100	90's	80's	70's	60's	< 60
0	6	14	9	14	6

Assessment 2 (10 or 11 am)
Dusty Wilson
Math 220

$\bar{x} = 72.9\%$

No work = no credit

No calculator

med = 73.1%

Name (first & last): Key

Unfortunately what is little recognized is that the most worthwhile scientific books are those in which the author clearly indicates what he does not know; for an author most hurts his readers by concealing difficulties.

Évariste Galois
1811 – 1832 (French mathematician)

Warm-ups (1 pt each): $\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\quad -\frac{4}{0} =$ undefined $\quad \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

1.) (1 pt) According to Galois (above), what most hurts scientific books? Answer using complete English sentences.

Authors should not hide or obscure what they do not know.

2.) (8 pts) Let $\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\vec{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\vec{y}_1 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$, and $\vec{y}_1 = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$, and let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that maps $\vec{e}_1$ into $\vec{y}_1$ and $\vec{e}_2$ into $\vec{y}_2$. Find the image of $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$.

$T(\vec{x}) = A\vec{x}$

w/ $A = \begin{bmatrix} 2 & -1 \\ 5 & 6 \end{bmatrix}$

And $T\left(\begin{bmatrix} -2 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2 & -1 \\ 5 & 6 \end{bmatrix}\begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} -5 \\ -4 \end{bmatrix}$

3.) (8 pts) Determine whether the columns of $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ are linearly independent. If not, find a linear dependence relation among its columns $\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$

$$\text{rref}([A \mid \vec{0}]) = \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{aligned} x_1 &= x_3 \\ x_2 &= -2x_3 \\ x_3 &= x_3 \end{aligned} \Rightarrow \vec{x} = x_3 \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$$\Rightarrow \vec{a}_1 - 2\vec{a}_2 + \vec{a}_3 = \vec{0}$$

2 pts.

4.) (6 pts) True or False (circle one). Justify your answer.

a.) (T or F) If A is an $m \times n$ matrix, then the range of the transformation $\vec{x} \to A\vec{x}$ is $\mathbb{R}^m$.

False. The codomain is $\mathbb{R}^m$.

b.) (T or F) If a set in $\mathbb{R}^n$ is linearly dependent, then the set contains more vectors than there are entries in each vector.

False. The set could, for example, just include the zero vector.

c.) (T or F) The homogeneous equation $A\vec{x} = \vec{0}$ has the trivial solution if and only if the equation has at least one free variable.

False. $A\vec{x} = \vec{0}$ always has the trivial soln.

5.) (5 pts) Complete the proof by filling in the missing pieces.

claim: An indexed set $S = \{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ of two or more vectors is linearly dependent if and only if at least one of the vectors in S is a linear combination of the others. In fact, if S is linearly dependent and $\mathbf{v}_1 \neq \mathbf{0}$, then some $\mathbf{v}_j$ (with $j > 1$) is a linear combination of the preceding vectors, $\mathbf{v}_1, \ldots, \mathbf{v}_{j-1}$.

proof,

($\Rightarrow$) Assume S is linearly dependent.

If $\vec{v}_1 = \vec{0}$ then it is a linear combination of the other vectors: $0\vec{v}_2 + \ldots + 0\vec{v}_p = \vec{v}_1$.

If $\vec{v}_1 \neq \vec{0}$ then $c_1\vec{v}_1 + \ldots + c_{j-1}\vec{v}_{j-1} + c_j\vec{v}_j + c_{j+1}\vec{v}_{j+1} + \ldots + c_p\vec{v}_p = \vec{0}$ where not all of $c_1, \ldots, c_p$ are zero. Suppose $c_j \neq 0$ and $c_{j+1} = \ldots = c_p = 0$.

$\Rightarrow c_1\vec{v}_1 + \ldots + c_j\vec{v}_j + 0\vec{v}_{j+1} + \ldots + 0\vec{v}_p = \vec{0}$

$\Rightarrow \vec{v}_j = -\frac{c_1}{c_j}\vec{v}_1 - \ldots - \frac{c_{j-1}}{c_j}\vec{v}_{j-1}$

thus at least one vector in S is a linear combination of the others.

($\Leftarrow$) Assume at least one vector in S is a linear comb of the others (Note: we can always reorder if need be to make this true).

$\Rightarrow \exists\, c_1, \ldots, c_{j-1} \in \mathbb{R}$ s.t. $\vec{v}_j = c_1\vec{v}_1 + \ldots + c_{j-1}\vec{v}_{j-1}$

$\Rightarrow c_1\vec{v}_1 + \ldots + c_{j-1}\vec{v}_{j-1} - \vec{v}_j + 0\vec{v}_{j+1} + \ldots + 0\vec{v}_p = \vec{0}$

not zero

$\Rightarrow$ S is linearly dependent since there is a non-trivial soln. to the homogeneous eqn.

$\therefore$ S is linearly dependent iff at least one of its vecs is a lin. comb. of the others.

6.) (8 pts) Consider the homogeneous "system" $3x_1 - \overset{6}{\cancel{4}}x_2 + \overset{9}{\cancel{5}}x_3 = 0$. Describe all solutions algebraically (in vector form) and geometrically (use words to describe the graph).

$$\left[\begin{array}{ccc|c} 3 & -6 & 9 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \end{array}\right]$$

$$\Rightarrow \quad x_1 = 2x_2 - 3x_3$$

$$x_2 = x_2$$

$$x_3 = x_3$$

$$\Rightarrow \quad \vec{x} = x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$$

1 pt.

$\text{2 pts.}\left\{\begin{array}{l}\text{which is a plane thru the}\\ \text{origin and parallel to } \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}.\end{array}\right.$