Assessment 4
Dusty Wilson
Math 220

No work = no credit

Name: New

May not music be described as the mathematics of the sense, mathematics as music of the reason? The musician feels mathematics, the mathematician thinks music: music the dream, mathematics the working life.

James Joseph Sylvester
1814 - 1897 (English mathematician)

Warm-ups (1 pt each): Note: Assume  $\vec{e}_1, \vec{e}_2 \in \mathbb{R}^2$ 

 $\operatorname{rref}(I) = \underline{\mathcal{I}}$ 

 $AA^{-1} =$  I

 $\vec{e}_2 \vec{e}_1^T =$ 

1.) (1 pt) The quote above is by Sylvester who was one of the inventers of linear algebra. What connection did Sylvester draw between math and music? Answer using complete English sentences

physical mandefestation of the metaphos

2.) (3 pts) A linear transformation T from a vector space V into a vector space W is a rule that assigs to each vector  $\bar{x}$  in V a unique vector  $T(\bar{x})$  in W, such that

(i.) T(x)+T(z)=T(x+z) for all x, z & V

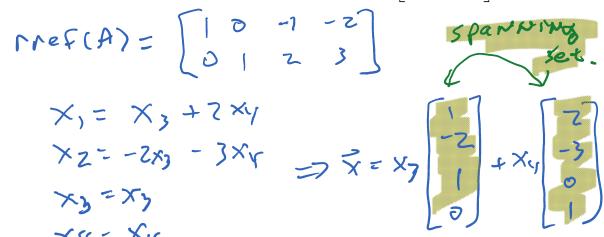
(ii.) T(cを)=cT(学) for all だもV and ce好

3.) (2 pts) Define a linear transformation  $T: P_2 \to \mathbb{R}^2$  by  $T(p) = \begin{bmatrix} p(0) \\ p(0) \end{bmatrix}$ . Find polynomials  $p_1$  and  $p_2$  in  $P_2$  that span the range of T.

Pi'N Pz looks like on+bt+ct2 | a => a=0 if P is to be in the heavel.

Let 
$$p_1 = t$$
 and  $p_2 = t^2$  and learnel(T) = Span  $\{t, t^2, t^3\}$ 
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4.) (5 pts) Find a spanning set for the null space of the matrix  $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}$ 



- 5.) (5 pts) Given  $\vec{v}_1$  and  $\vec{v}_2$  in a vector space V, let  $H = \text{Span}\{\vec{v}_1, \vec{v}_2\}$ .
  - a.) What properties must H satisfy if it is to be a subspace?

- b.) Show that H is a subspace of V.
  - 0 6 = 0√, + 0√z ∈ Span {√1; √z}= H.
  - D Let M, WEH be given.

    Differe are a,b,c,d ERs.t. M=av,+bvz

- There are  $a,b,c,d \in \mathbb{R}$  s.  $\star$ .  $\pi = a\vec{v}_1 + b\vec{v}_2$ White  $\vec{v} = (a\vec{v}_1 + b\vec{v}_2) + (c\vec{v}_1 + d\vec{v}_2) + c\vec{v}_1 + c\vec{v}_2$   $= (a+c)\vec{v}_1 + (b+d)\vec{v}_2$   $= span \{\vec{v}_1, \vec{v}_2\vec{v}_3 = H$ (3) Let  $\pi \in H$  be given and  $\pi \in H$  be given  $\vec{v} = c\vec{v}_1 + \beta\vec{v}_2$   $\Rightarrow c \vec{v} = c (d\vec{v}_1 + \beta\vec{v}_2)$   $= (cd)\vec{v}_1 + (c\beta)\vec{v}_2$   $= gan 2\vec{v}_1, \vec{v}_2\vec{v}_3 = H$ 
  - .. H is a subspace.