Thursday, February ... 2023 8:43 AM

...oordinates-220

to the homogeneous equation since basis vectors are LI.

coordinate vector of $\vec{x}$ relative to basis B

B - coordinate vector of $\vec{x}$

coordinate mapping

$\vec{x} = -2\begin{bmatrix}1\\0\end{bmatrix} + 3\begin{bmatrix}1\\2\end{bmatrix}$

$[\vec{x}]_B = \vec{x}$

NEW

$1\begin{bmatrix}1\\0\end{bmatrix} + 6\begin{bmatrix}0\\1\end{bmatrix}$ $1\vec{e}_1 + 6\vec{e}_2$

$[\vec{x}]_B = \begin{bmatrix}-2\\3\end{bmatrix}$

1 Standard graph paper.

on page 219.

...inate vector $[\mathbf{x}]_B$ of $\mathbf{x}$ rel...

... reduction

$[\vec{x}]_B = \begin{bmatrix}c_1\\c_2\end{bmatrix}$

$c_1\begin{bmatrix}2\\1\end{bmatrix} + c_2\begin{bmatrix}-1\\1\end{bmatrix} = \begin{bmatrix}4\\5\end{bmatrix}$

$\Rightarrow \begin{bmatrix}2 & -1\\1 & 1\end{bmatrix}\begin{bmatrix}c_1\\c_2\end{bmatrix} = \begin{bmatrix}4\\5\end{bmatrix}$

$\Rightarrow \left[\begin{array}{cc|c}2 & -1 & 4\\1 & 1 & 5\end{array}\right]$

$\sim \left[\begin{array}{cc|c}1 & 0 & 3\\0 & 1 & 2\end{array}\right]$ so $[\vec{x}]_B = \begin{bmatrix}3\\2\end{bmatrix}$

$\vec{x} \xrightarrow{P_B} [\vec{x}]_B$

P_B^{-1} change of basis matrix: P_B^{-1}

$P_B[\vec{x}]_B = \vec{x} \iff [\vec{x}]_B = P_B^{-1}\vec{x}$

$c \quad d$ ... d ... $ad-bc$ ... $-c \quad a$

basis

① L.I.

② span

$\vec{x} \xrightarrow{P_B} [\vec{x}]_B$ $T(\vec{x}) = A\vec{x}$

$[\vec{x}]_B = \begin{bmatrix}c_1\\\vdots\\c_n\end{bmatrix}$

$\vec{x} \xrightarrow{A} T(\vec{x}) = A\vec{x}$

matrix mult. (left) gives a linear transformation

P_B = matrix

$[\vec{x}]_B$ = vector

$\mathbb{P}_3$ $\mathbb{R}^4$

FIGURE 5 The coordinate mapping from V onto $\mathbb{R}^n$.

Theorem 8
Let $B = \{\mathbf{b}_1, \dots, \mathbf{b}_n\}$ be a basis for a vector space V. Then the coordinate mapping $\mathbf{x} \mapsto [\mathbf{x}]_B$ is a one-to-one linear transformation from V onto $\mathbb{R}^n$.

$\mathbb{P}_3$ $\mathbb{R}^4$

isomorphism between the vector spaces.

$\mathbb{P}_3 \Longleftrightarrow \mathbb{R}^4$

A one-to-one linear transformation from a vector space V onto a vector space W is called an isomorphism from V onto W.

Essentially, these two vector spaces are indistinguishable.

Ex 4: Let B be the standard basis of the space $\mathbb{P}_3$ of polynomials; that is, let $B = \{1, t, t^2, t^3\}$. A typical element $\mathbf{p}$ of $\mathbb{P}_3$ has the form

not a vector $\rightarrow$ $\mathbf{p}(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$

Since p is a linear combination of the standard basis vectors, then $[\mathbf{p}]_B = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}$ $\leftarrow$ is a traditional vector.

$\begin{bmatrix} 1 \\ 0 \\ 2 \\ 3 \end{bmatrix} \Longleftrightarrow 1 + 0t + 2t^2 + 3t^3$

So $\mathbf{p} \mapsto [\mathbf{p}]_B$ is an isomorphism from $\mathbb{P}_3$ onto $\mathbb{R}^4$.

Ex 5: Use coordinate vectors to test the linear independence of the sets of polynomials

a) $1 + 2t^3, 2 + t - 3t^2, -t + 2t^2 - t^3$

$A\vec{x} = \vec{0}$ are there non-triv. sol.

$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 2 \end{bmatrix}$ $\begin{bmatrix} 2 \\ 1 \\ -3 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 0 \\ -1 \\ 2 \\ -1 \end{bmatrix}$

$[p_1]_B$ $[p_2]_B$ $[p_3]_B$

$\rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -3 & 2 & 0 \\ 2 & 0 & -1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

can't make one poly from the others.

vectors are L.I.

$\Leftrightarrow$ polys are L.I.

