100	90's	80's	70's	60's	< 60
2	10	11	4	8	8

Assessment 8 (10 or 11 am)
Dusty Wilson
Math 220

$\bar{x} = 75.5\%$
med $= 77.5\%$

No work = no credit

Name (first & last): key

But just because geometry is so eminently fitted for the youthful mind it should be at first presented in such form as to be in accord with the general views of laymen, when these are not in direct opposition to the truth.

Charlotte Angas Scott
1858-1931 (English mathematician)

Warm-ups (1 pt each): $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}^5 = \begin{bmatrix} 1 & 0 \\ 0 & 32 \end{bmatrix}$ $\quad \begin{bmatrix} 1 & 0 \\ 0 & 0.2 \end{bmatrix}^{500} = \begin{bmatrix} 1 & 0 \\ 0 & .2^{500} \end{bmatrix}$ $\quad \frac{\sqrt{3}}{\sqrt{12}} = \frac{1}{2}$

1.) (1 pt) According to Scott (above), geometry should be taught in a way that makes sense to ordinary people. How could linear algebra be taught so as to make sense to more people? Please answer using complete sentences.

Perhaps teaching w/more applications would be better.

2.) (4 pts) Consider the matrix $A = \begin{bmatrix} 3.4 & -4 \\ 1.5 & -1.5 \end{bmatrix}$ which has eigenvalues 1 and 0.9 with respective eigenvectors $\begin{bmatrix} 5 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 8 \\ 5 \end{bmatrix}$.

a.) Diagonalize A.

$$A = \underbrace{\begin{bmatrix} 5 & 8 \\ 3 & 5 \end{bmatrix}}_{P} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0.9 \end{bmatrix}}_{D} \underbrace{\begin{bmatrix} 5 & -8 \\ -3 & 5 \end{bmatrix}}_{P^{-1}}$$

b.) Explain how you can use the result of (a.) to find ~~powers of the matrix~~ A^k.

$A^k = PD^kP^{-1}$ and we know $D^k = \begin{bmatrix} 1 & 0 \\ 0 & .9^k \end{bmatrix}$. so this makes finding A^k easy.

c.) Use diagonalization to find $\lim_{k\to\infty} A^k$

$$\lim_{k\to\infty} A^k = \begin{bmatrix} 5 & 8 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 5 & -8 \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} 25 & -40 \\ 15 & -24 \end{bmatrix}$$

$$\left[\begin{matrix} a & -b \\ b & a \end{matrix}\right]$$

3.) (4 pts) The matrix $A = \begin{bmatrix} \sqrt{3} & -3 \\ 3 & \sqrt{3} \end{bmatrix}$ is a rotation-scaling matrix.

a.) Find the scaling factor of A (Hint: It is irrational).

$$r = \sqrt{a^2+b^2} = \sqrt{3+9} = \sqrt{12}$$

b.) Find the angle of rotation of A (Hint: It is a familiar angle).

$$\begin{bmatrix} \sqrt{3} & -3 \\ 3 & \sqrt{3} \end{bmatrix} = \sqrt{12}\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

$\cos\varphi = \frac{1}{2}$

$\Rightarrow \varphi = \frac{\pi}{3}$

c.) Suppose $\vec{x}_0 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, find (i.) the angle between $\vec{x}_0$ and $A^2\vec{x}_0$. Also find (ii.) the ratio of the two magnitudes. That is, find: $\frac{\|A^2\vec{x}_0\|}{\|\vec{x}_0\|}$. (Hint: How does this connect to parts (a.) and (b.)?).

(i) $2 \cdot \frac{\pi}{3} = \frac{2\pi}{3}$

(ii) $\frac{60}{5} = 12 \leftarrow (\sqrt{12})^2$

check: $x_1 = \begin{bmatrix} 3\sqrt{3}-12 \\ 9+4\sqrt{3} \end{bmatrix}$

$\vec{x}_2 = \begin{bmatrix} -18-24\sqrt{3} \\ 18\sqrt{3}-24 \end{bmatrix}$

$\|x_0\| = 5$, $\|\vec{x}_2\| = 60$ $\quad \frac{\|x_2\|}{\|x_0\|} = 12$

Angle

$\vec{x}_0 \cdot \vec{x}_2 = \|\vec{x}_0\|\|\vec{x}_2\|\cos\theta$

$\Rightarrow \frac{-150}{5 \cdot 60} = \cos\theta$

$\Rightarrow \theta = \frac{2\pi}{3}$

4.) (4 pts) Find the eigenvalue(s) and eigenvector(s) of $A = \begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix}$

