

Math 220
Winter 2024
Assessment 6
Dusty Wilson

Name: _____

We have to push the boundaries, take the risks. Without that, there is no science. No medicine.

Dr. Michael Morbius in *Morbius* (2022)

No work = no credit

1. Warm-ups

(a) (1 point) \mathbb{P}_n isomorphic with \mathbb{R}

(b) (1 point) min rank of $A_{2 \times 3}$

(c) (1 point) max rank of $A_{2 \times 3}$

2. (1 point) In reference to the quote above, what is required for science and medicine to exist? Answer using complete English sentences.

3. (4 points) Prove (or disprove) the Unique Representation Theorem.

Claim: Let $B = \{\vec{b}_1, \dots, \vec{b}_n\}$ be a basis for a vector space V . Then for each $\vec{x} \in V$, there exists a unique set of scalars c_1, \dots, c_n such that $\vec{x} = c_1\vec{b}_1 + \dots + c_n\vec{b}_n$.

4. (4 points) Consider the subspace $H = \{(a, b, c) : a - 3b + c = 0, b - 2c = 0, a - 2b - c = 0\}$. Find a basis and state the dimension of H .

5. (4 points) Find $[\vec{x}]_{\mathcal{B}}$ if $\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ -9 \end{bmatrix}, \begin{bmatrix} 1 \\ 8 \end{bmatrix} \right\}$ and $\vec{x} = \begin{bmatrix} 13 \\ 4 \end{bmatrix}$

6. (2 points) True or False: The number of variables in the equation $A\vec{x} = \vec{0}$ equals the nullity of A . Justify your answer.

7. (4 points) Let $\mathcal{B} = \left\{ \begin{bmatrix} 7 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right\}$ and $\mathcal{C} = \left\{ \begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \end{bmatrix} \right\}$ be bases for \mathbb{R}^2 . Find the change-of-coordinates matrix from \mathcal{B} to \mathcal{C} and find the \mathcal{C} -coordinates of $\begin{bmatrix} 1 \\ 1 \end{bmatrix}_{\mathcal{B}}$