A2

Monday, January 30, 2023 3:17 PM

Assessment 2
Dusty Wilson
Math 220

No work = no credit

Name: Key

He is like the fox, who effaces his tracks in the sand with his tail.

Niels Henrik Abel
1802 - 1829 (Norwegian mathematician)

Warm-ups (1 pt each): $\vec{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ 6/0 $\cancel{\frac{0}{6}} =$ undefined $-5^2 =$ -25

1.) (1 pt) In the quote above, Abel talks about Gauss' writing style.

According to Abel, how easy was it to understand Gauss' work? Answer using complete English sentences. Abel said Gauss was hard to understand; like a sly fox.

2.) (5 pts) Describe all solutions of $A\vec{x} = \vec{b}$ in parametric vector form, where A is row equivalent to the matrix:

$$\begin{bmatrix} 1 & -4 & -2 & 0 & 3 & -5 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad R_1 + 2R_2 \to R_1$$

$$\sim \begin{bmatrix} 1 & -4 & 0 & 0 & 3 & -7 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad R_1 - 3R_3 \to R_1$$

$$\sim \begin{bmatrix} 1 & -4 & 0 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 = 4x_2 - 5x_6$
$x_2 = x_2$
$x_3 = x_6$
$x_4 = x_4$
$x_5 = 4x_6$
$x_6 = x_6$

$$\Rightarrow \vec{x} = x_2 \begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_6 \begin{bmatrix} -5 \\ 0 \\ 1 \\ 0 \\ 4 \\ 1 \end{bmatrix}$$

3.) (6 pts) True or False. Please provide a short justification for your answer.

a.) (T/F) If a set contains fewer vectors than there are entries in the vectors, then the set is linearly independent.

F ex: $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \right\}$

b.) (T/F) The columns of a matrix A are linearly independent if the equation $A\vec{x} = \vec{0}$ has the trivial solution.

F $A\vec{x} = \vec{0}$ always has the trivial solution. The columns of A are L.I. iff $A\vec{x} = \vec{0}$ has only the trivial solution.

c.) (T/F) If $\vec{x}$ and $\vec{y}$ are linearly independent, and if $\{\vec{x}, \vec{y}, \vec{z}\}$ is linearly dependent, then $\vec{z}$ is in Span$\{\vec{x}, \vec{y}\}$.

T $c_1\vec{x} + c_2\vec{y} = \vec{0}$ iff $c_1 = c_2 = 0$

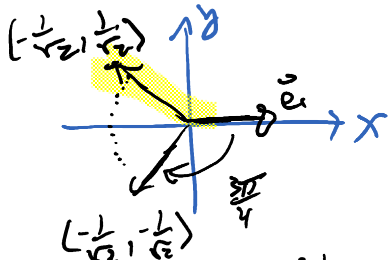
And $a_1\vec{x} + a_2\vec{y} + a_3\vec{z} = \vec{0}$ w/ at least one $a_i \neq 0$.

$\Rightarrow a_3 \neq 0. \Rightarrow a_1\vec{x} + a_2\vec{y} = -a_3\vec{z}$

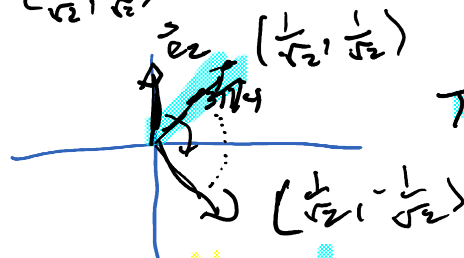
$\therefore z \in \text{span}\{\vec{x}, \vec{y}\}$.

4.) (5 pts) Find the standard matrix A of the linear transformation T where $T : \mathbb{R}^2 \to \mathbb{R}^2$ first rotates points clockwise $\frac{3\pi}{4}$ and then reflects points through the horizontal $x_1 -$axis.

Hint: $T(\vec{e}_1) = \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$ ✓



$T(\vec{e}_1) = \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$ ✓



$T(\vec{e}_2) = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$

Therefore: $A = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} T(\vec{e}_1) & T(\vec{e}_2) \end{bmatrix}$