A4

Thursday, February 16, 2023 8:21 AM

Assessment 4
Dusty Wilson
Math 220

No work = no credit

Name: Key

May not music be described as the mathematics of the sense, mathematics as music of the reason? The musician feels mathematics, the mathematician thinks music: music the dream, mathematics the working life.

James Joseph Sylvester
1814 - 1897 (English mathematician)

$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix}$

Warm-ups (1 pt each):
Note: Assume $\vec{e}_1, \vec{e}_2 \in \mathbb{R}^2$

$\text{rref}(I) = I$ $\qquad AA^{-1} = I$ $\qquad \vec{e}_2 \vec{e}_1^T = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

1.) (1 pt) The quote above is by Sylvester who was one of the inventers of linear algebra. What connection did Sylvester draw between math and music? Answer using complete English sentences.

Music is a metaphor. Math is a physical manifestation of the metaphor.

2.) (3 pts) A linear transformation T from a vector space V into a vector space W is a rule that assigs to each vector $\vec{x}$ in V a unique vector $T(\vec{x})$ in W, such that

(i.) $T(\vec{x}) + T(\vec{y}) = T(\vec{x}+\vec{y})$ for all $\vec{x}, \vec{y} \in V$

(ii.) $T(c\vec{x}) = c\,T(\vec{x})$ for all $\vec{x} \in V$ and $c \in \mathbb{R}$

The null space a linear transformation is called the kernel.

3.) (2 pts) Define a linear transformation $T: P_2 \to \mathbb{R}^2$ by $T(p) = \begin{bmatrix} p(0) \\ p(0) \end{bmatrix}$. Find polynomials p_1 and p_2 in P_2 that span the ~~range~~ kernel of T.

p in P_2 looks like $a + bt + ct^2 \Big|_{t=0} \; a$

$\Rightarrow a = 0$ if p is to be in the kernel.

$\Rightarrow a = 0$ if p is to be in the kernel.

Let $p_1 = t$ and $p_2 = t^2$ and

kernel$(T) = \text{Span}\{t, t^2\}$

4.) (5 pts) Find a spanning set for the null space of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}$

$\text{rref}(A) = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \end{bmatrix}$

spanning set.

$x_1 = x_3 + 2x_4$

$x_2 = -2x_3 - 3x_4$

$x_3 = x_3$

$x_4 = x_4$

$\Rightarrow \vec{x} = x_3 \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 2 \\ -3 \\ 0 \\ 1 \end{bmatrix}$

5.) (5 pts) Given $\vec{v}_1$ and $\vec{v}_2$ in a vector space V, let $H = \text{Span}\{\vec{v}_1, \vec{v}_2\}$.

a.) What properties must H satisfy if it is to be a subspace?

① $\vec{0} \in H$

② $\vec{v}_1, \vec{v}_2 \in H \Rightarrow a\vec{v}_1 + b\vec{v}_2 \in H$ for all $a, b \in \mathbb{R}$

③ $\vec{v}_1 \in H \Rightarrow c\vec{v}_1 \in H$ for all $c \in \mathbb{R}$

b.) Show that H is a subspace of V.

① $\vec{0} = 0\vec{v}_1 + 0\vec{v}_2 \in \text{Span}\{\vec{v}_1, \vec{v}_2\} = H$.

② Let $\vec{u}, \vec{w} \in H$ be given.

$\Rightarrow$ there are $a, b, c, d \in \mathbb{R}$ s.t. $\vec{u} = a\vec{v}_1 + b\vec{v}_2$ and

② Let $u, w \in H$ be given

$\Rightarrow$ there are $a, b, c, d \in \mathbb{R}$ s.t. $\vec{u} = a\vec{v}_1 + b\vec{v}_2$ and $\vec{w} = c\vec{v}_1 + d\vec{v}_2$

$$u + \vec{w} = (a\vec{v}_1 + b\vec{v}_2) + (c\vec{v}_1 + d\vec{v}_2)$$

$$= (a+c)\vec{v}_1 + (b+d)\vec{v}_2$$

$$\in \operatorname{span}\{\vec{v}_1, \vec{v}_2\} = H$$

③ Let $\vec{u} \in H$ be given and $c \in \mathbb{R}$ be given

$\Rightarrow$ there $\alpha, \beta \in \mathbb{R}$ s.t. $\vec{u} = \alpha\vec{v}_1 + \beta\vec{v}_2$

$$\Rightarrow c\vec{u} = c(\alpha\vec{v}_1 + \beta\vec{v}_2)$$

$$= (c\alpha)\vec{v}_1 + (c\beta)\vec{v}_2$$

$$\in \operatorname{span}\{\vec{v}_1, \vec{v}_2\} = H$$

$\therefore$ H is a subspace.