Name: ______________________________

Math 220
Winter 2024
Assessment 7
Dusty Wilson

No work = no credit

One time [musician] Robert Plant was set to check into the same room after I checked out, so I removed every light bulb and ordered up a bunch of stinky cheese and put it under the mattress.
Richard Marx *singer*

1. Warm-ups

   (a) (1 point) If $A\vec{v} = \lambda\vec{v}$, what is $A^2\vec{v}$

   (b) (1 point) Eigenvalue(s) of I_{2x2}

   (c) (1 point) If A is singular, then an eigenvalue is:

2. (1 point) In reference to the quote above, what is the best practical joke you have taken part in? Answer using complete English sentences.

3. (4 points) Find the eigenspace of $A = \begin{bmatrix} 4 & 2 & 3 \\ -1 & 1 & -3 \\ 2 & 4 & 9 \end{bmatrix}$ with associated eigenvalue $\lambda = 3$. Is every vector in this eigenspace an eigenvector?

4. (2 points) True or False: If A is diagonalizable, then A is invertible. Justify your answer.

5. (4 points) Find the eigenvalue(s) and eigenvector(s) of matrix $A = \begin{bmatrix} 6 & -2 & 0 \\ -2 & 9 & 0 \\ 5 & 8 & 3 \end{bmatrix}$.

6. (4 points) The matrix $A = \begin{bmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{bmatrix}$ has eigenvalues $\lambda = 5, 1$. Diagonalize A.

7. (4 points) Matrix $A = \begin{bmatrix} -2 & 12 \\ -1 & 5 \end{bmatrix}$ has eigenvectors $\vec{v}_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$. The associated eigenvalues are $\lambda = 2, 1$.

   If $x_0 = \begin{bmatrix} 8 \\ 2 \end{bmatrix}$ and $\vec{x}_t = A^t \vec{x}_0$, find a closed form expression for $\vec{x}_t$ and $\vec{x}_{equ}$

8. (4 points) Find the eigenvalues and a basis for each eigenspace of $\begin{bmatrix} 0 & 1 \\ -8 & 4 \end{bmatrix}$