100	90's	80's	70's	60's	<60
0	4	4	4	2	3

Assessment 4 mean = 76.6%
Dusty Wilson
Math 220 median = 74.1%

Name: Key

As for everything else, so for a mathematical theory: beauty can be perceived but not explained.

Arthur Cayley
1821 - 1895 (English mathematician)

No work = no credit
Non CAS Calculators allowed

Warm-ups (1 pt each): $\begin{bmatrix}3 & 1\end{bmatrix}\begin{bmatrix}4\\1\end{bmatrix} = \begin{bmatrix}13\end{bmatrix}$ $\begin{bmatrix}4\\1\end{bmatrix}\begin{bmatrix}3 & 1\end{bmatrix} = \begin{bmatrix}12 & 4\\3 & 1\end{bmatrix}$

1.) (1 pt) According to Cayley (above), in what sense to do we understand the beauty of mathematics? Answer using complete English sentences.

We can sense beauty but can't articulate it.

2.) (4 pts) Calculate $\begin{bmatrix}-1 & 2\\5 & 4\\2 & -3\end{bmatrix} - 2\begin{bmatrix}4 & -2\\-3 & 0\\3 & 5\end{bmatrix}$. $= \begin{bmatrix}-9 & 6\\11 & 4\\-4 & -13\end{bmatrix}$

3.) (4 pts) Calculate $\underset{3\times 2}{\begin{bmatrix}-1 & 2\\5 & 4\\2 & -3\end{bmatrix}}\underset{2\times 4}{\begin{bmatrix}3 & 5 & -5 & 1\\-1 & 4 & 3 & 1\end{bmatrix}} = \underset{3\times 4}{\begin{bmatrix}-5 & 3 & 11 & 1\\11 & 41 & -13 & 9\\9 & -12 & -19 & -1\end{bmatrix}}$

4.) (4 pts) Is $A = \begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{bmatrix}$ singular? If possible, find A^{-1}.

A is singular.

7 5.) (2 pts) How many rows does B have if BC is a 3x4 matrix?

$(BC)_{3\times 4} = B_{3\times n} C_{n\times 4}$ B has 3 rows.

6.) (8 pts) Explaining linear algebra in words

8 a.) Explain the process for finding the inverse of an n x n matrix A.

① augment A w/ I

② row reduce $[A|I]$

③ in the result $[I|A^{-1}]$ the right side is A^{-1}. (assuming the left reduces to I).

9 b.) Interpret the linear transformation $T(\vec{x}) = \begin{bmatrix} \cos\frac{\pi}{3} & -\sin\frac{\pi}{3} \\ \sin\frac{\pi}{3} & \cos\frac{\pi}{3} \end{bmatrix} \vec{x}$ geometrically. What would the inverse T^{-1} of this transformation accomplish? Answer using complete sentences.

T rotates $\vec{x}$ $\frac{\pi}{3}$ radians C.C.W.

T^{-1} rotates $\vec{x}$ $\frac{\pi}{3}$ radians C.W. (opposite direction

10 c.) In general, how do you prove an if and only if claim "A if and only if B"?

($\Rightarrow$) Assume A show B.

($\Leftarrow$) Assume B show A.

11 d.) In general, how do you a uniqueness claim: "A uniquely satisfies condition C"?

Assume A and B satisfy condition C and show A = B

12 7.) (4 pts) Prove the following:

Claim: If A is an invertible n x n matrix, then for each $\vec{b} \in \mathbb{R}^n$, the equation $A\vec{x} = \vec{b}$ has the unique solution $\vec{x} = A^{-1}\vec{b}$.

proof.

existence.

let $\vec{b} \in \mathbb{R}^n$ and invertible $n \times n$ A be given.

solve $A\vec{x} = \vec{b} \Rightarrow A^{-1}A\vec{x} = A^{-1}\vec{b} \Rightarrow I\vec{x} = A^{-1}\vec{b} \Rightarrow \vec{x} = A^{-1}\vec{b}$.

$\therefore A\vec{x} = \vec{b}$ has a solution.

uniqueness

suppose $A\vec{x} = \vec{b}$ has two solutions $\vec{u}, \vec{v} \in \mathbb{R}^n$.

$\Rightarrow A\vec{u} = A\vec{v} = \vec{b} \Rightarrow A^{-1}A\vec{u} = A^{-1}A\vec{v} = A^{-1}\vec{b}$

$\Rightarrow \vec{u} = \vec{v} = A^{-1}\vec{b}$

$\Rightarrow \vec{u} = \vec{v}$

Q.E.D.