

**Math 220**  
**Winter 2024**  
**Assessment 5**  
**Dusty Wilson**

Name: \_\_\_\_\_

*Have you ever had a dream, Neo, that you were so sure was real? What if you were unable to wake from that dream? How would you know the difference between the dream world and the real world?*  
Morpheus in *The Matrix* (1999)

**No work = no credit**

1. Warm-ups

(a) (1 point) Trivial solution:

(b) (1 point) Homogeneous equation:

(c) (1 point) A basis is:

2. (1 point) In reference to the quote above, how do you know whether you are awake or dreaming? Answer using complete English sentences.

3. (7 points) Prove (or disprove) the following claim.

Claim: The set  $H$  of polynomials  $p(t) = 2at + at^2$ , where  $a$  is in  $\mathbb{R}$ , is a subspace of  $\mathbb{P}_2$

4. (10 points) Consider matrix  $C = \begin{bmatrix} 1 & 1 & 2 & -1 & 2 & 3 \\ 0 & -5 & -5 & 10 & 0 & 0 \\ 2 & 2 & 4 & -2 & 5 & 4 \\ 3 & -2 & 1 & 7 & 7 & 8 \end{bmatrix}$ .

(a) (2 points) Find a basis for the column space of  $C$

(b) (2 points) The rank of  $C$

(c) (2 points) The null space of  $C$

(d) (2 points) A basis for  $\text{row}(C)$

(e) (2 points) Choose any non-zero  $\vec{u} \in \text{nul}(C)$  and  $\vec{v} \in \text{row}(C)$  and then find  $\vec{v} \vec{u}$

5. (2 points) True or False:  $\text{Col}A$  is the set of all solutions of  $A\vec{x} = \vec{b}$ . Justify your answer.

6. (4 points) Give an example of a space that is not a subspace.

You can have fame, fortune, and an extra credit point if your example satisfies one of the three conditions of a subspace; two points if it satisfies two of the conditions.

7. (4 points) Find two matrices  $A$  and  $B$  such that  $\left\{ \begin{bmatrix} b - c \\ 2b + c + d \\ 5c - 4d \\ d \end{bmatrix} : b, c, d \in \mathbb{R} \right\}$  is the column space of  $A$  and also the column space of  $B$