$\mathbb{P}_3$

- 1
- t
- t^2
- t^3

} standard basis

$\dim(\mathbb{P}_3) = 4$

$\mathbb{R}^3$

$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \vec{e}_2, \vec{e}_3$

$\dim(\mathbb{R}^3) = 3$

b) Is this a basis $\mathbb{P}_3$? $(1-t)^2, t - 2t^2 + t^3, (1-t)^3$ $\Longleftrightarrow$ 3 L.I. vectors in $\mathbb{R}^4$

No

3 polys can't span $\mathbb{P}_3$.

Does not span.

$\Rightarrow$ vectors did not span $\mathbb{R}^4$.

$\therefore$ not a basis for $\mathbb{R}^4$

Ex 6: Let $\mathbf{v}_1 = \begin{bmatrix} 3 \\ 6 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} 3 \\ 12 \\ 7 \end{bmatrix}$,

and $B = \{\mathbf{v}_1, \mathbf{v}_2\}$. Then B is a basis for $H = \mathrm{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$. Determine if $\mathbf{x}$ is in H, and if it is, find the coordinate vector of $\mathbf{x}$ relative to B.

① row reducing

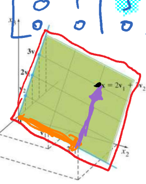
$P_B [\vec{x}]_B = \vec{x}$

$\left[\begin{array}{cc|c} 3 & -1 & 3 \\ 6 & 0 & 12 \\ 2 & 1 & 7 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right] \leftarrow$ consistent $\vec{x} \in H$

$[\vec{x}]_B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$\vec{x} = 2 \begin{bmatrix} 3 \\ 6 \\ 2 \end{bmatrix} + 3 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$2\vec{v}_1 + 3\vec{v}_2$



$\vec{x} = \begin{bmatrix} 3 \\ 12 \\ 7 \end{bmatrix}$ ② Graphing.

$[\vec{x}]_B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$\boxed{③\ P_B^{-1} \text{ fails}}$

Practice Problems

1. Let $\mathbf{b}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \mathbf{b}_2 = \begin{bmatrix} -3 \\ 4 \\ 0 \end{bmatrix}, \mathbf{b}_3 = \begin{bmatrix} 3 \\ -6 \\ 3 \end{bmatrix}$, and $\mathbf{x} = \begin{bmatrix} -8 \\ 2 \\ 3 \end{bmatrix}$.

a. Show that the set $B = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ is a basis of $\mathbb{R}^3$.

b. Find the change-of-coordinates matrix from B to the standard basis.

c. Write the equation that relates $\mathbf{x}$ in $\mathbb{R}^3$ to $[\mathbf{x}]_B$.

d. Find $[\mathbf{x}]_B$, for the $\mathbf{x}$ given above.

(a.) $P_B = \begin{array}{c} \begin{array}{ccc} \vec{b}_1 & \vec{b}_2 & \vec{b}_3 \end{array} \\ \begin{bmatrix} 1 & -3 & 3 \\ 0 & 4 & -6 \\ 0 & 0 & 3 \end{bmatrix} \end{array}$ has a pivot in every column. Three L.I. vectors in $\mathbb{R}^3$ must form a basis for $\mathbb{R}^3$.

(b.) $\vec{x} \underset{P_B^{-1}}{\overset{P_B}{\rightleftarrows}} [\vec{x}]_B$ so the P_B matrix above does the trick. $P_B : [\vec{x}]_B \longmapsto \vec{x}$

(c.) $P_B [\vec{x}]_B = \vec{x}$

OR

$P_B^{-1} \vec{x} = [\vec{x}]_B$

(d.) $\begin{bmatrix} 1 & 3/4 & 1/2 \\ 0 & 1/4 & 1/2 \\ 0 & 0 & 1/3 \end{bmatrix} \underbrace{\begin{bmatrix} -8 \\ 2 \\ 3 \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} -5 \\ 2 \\ 1 \end{bmatrix}_B}_{[\vec{x}]_B}$

2. The set $B = \{1 + t, 1 + t^2, t + t^2\}$ is a basis for $\mathbb{P}_2$. Find the coordinate vector of $\mathbf{p}(t) = 6 + 3t - t^2$ relative to B.

$\mathbb{P}_2 \longleftrightarrow \mathbb{R}^3$

so let $P_B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

And $\vec{x} = \begin{bmatrix} 6 \\ 3 \\ -1 \end{bmatrix}$

Now, $[\vec{x}]_B = P_B^{-1} \vec{x}$

$= \begin{bmatrix} 5 \\ 1 \\ -2 \end{bmatrix}_B$

Thus $p(t) = 5(1+t) + 1(1+t^2) + -2(t+t^2)$.

so Let $P_B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

Thus $p(t) = 5(1+t) + 1(1+t^2) + -2(t+t^2)$.

AND $\vec{x} = \begin{bmatrix} \text{[illegible]} \\ \text{[illegible]} \\ \text{[illegible]} \end{bmatrix}$