

Assessment 6 (10 or 11 am)
 Dusty Wilson
 Math 220

Name (first & last): Key

*Either mathematics is too big for the human mind or
 the human mind is more than a machine.*

No work = no credit

Kurt Gödel
 1906-1978 (Austrian mathematician)

Warm-ups (1 pt each):

$\vec{e}_2 + \vec{e}_1^T =$ *undefined mismatched dim.* $\vec{e}_2 \vec{e}_1^T = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} - 3^2 = -9$

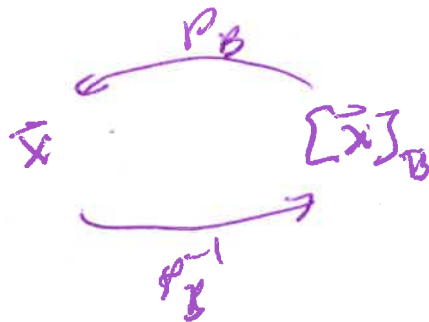
1.) (1 pt) Based upon the quote by Gödel (above), did Gödel believe that all mathematics could be performed by a machine? Please answer using complete sentences.

Gödel thought math is more than machines.

2.) (4 pts) Show that $B = \{1+t, 1+t^2, t+t^2\}$ is a basis for P_2 and then find the coordinate vector of $p(t) = 6+3t-t^2$ relative to B .

$B \leftrightarrow \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$

And $P_B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \sim I$ so the vectors are L.I. and form a basis.



$P_B^{-1} \begin{bmatrix} 6 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ -2 \end{bmatrix}_B$

OR $p(t) = 5(1+t) + 1(1+t^2) - 2(t+t^2)$

3.) (8 pts) Given the matrix $A = \begin{bmatrix} 1 & 1 & 3 & 1 \\ 2 & 1 & 5 & 4 \\ 1 & 2 & 4 & -1 \end{bmatrix}$, answer the following.

a) Find a basis for the column space of A .

$$A \sim \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{basis for col } A = \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \right\}$$

b) Find $\text{Nul } A$

$$\text{Nul } A = \text{span} \left\{ \begin{bmatrix} -2 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\}$$

c) What is the dimension of the null space of A and what is the rank of A ?

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d) The zero vector is in both the null space and column space. List all vectors (if any) in the intersection (overlap) of the two subspaces.

$$\begin{aligned} \text{col } A &\subset \mathbb{R}^3 \\ \text{Nul } A &\subset \mathbb{R}^4 \end{aligned}$$

the intersection
is the empty set.

4.) (4 pts) Complete the proof of the Unique Representation Theorem

Claim: Let $B = \{\vec{b}_1, \dots, \vec{b}_n\}$ be a basis for a vector space V . Then for each \vec{x} in V , there exists a unique set of scalars c_1, \dots, c_n such that $\vec{x} = c_1\vec{b}_1 + \dots + c_n\vec{b}_n$.

Proof.

Let $\vec{x} \in V$ be given. Assume there are two representations for \vec{x} with respect to B ,

$$\text{so } \vec{x} = c_1\vec{b}_1 + \dots + c_p\vec{b}_p \text{ and}$$

$$\vec{x} = d_1\vec{b}_1 + \dots + d_p\vec{b}_p \text{ for scalars } c_1, \dots, c_p \text{ and } d_1, \dots, d_p$$

$$\Rightarrow \vec{0} = \vec{x} - \vec{x} = (c_1 - d_1)\vec{b}_1 + \dots + (c_p - d_p)\vec{b}_p$$

but $\vec{b}_1, \dots, \vec{b}_p$ is a basis so there are no non-trivial solns. to the homogeneous eqn.

$$\Rightarrow c_i - d_i = 0 \text{ or } c_i = d_i \text{ for } i = 1, \dots, p$$

$\therefore \vec{x}$ has a unique representation.

5.) (3 pts) True or False (circle one). Justify your answer.

a.) (T or F) If $H = \text{Span}\{\vec{b}_1, \dots, \vec{b}_p\}$, then $\{\vec{b}_1, \dots, \vec{b}_p\}$ is a basis for H ?

False. $\vec{b}_1, \dots, \vec{b}_p$ must also be LI to be a basis.

b.) (T or F) The dimension of the vector space P_4 is 4.

False. $\dim(P_4) = 5$

c.) (T or F) The dimensions of the row space and the column space of A are the same, even if A is not square.

True. $\dim(\text{row } A) = \dim(\text{col } A) = \# \text{ of pivots in } A$.