100	90's	80's	70's	60's	< 60
0	2	5	8	10	7

Math 220
Winter 2024
Assessment 2
Dusty Wilson

Name: KEY

max = 95.7%

$\bar{x}$ = 68.9%

med = 69.4%

If others would but reflect on mathematical truths as deeply and continuously as I have, they would make my discoveries.
Carl Friedrich Gauss (1777-1855)
German mathematician

No work = no credit

1. Warm-ups

   (a) (1 point) $\vec{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

   (b) (1 point) $\vec{e}_2\,\vec{e}_2^{\,T} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

   (c) (1 point) $\vec{e}_2^{\,T}\vec{e}_2 = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$

2. (1 point) What does Gauss see as the key to his mathematical discoveries? Answer using complete English sentences.

   Deep and continuous reflection was Gauss' key to mathematical discovery

3. (7 points) Answer the following in regards to the system below.

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 0 \\ -4x_1 - 9x_2 + 2x_3 &= 0 \\ -3x_2 - 6x_3 &= 0 \end{aligned}$$

   (a) (1 point) Write the system as a vector equation.

$$x_1 \begin{bmatrix} 1 \\ -4 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 3 \\ -9 \\ -3 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 2 \\ -6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

   (b) (1 point) Write the system as a matrix equation.

$$\begin{bmatrix} 1 & 3 & 1 \\ -4 & -9 & 2 \\ 0 & -3 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

   (c) (4 points) Find all solution(s) to this system. Write your answer in vector form

$$\left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ -4 & -9 & 2 & 0 \\ 0 & -3 & -6 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -5 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

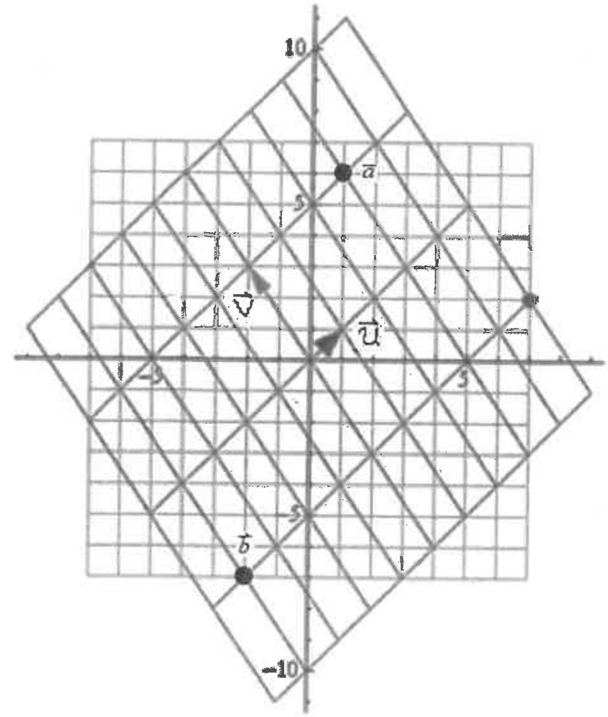
$$\Rightarrow \vec{x} = x_3 \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$$

   (d) (1 point) The process you used to find solution(s) to this system are an example of seeking non-trivial solutions to the homogeneous equation.

4. (4 points) Write vectors $\vec{a}$ and $\vec{b}$ as linear combinations of $\vec{u}$ and $\vec{v}$.

$\vec{a} = 3\vec{u} + \vec{v}$

$\vec{b} = -4\vec{u} - \vec{v}$



5. (6 points) Answer the following:

(a) (2 points) What condition must $\vec{v}_1, \vec{v}_2, ..., \vec{v}_m$ in $\mathbb{R}^n$ satisfy for the set to be linearly independent?

$c_1\vec{v}_1 + \ldots + c\vec{v}_m = \vec{0}$ must only have the trivial solution

(b) (2 points) What conditions must $T(\vec{x}) = A\vec{x}$ satisfy in order to be a linear transformation?

$T(\vec{x} + \vec{y}) = T(\vec{x}) + T(\vec{y})$

AND

$T(c\vec{x}) = cT(\vec{x})$

(c) (2 points) True or False: If a set in $\mathbb{R}^n$ is linearly dependent, then the set contains more vectors than there are entries in each vector. Justify your answer.

$\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$ is a L.D. set

6. (4 points) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}) = \begin{bmatrix} x_1 + x_2 \\ 4x_1 + 5x_2 \end{bmatrix}$. Find $\vec{x}$ such that $T(\vec{x}) = \begin{bmatrix} 3 \\ 8 \end{bmatrix}$.

$$T(\vec{x}) = \begin{bmatrix} 1 & 1 \\ 4 & 5 \end{bmatrix} \vec{x}$$

solve $\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 4 & 5 & 8 \end{array}\right]$

$$\sim \left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & -4 \end{array}\right]$$

$$\Rightarrow \vec{x} = \begin{bmatrix} 7 \\ -4 \end{bmatrix}$$

7. (4 points) Prove the following.

Claim: For all $\vec{u}$ in $\mathbb{R}^n$ and all scalars c and d: $(c+d)\vec{u} = c\vec{u} + d\vec{u}$

proof.

Let $\vec{u} \in \mathbb{R}^n$ and scalars c, d be given.

$$(c+d)\vec{u} = (c+d)\begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}$$

$$= \begin{bmatrix} (c+d)u_1 \\ \vdots \\ (c+d)u_n \end{bmatrix}$$

$$= \begin{bmatrix} cu_1 + du_1 \\ \vdots \\ cu_n + du_n \end{bmatrix}$$

$$= \begin{bmatrix} cu_1 \\ \vdots \\ cu_n \end{bmatrix} + \begin{bmatrix} du_1 \\ \vdots \\ du_n \end{bmatrix}$$

$$= c\begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix} + d\begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}$$

$$= c\vec{u} + d\vec{u}$$

$$\therefore (c+d)\vec{u} = c\vec{u} + d\vec{u}$$