Math 220
Spring 2024
Assessment 4a
Dusty Wilson

Name: Key

The total number of Dirichlet's publications is not large: jewels are not weighed on a grocery scale.
Carl Friedrich Gauss (1777-1855)
German mathematician

No work = no credit

1. (4 points) Find the determinant of $A = \begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix}$. Is A invertible? Why or why not.

$|A| = 15 - 8 = 7$ (A invertible since $|A| \neq 0$)

2. (4 points) Find the determinant of $B = \begin{bmatrix} 3 & 2 & 4 \\ 1 & 5 & 1 \\ 2 & -3 & 3 \end{bmatrix}$. Is B invertible? Why or why not.

$$|B| = 3\begin{vmatrix} 5 & 1 \\ -3 & 3 \end{vmatrix} - 2\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} + 4\begin{vmatrix} 1 & 5 \\ 2 & -3 \end{vmatrix}$$

$$= 3(15 + 3) - 2(1) + 4(-13)$$

$$= 54 - 2 - 52$$

$$= 0$$

B is singular since $|B| = 0$.

3. (4 points) Find the determinant of $C = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 5 & 0 \\ 4 & 3 & 2 & 1 \\ 0 & 0 & 3 & -2 \end{bmatrix}$. Is ~~B~~ C invertible? Why or why not.

$$|C| = -5 \begin{vmatrix} 1 & 2 & 4 \\ 4 & 3 & 1 \\ 0 & 0 & -2 \end{vmatrix}$$

$$= -5(-2) \begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix}$$

$$= -5(-2)(-5)$$

$$= -50.$$

C is invertible since $|C| \neq 0$

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1. (4 points) Find the determinant of $A = \begin{bmatrix} 6 & 3 \\ 10 & 5 \end{bmatrix}$. Is A invertible? Why or why not.

$|A| = 30 - 30$

$= 0$. A is singular since $\det(A) = 0$.

2. (4 points) Find the determinant of $B = \begin{bmatrix} 3 & 2 & 4 \\ 1 & 5 & 1 \\ 2 & -3 & 1 \end{bmatrix}$. Is B invertible? Why or why not.

$|B| = 3\begin{vmatrix} 5 & 1 \\ -3 & 1 \end{vmatrix} - 2\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} + 4\begin{vmatrix} 1 & 5 \\ 2 & -3 \end{vmatrix}$

$= 3(8) - 2(-1) + 4(-13)$

$= 24 + 2 - 52$

$= -26$

B is invertible since $|B| \neq 0$.

3. (4 points) Find the determinant of $C = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 4 & 0 \\ 4 & 3 & 2 & 1 \\ 0 & 0 & 3 & -3 \end{bmatrix}$. Is ~~$B$~~ C invertible? Why or why not.

$$|C| = -4 \begin{vmatrix} 1 & 2 & 4 \\ 4 & 3 & 1 \\ 0 & 0 & -3 \end{vmatrix}$$

$$= -4(-3) \begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix}$$

$$= -4(-3)(-5)$$

$$= -60$$

C is invertible since $|C| \neq 0$.

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1. Warm-ups

   (a) (1 point) $\det(AA^{-1})$ $= 1$

   (b) (1 point) $\vec{e}_2\,\vec{e}_2^{\,T}$ $= \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

   (c) (1 point) $\vec{e}_2^{\,T}\vec{e}_1$ $= \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 0$

2. (1 point) Dirichlet was a mathematician around the same time as Gauss. Based upon the quote by Gauss (above), what was his view of Dirichlet's work? Answer using complete English sentences.

   Gauss believed that Dirichlet didn't write much, but what he did write was important

3. (2 points) True or false: The null space of A is the solution set of the equation $A\vec{x} = \vec{0}$. Justify your answer.

   True, this is the defn. and also equiv. to saying the null space is the set of all solutions to the homogeneous equation.

4. (4 points) Prove the following:

   Claim: The null space of an $m \times n$ matrix A is a subspace of $\mathbb{R}^n$.

proof.

Let $\vec{u}, \vec{v} \in \text{Null}A$ and scalar c be given.

① $A\vec{0} = \vec{0} \Rightarrow \vec{0} \in \text{Null}(A)$

② $A(\vec{u}+\vec{v}) = A\vec{u} + A\vec{v} = \vec{0} + \vec{0} = \vec{0}$
so $\vec{u}+\vec{v} \in \text{Null}(A)$

③ $A(c\vec{u}) = cA\vec{u} = c\vec{0} = \vec{0}$ so $c\vec{u} \in \text{Null}(A)$

$\therefore$ Null(A) is a subspace.

5. (2 points) True or false. If $\det(A) = 0$, then two rows or two columns are the same, or a row or a column is zero. Justify your answer.

$\begin{vmatrix} 2 & 4 \\ 1 & 2 \end{vmatrix} = 0$ but neither col/row is zero or identical.

6. (4 points) Match the following with the analogous term from pre-calculus:

(a.) null space
(b.) column space

domain
range
codomain
x-intercept(s)
y-intercepts(s)
the origin

7. (4 points) What condition(s) must a space W satisfy to be a subspace? You may use words or symbols. (+1 if you are able to correctly do both).

① $\vec{0} \in W$
zero is in the space

② If $\vec{u}, \vec{v} \in W$ then $\vec{u} + \vec{v} \in W$
the space is closed under addition.

③ If $\vec{u} \in W$ and c is a scalar, then $c\vec{u} \in W$.
the space is closed under scalar multiplication.

8. (4 points) Define $T : \mathbb{P}^2 \to \mathbb{R}^2$ by $T(p) = \begin{bmatrix} p(0) \\ p(0) \end{bmatrix}$.

(a) (2 points) Find a non-trivial polynomial in the kernel.

$T(t) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ In kernel.

$T(t^2) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ OR more generally $T(at+bt^2) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(b) (2 points) Find a non-zero vector in the range.

$T(1) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ so $\begin{bmatrix} 1 \\ 1 \end{bmatrix} \in$ Range.

9. (4 points) Let W be the set of all vectors of the form $\begin{bmatrix} -a+1 \\ a-6b \\ 2b+a \end{bmatrix}$ where a ~~abd~~ and b represent arbitrary real numbers. Find a set S of vectors that spans W or given an example to show that W is not a subspace.

$\vec{0} \notin W$. so W is not a subspace.

$$\begin{bmatrix} 1-a \\ a-6b \\ 2b+a \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow a = 1$ (1st component)

$\Rightarrow 1 - 6b = 0 \Rightarrow b = \frac{1}{6}$ (2nd component)

$\Rightarrow 2\left(\frac{1}{6}\right) + 1 \neq 0$ (3rd component)