

To get Jordan normal form of A we first find the eigenvalues and eigenvectors.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The characteristic polynomial is $(\lambda - 1)^3 = 0$, so the eigenvalue is $\lambda = 1$ with multiplicity 3.

To find the Jordan normal form, we need to find the generalized eigenvectors.

$$(A - I)v = 0 \implies \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} v = 0$$

This gives us a 3-dimensional eigenspace. We can choose a basis for the eigenspace as $\{v_1, v_2, v_3\}$.

3.) (8 pts) Solve the augmented matrix and express your solution in vector form.

$$\left[\begin{array}{ccccc|c} 2 & -4 & 3 & -4 & -11 & 28 \\ -1 & 2 & -1 & 2 & 5 & -13 \\ 0 & 0 & -3 & 1 & 6 & -10 \\ 3 & -6 & 10 & -8 & -28 & 61 \end{array} \right] \sim \left[\begin{array}{ccccc|c} 1 & -2 & 0 & 0 & 2 & 3 \\ 0 & 0 & -1 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$x_1 = 3 + 2x_2 + 2x_5$$

$$x_2 = x_2 \text{ (free)}$$

$$x_3 = 2 + x_5$$

$$x_4 = -4 - 3x_5$$

$$x_5 = x_5 \text{ (free)}$$

$$\Rightarrow \vec{x} = \begin{bmatrix} 3 \\ 0 \\ 2 \\ -4 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 2 \\ 0 \\ 1 \\ -3 \\ 1 \end{bmatrix}$$

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No Calculator

1.) (8 pts) Solve the linear system

$$\begin{aligned} x_1 - 3x_3 &= 8 \\ 2x_1 + 2x_2 + 9x_3 &= 7 \\ x_2 + 5x_3 &= -2 \end{aligned} \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{array} \right] \quad R_2 - 2R_1 \rightarrow R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 2 & 15 & -9 \\ 0 & 1 & 5 & -2 \end{array} \right] \quad R_2 \leftrightarrow R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 2 & 15 & -9 \end{array} \right] \quad R_3 - 2R_2 \rightarrow R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 5 & -5 \end{array} \right] \quad \frac{1}{5} R_3 \rightarrow R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 1 & -1 \end{array} \right] \quad \begin{array}{l} R_1 + 3R_3 \rightarrow R_1 \\ R_2 - 5R_3 \rightarrow R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array} \right] \Rightarrow \vec{X} = \begin{bmatrix} 5 \\ 3 \\ -1 \end{bmatrix}$$