***Partial Derivatives***

The process of finding the derivative of a function with respect to one independent variable while holding the other variables constant is called **partial differentiation**. Each derivative is called a **partial derivative**.

Note: The notation of partial derivatives that we are about to learn looks intimidating. But this is actually very straight-forward and no more difficult than ordinary derivatives. Similarly, if you understand derivatives graphically, you should have no trouble understanding the graphical interpretation of partial derivatives.

* **Partial Derivatives**

Functions of several variables have several derivatives, one for each variable. In the following example, we find these **partial derivatives** without (yet) having them clearly defined.

**Example 1**: If  find the following.

1.   Treat *y* like a constant.

1.   Treat *x* like a constant. Then evaluate the **partial derivative** when 

* **Interpretation of Partial Derivatives**

The graph of  is a surface in space.

Short version:  is the slope in the *x*-direction and  is the slope in the *y*-direction.

Long version:

|  |  |
| --- | --- |
| If the variable  is fixed at , then  is a function of one variable. The graph of this function is the curve of intersection between  and plane .  represents the slope of the tangent line to this curve or **the slope of the surface**  in the **-direction** or the **rate of change of  in the direction of.** |  |
| Similarly, represents the **slope of the surface**  **in the -direction** or the **rate of change of  in the direction of .** |  |

**Example 1b revisited**: If  interpret .

* **Notation for first partial derivatives**
* First partial derivatives of :
  + with respect to :
    - 
  + with respect to :
    - 
* The values of the first partial derivatives at the point  are detonated by
  + 
  + 

To calculate/evaluate an expression such as , first differentiate with respect to , and then evaluate the resulting expression at .

The concept of a partial derivative can be extended to functions of 3 or more variables. The function  has 3 partial derivatives, each of which can be found by holding 2 of the variables constant.

**Example 2**: If  find the following.

1. 
2. 

* ***Higher-Order Partial Derivatives***

You can differentiate a function more than once to find partial derivatives of second, third or higher order.

Second Partial Derivatives of :

* Twice with respect to : 
* Twice with respect to : 
* First with respect to  and then with respect to : 
* First with respect to  and then with respect to : 

The last two cases are called **mixed partial derivatives**.

Caution: Notice the order in which *x* and *y* are listed in the mixed partials in the two notations. In Leibniz notation, the order reminds us of process where as with subscripts the order matches the order of the partials.

Partial derivative of order three or higher can also be defined. For example:

.

**Example 3**: Find the second-order partial derivatives of .

Clairaut’s Theorem: Assuming *f* is defined on an open set *D* of , and that  and  are continuous throughout *D*. Then  at all points of *D*.

**Example 4**: Let . Find .

**Example 5**: If , find .

**Example 6**: Suppose that a point  moves along the intersection of the sphere  with the plane . At what rate is  changing with respect to  when the point is at ?