 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 7 \end{bmatrix} \right\}$

   No, $\vec{0} \in E_{\lambda=1}$ but is not an eigenvector,
   so there are vecs in $E_{\lambda=1}$ that aren't eigenvectors.

5. (4 points) Find the eigenvalue(s) and corresponding eigenvector(s) of matrix $A = \begin{bmatrix} -3 & 15 \\ -6 & 16 \end{bmatrix}$.

Solve $0 = \begin{vmatrix} -3-\lambda & 15 \\ -6 & 16-\lambda \end{vmatrix}$

$= \lambda^2 - 13\lambda + 42$

$= (\lambda - 6)(\lambda - 7)$

$\lambda = 6$ OR $\lambda = 7$

$A - 6I \sim \begin{bmatrix} 1 & -5/3 \\ 0 & 0 \end{bmatrix}$ eigenvec $\vec{v}_1 = \begin{bmatrix} 5/3 \\ 1 \end{bmatrix}$ or $\begin{bmatrix} 5 \\ 3 \end{bmatrix}$

$A - 7I \sim \begin{bmatrix} 1 & -\frac{3}{2} \\ 0 & 0 \end{bmatrix}$ $\vec{v}_2 = \begin{bmatrix} 3/2 \\ 1 \end{bmatrix}$ or $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$

so A is diagonalizabe

$\Rightarrow A = PDP^{-1}$

where

$P = \begin{bmatrix} 5/3 & 3/2 \\ 1 & 1 \end{bmatrix}$ and $D = \begin{bmatrix} 6 & 0 \\ 0 & 7 \end{bmatrix}$

6. (2 points) True or False: If A is invertible, then A is diagonalizable. Justify your answer.

False: $\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix}$ is invertible, but we showed in class that it isn't diagonalizable.