Assessment 1
Dusty Wilson
Math 220

No work = no credit

Name: key

Suppose a contradiction were to be found in the axioms of set theory. Do you seriously believe that a bridge would fall down?

Frank Ramsey
1903 – 1930 (English mathematician)

Warm-ups (1 pt each): $9+10=$ 19 $-\frac{0}{4}=$ 0 $-1^2=$ -1

1.) (1 pt) In addition to infinity, one of the topics in the philosophy of math is called "axiomatic set theory." According to Ramsey (above), how seriously ought we be concerned by the possibility of a contradiction arising in set theory? Answer using complete English sentences.

We should not worry about contradictions (bridges won't fall).

2.) (8 pts) Let $\vec{\mathbf{a}}_1 = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$, $\vec{\mathbf{a}}_2 = \begin{bmatrix} 3 \\ 10 \\ -4 \end{bmatrix}$, and $\vec{\mathbf{b}} = \begin{bmatrix} -1 \\ 4 \\ 2 \end{bmatrix}$. $\text{Span}\{\vec{\mathbf{a}}_1, \vec{\mathbf{a}}_2\}$ is a plane in $\mathbb{R}^3$.

Is $\vec{b}$ in that plane? Explain/justify your response.

Is $\vec{b}$ a linear combination of $\vec{a}_1, \vec{a}_2$?

$$\text{rref}\left(\left[\begin{array}{cc|c} 1 & 3 & -1 \\ 3 & 10 & 4 \\ -2 & -4 & 2 \end{array}\right]\right) \sim \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]$$

system is inconsistent and thus $\vec{b}$ is <u>not</u> in the span (or on the plane).

3.) (8 pts) Solve the augmented matrix and express your solution in vector form.

$$\left[\begin{array}{ccccc|c} 2 & -4 & 3 & -4 & -11 & 28 \\ -1 & 2 & -1 & 2 & 5 & -13 \\ 0 & 0 & -3 & 1 & 6 & -10 \\ 3 & -6 & 10 & -8 & -28 & 61 \end{array}\right] \sim \left[\begin{array}{ccccc|c} 1 & -2 & 0 & 0 & 2 & 3 \\ 0 & 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$x_1 = 3 + 2x_2 + 2x_5$$
$$x_2 = x_2 \text{ (free)}$$
$$x_3 = 2 + x_5$$
$$x_4 = -4 - 3x_5$$
$$x_5 = x_5 \text{ (free)}$$

$$\Rightarrow \vec{x} = \begin{bmatrix} 3 \\ 0 \\ 2 \\ -4 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 2 \\ 0 \\ 1 \\ -3 \\ 1 \end{bmatrix}$$

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Math 220

No Calculator

1.) (8 pts) Solve the linear system

$$\begin{aligned} x_1 \qquad -3x_3 &= 8 \\ 2x_1+2x_2+9x_3 &= 7 \\ x_2+5x_3 &= -2 \end{aligned}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{array}\right] \quad R_2 - 2R_1 \to R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 2 & 15 & -9 \\ 0 & 1 & 5 & -2 \end{array}\right] \quad R_2 \leftrightarrow R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 2 & 15 & -9 \end{array}\right] \quad R_3 - 2R_2 \to R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 5 & -5 \end{array}\right] \quad \tfrac{1}{5}R_3 \to R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 1 & -1 \end{array}\right] \quad \begin{array}{l} R_1 + 3R_3 \to R_1 \\ R_2 - 5R_3 \to R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right] \Rightarrow \vec{x} = \begin{bmatrix} 5 \\ 3 \\ -1 \end{bmatrix}$$