Math 220
Spring 2024
Assessment 5
Dusty Wilson

Name: ______________________________

Sophie Germain proved to the world that even a woman can accomplish something in the most rigorous and abstract of sciences.
Carl Friedrich Gauss (1777-1855)
German mathematician

No work = no credit

1. Warm-ups

   (a) (1 point) $\vec{e}_1^{\,T}\vec{e}_1$

   (b) (1 point) $\vec{e}_1\,\vec{e}_2^{\,T}$

   (c) (1 point) If $A_{n\times m}$ then rank+nullity = _____

2. (1 point) Based upon the quote by Gauss (above), name a famous female mathematician? Answer using complete English sentences.

3. (2 points) True or False. A plane in $\mathbb{R}^3$ is a two-dimensional subspace of $\mathbb{R}^3$. Justify your answer.

4. (4 points) The set $\mathcal{B} = \{1+2t, 3+5t\}$ is a basis for $\mathbb{P}_1$. Find the coordinate vector of $p(t) = 1+t$ relative to $\mathcal{B}$.

5. (2 points) The subspace $H = \text{span}\{\vec{v}_1, \vec{v}_2\}$. List all possible value(s) for $\dim(H)$.

6. (7 points) Consider matrix $C = \begin{bmatrix} 1 & 0 & 3 & 4 & 2 & -1 \\ 2 & 0 & 1 & 3 & -1 & 3 \\ 3 & 0 & 4 & 7 & 1 & 2 \\ 4 & 0 & 1 & 5 & -3 & 7 \end{bmatrix}$.

(a) (2 points) Find a basis for the column space of C

(b) (1 point) The rank of C

(c) (2 points) The null space of C

(d) (1 point) A basis for row(C)

(e) (1 point) Choose any non-zero $\vec{u} \in \text{nul}(C)$ and $\vec{v} \in \text{row}(C)$ and then find $\vec{v}\,\vec{u}$

7. (4 points) Prove (or disprove) the Unique Representation Theorem.

   Claim: Let $B = \left\{\vec{b}_1, ..., \vec{b}_n\right\}$ be a basis for a vector space V. Then for each $\vec{x} \in V$, there exists a unique set of scalars $c_1, ..., c_n$ such that $\vec{x} = c_1\vec{b}_1 + ... + c_n\vec{b}_n$.

8. (6 points) Let $\mathcal{B} = \left\{\begin{bmatrix}2\\2\end{bmatrix}, \begin{bmatrix}0\\4\end{bmatrix}\right\}$ and $\mathcal{C} = \left\{\begin{bmatrix}3\\-2\end{bmatrix}, \begin{bmatrix}-1\\7\end{bmatrix}\right\}$ be bases for $\mathbb{R}^2$.

   (a) (2 points) If $[\vec{y}]_{\mathcal{B}} = \begin{bmatrix}-1\\1\end{bmatrix}$, find $\vec{y}$

   (b) (2 points) Find the change-of-coordinates matrix from $\mathcal{B}$ to $\mathcal{C}$

   (c) (2 points) Find the $\mathcal{C}$-coordinates of $[\vec{x}]_{\mathcal{B}} = \begin{bmatrix}-7\\9\end{bmatrix}$