

Math 220
Spring 2024
Assessment 7
Dusty Wilson

No work = no credit

Name: _____

You know that I write slowly. This is chiefly because I am never satisfied until I have said as much as possible in a few words, and writing briefly takes far more time than writing at length.

Carl Friedrich Gauss (1777-1855)
German mathematician

1. Warm-ups

(a) (1 point) If $\vec{u} \perp \vec{v}$, what is $\vec{u} \cdot \vec{v}$

(b) (1 point) Real valued A has $\lambda = 7 - 9i$.
Another λ is:

(c) (1 point) If $\vec{u} \perp \vec{v}$, what is $\vec{u}^T \vec{v}$

2. (1 point) Based upon the quote (above), why did Gauss write slowly? Answer using complete English sentences.

3. (4 points) Find a unit vector in the direction of $\vec{v} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}$.

4. (4 points) Explain how you would show that three vectors $\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ form an orthonormal set

5. (4 points) Let $\vec{y} = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$ and $\vec{u} = \begin{bmatrix} 1 \\ -5 \end{bmatrix}$. Write \vec{y} as the sum of two orthogonal vectors, one in $\text{Span}\{\vec{u}\}$ and one orthogonal to \vec{u} .

6. (2 points) True or False: If \vec{y} is a linear combination of nonzero vectors from an orthogonal set, then the weights in the linear combination can be computed without row operations. Justify your answer.
7. (2 points) True or False: An orthogonal matrix is invertible. Justify your answer.
8. (6 points) Suppose $A = \begin{bmatrix} 19 & -15 \\ 24 & -17 \end{bmatrix}$ and $A = \rho D \rho^{-1}$ where $\rho = \begin{bmatrix} 3+i & 3-i \\ 4 & 4 \end{bmatrix}$ and $D = \begin{bmatrix} 1+6i & 0 \\ 0 & 1-6i \end{bmatrix}$.
- (a) (4 points) Find a new invertible P and rotation-scaling matrix C such that $A = PCP^{-1}$.
- (b) (1 point) What is the scaling factor of C ?
- (c) (1 point) What is the angle (in degrees) of the rotation?

9. (4 points) Find the eigenvalue(s) and corresponding eigenvectors for the matrix $A = \begin{bmatrix} 16 & -10 \\ 18 & -8 \end{bmatrix}$.