Name: ______________________________

**Math 220**
**Winter 2024**
**Assessment 4**
**Dusty Wilson**

*What is real? How do you define 'real'? If you're talking about what you can feel, what you can smell, what you can taste and see, then 'real' is simply electrical signals interpreted by your brain.*
Morpheus in *The Matrix* (1999)

**No work = no credit**

1. Warm-ups

   (a) (1 point) $AA^{-1}$

   (b) (1 point) $\vec{e}_2\,\vec{e}_2^{\,T}$

   (c) (1 point) $\vec{e}_2^{\,T}\vec{e}_2$

2. (2 points) In light of the quote by Morpheus, what is real? Answer using complete English sentences.

3. (4 points) Calculate $\begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix}$. Is this determinant invertible? Why or why not.

4. (4 points) Find the determinant of $A = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. Is $A$ invertible? Why or why not.

5. (4 points) Find the determinant of $B = \begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & -5 & 0 & 0 \\ 2 & 2 & 5 & 4 \\ 0 & 3 & 0 & 1 \end{bmatrix}$. Is $B$ invertible? Why or why not.

6. (7 points) For the matrix $A_{n \times n}$, there are at least 13 statements equivalent to, "$A$ is invertible." List at least seven of them. List more for extra credit (2 points max).

| | |
|---|---|
| i.) $A$ is invertible | vi.) |
| ii.) | vii.) |
| iii.) | viii.) |
| iv.) | xi.) (1 pt extra credit) |
| v.) | x.) (1 pt extra credit) |

What is your name (for the calculator portion):

7. (2 points) True or False: The determinant of $A$ is the product of the diagonal entries in $A$. Justify your answer.

8. (4 points) One interpretation of the determinant is as the scaling factor of a linear transformation. For example, "A region with area $X$ becomes a region of area $X * det(A)$ under the linear transformation $T(\vec{x}) = A\vec{x}$"

If $A = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$, (a.) find and (b.) interpret the determinants of $A$ and $A^{-1}$ using the language of scaling factors.

9. (4 points) Given the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{bmatrix}$ and vector $\vec{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, solve the matrix equation $A\vec{x} = \vec{b}$ using the matrix inverse. You may use a calculator, but show enough work so that it is clear that you could do this by hand if necessary.

10. (4 points) Prove the following.

    Claim: If $A$ is an invertible $n \times n$ matrix, then for each $\vec{b}$ in $\mathbb{R}^n$, the equation $A\vec{x} = \vec{b}$ has the unique solution $\vec{x} = A^{-1}\vec{b}$.