solve $0 = \begin{vmatrix} 1-\lambda & 5 \\ -2 & 3-\lambda \end{vmatrix}$

$= (1-\lambda)(3-\lambda) + 10$

$= \lambda^2 - 4\lambda + 13$

$\lambda = \frac{4 \pm \sqrt{16-4(1)(13)}}{2(1)}$

$= \frac{4 \pm \sqrt{-36}}{2}$

$= 2 \pm 3i$

$A - (2+3i)I = \begin{bmatrix} -1-3i & 5 \\ -2 & 1-3i \end{bmatrix} \sim \begin{bmatrix} -2 & 1-3i \\ -1-3i & 5 \end{bmatrix}$

so the eigen vector is

$\begin{bmatrix} 1-3i \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + i\begin{bmatrix} -3 \\ 0 \end{bmatrix}$

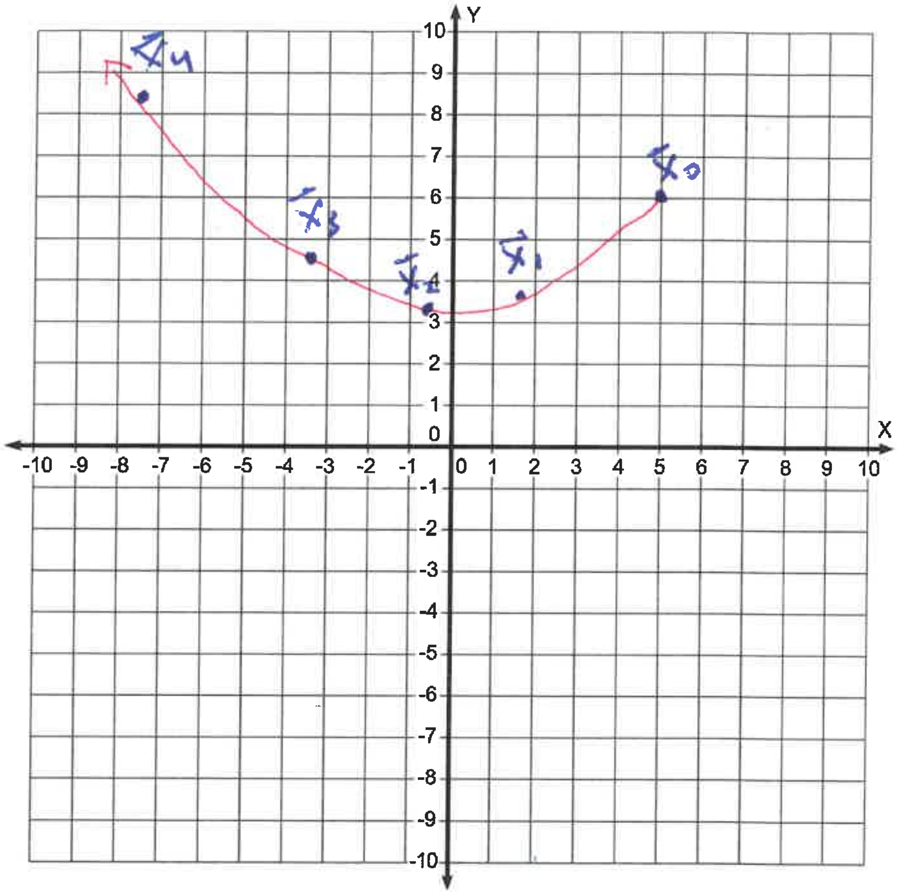
and also $\begin{bmatrix} 1 \\ 2 \end{bmatrix} - i\begin{bmatrix} -3 \\ 0 \end{bmatrix}$

eigenvecs

$\begin{bmatrix} \frac{1}{2} \pm \frac{3}{2}i \\ 1 \end{bmatrix}$

5.) (4 pts) Consider the dynamical system $A = \begin{bmatrix} 1.25 & -0.75 \\ -0.75 & 1.25 \end{bmatrix}$ with initial state $\vec{x}_0 = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$.

Carefully plot $\vec{x}_0, \vec{x}_1, \vec{x}_2, \vec{x}_3, \vec{x}_4$, ~~and $\vec{x}_5$~~ and the trajectory curve that begins at $\vec{x}_0$.



b.) Explain the process you would use to find a nice formula for $\vec{x}_k$ involving k and the eigenvalues/eigenvectors of A.

1. find eigenvalues,
2. find eigenvectors.
3. diagonalize (if possible)

$A = P D P^{-1}$

eigen vecs (for P) $\quad$ λ's (for D)

4. $\vec{x}_k = P D^k P^{-1} \vec{x}_0$

λ's to k^{th} power