100+	90's	80's	70's	60's	< 60
4	15	12	8	5	4

Assessment 5 (10 or 11 am) **Name (first & last):** key
Dusty Wilson
Math 220

$\bar{x} = 81.1\%$

It is true that a mathematician who is not also something of a poet will never be a perfect mathematician.

No work = no credit

med $= 82.5\%$

Karl Weierstrass
1815-1897 (German mathematician)

$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix}$

Warm-ups (1 pt each): $\vec{e}_2\vec{e}_1^{\,T} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ $\quad \vec{e}_2^{\,T}\vec{e}_2 = 1$ $\quad \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = -2$

1.) (1 pt) According to Weierstrass (above), what is an additional (perhaps unexpected) quality required if one is to be a perfect mathematician? Please answer using complete sentences.

A perfect mathematician has a poetic eye for the abstract world of math.

2.) (7 pts) For the matrix $A_{n\times n}$, there are at least 13 statements equivalent to, "A is invertible." List at least seven of them. List more for extra credit (2 points max).

i.) A is invertible.	vi.) the L.T. $\vec{x} \mapsto A\vec{x}$ is one-to-one
ii.) $A \sim I_{n\times n}$	vii.) The eq't $A\vec{x} = \vec{b}$ has at least one sol'n for each $\vec{b} \in \mathbb{R}^n$.
iii.) A has n pivot positions	viii.) The columns of A span $\mathbb{R}^n$.
iv.) $A\vec{x} = \vec{0}$ has only the trivial solution	ix.) (1 pt extra credit) the L.T. $\vec{x} \mapsto A\vec{x}$ maps $\mathbb{R}^n$ onto $\mathbb{R}^n$.
v.) The columns of A form a L.I. set.	x.) (1 pt extra credit) A^T is invertible.

There is $C_{n\times n}$ s.t. $CA = I$

There is $D_{n\times n}$ s.t. $AD = I$

$\det(A) \neq 0$.

3.) (3 pts) What properties must H satisfy if it is to be a subspace?

(1) $0 \in H$

(2) H is closed under addition

(3) H is closed under scalar multiplication.

4.) (4 pts) Prove/disprove that the set $\overset{W}{\cancel{S}} = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : 2x + 3y = 0 \right\}$ is a subspace.

(1) show $\vec{0}$ is in ~~B S~~. W.

$2(0) + 3(0) = 0 \Rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix} \in W$.

(2) show W is closed under addition.

Let $\vec{u}, \vec{v} \in W$ be given

$\Rightarrow \vec{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ w/ $2u_1 + 3u_2 = 0$

AND

$\vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ w/ $2v_1 + 3v_2 = 0$

$\Rightarrow \vec{u} + \vec{v} = \begin{bmatrix} u_1 + v_1 \\ u_2 + v_2 \end{bmatrix}$ w/ $2(u_1 + v_1) + 3(u_2 + v_2)$

$= 2u_1 + 2v_1 + 3u_2 + 3v_2$

$= (2u_1 + 3u_2) + (2v_1 + 3v_2)$

$= 0 + 0$

$= 0$

$\Rightarrow \vec{u} + \vec{v} \in W$

(3) show W is closed under scalar multiplication

Let ~~be~~ $\vec{u} \in W$ and scalar c be given

$\Rightarrow \vec{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ w/ $2u_1 + 3u_2 = 0$

$\Rightarrow c\vec{u} = \begin{bmatrix} cu_1 \\ cu_2 \end{bmatrix}$ w/ $2cu_1 + 3cu_2 = c(2u_1 + 3u_2)$

$= c(0)$

$= 0$

$\Rightarrow c\vec{u} \in W$.

$\therefore W$ is a subspace

union of Q2 & 4

5.) (4 pts) Prove/disprove that the set $W = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : xy \le 0 \right\}$ is a subspace.

$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \in W$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix} \in W$

but $\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \notin W$

so W isn't closed under addition.

$\therefore$ W is <u>not</u> a subspace

6.) (4 pts) Is the matrix $A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 1 & 2 \\ 0 & 4 & 5 \end{bmatrix}$ invertible? Justify your answer.

$$\det A = 2 \begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix}$$

$$= 2(5-8)$$

$$= -6$$

$$\neq 0$$

$\therefore$ A is invertible.

7.) (4 pts) Calculate the determinant $\begin{vmatrix} 1 & 0 & 2 & 6 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 2 & 0 & 0 & 7 & 1 \\ 0 & 0 & 1 & 8 & 2 \\ 1 & 2 & 3 & 4 & 5 \end{vmatrix}$

$$= -3 \begin{vmatrix} 1 & 0 & 2 & 0 \\ 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 1 & 2 & 3 & 5 \end{vmatrix}$$

$$= -3(2) \begin{vmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{vmatrix}$$

$$= -6 \left[1 \begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix} - 2 \begin{vmatrix} 2 & 1 \\ 0 & 2 \end{vmatrix} \right]$$

$$= -6 \left[-1 - 2(4) \right]$$

$$= 54$$

matrix [ ] half pt.

det | |