100	90's	80's	70's	60's	7 <60
3	2	4	4	1	1

Assessment 2
Dusty Wilson
Math 220

mean = 83.5%

med = 82.6%

Name: Key

If others would but reflect on mathematical truths as deeply and continuously as I have, they would make my discoveries.

Johann Carl Friedrich Gauss
1777 - 1855 (German mathematician)

No work = no credit
Non CAS Calculators allowed

Warm-ups (1 pt each): $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}3\\4\end{bmatrix} = [11]$ $\quad \begin{bmatrix}3\\4\end{bmatrix}\begin{bmatrix}1 & 2\end{bmatrix} = \begin{bmatrix}3 & 6\\4 & 8\end{bmatrix}$

1.) (1 pt) According to Gauss (above), what is the secret to making great mathematical discoveries? Answer using complete English sentences.

continuous and deep reflection is the key to discovery.

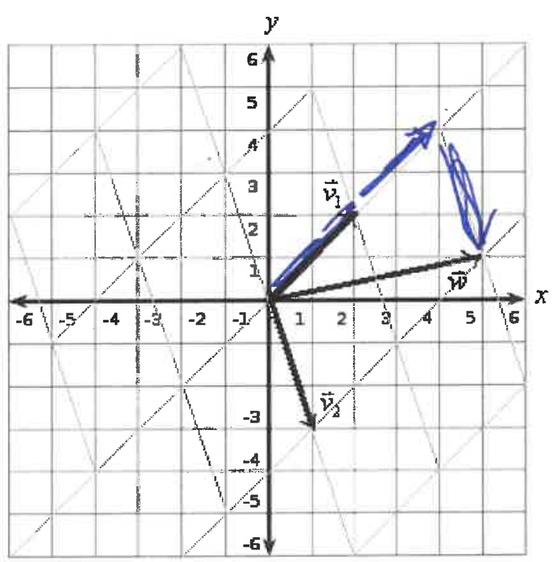
2.) (4 pts) Write the general solution of the linear system whose augmented matrix is reduced to:

$$\begin{bmatrix}\boxed{1} & 2 & 0 & 4 & 5\\ 0 & 0 & \boxed{1} & 6 & 7\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

$x_1 = 5 - 2x_2 - 4x_4$
$x_2 = x_2$ (free)
$x_3 = 7 - 6x_4$
$x_4 = x_4$ (free)

$$\Rightarrow \begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}5\\0\\7\\0\end{bmatrix} + x_2\begin{bmatrix}-2\\1\\0\\0\end{bmatrix} + x_4\begin{bmatrix}-4\\0\\-6\\1\end{bmatrix}$$

3.) (4 pts) Express $\vec{w}$ as a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$



$\vec{w} = 2\vec{v}_1 + \vec{v}_2$

4.) (2 pts) For what value(s) of h is $\mathbf{y}=\begin{bmatrix}-6\\h\\3\end{bmatrix}$ in the span of $\mathbf{v}_1=\begin{bmatrix}1\\2\\3\end{bmatrix}$ and $\mathbf{v}_2=\begin{bmatrix}4\\2\\5\end{bmatrix}$?

$$\begin{bmatrix}1&4&-6\\2&2&h\\3&5&3\end{bmatrix}\begin{matrix}R_2-2R_1\to R_2\\R_3-3R_1\to R_3\end{matrix}\sim\begin{bmatrix}1&4&-6\\0&-6&h+12\\0&-7&21\end{bmatrix}R_2\leftrightarrow R_3\sim\begin{bmatrix}1&4&-6\\0&-7&21\\0&-6&h+12\end{bmatrix}-\tfrac{1}{7}R_2\to R_2$$

$$\sim\begin{bmatrix}1&4&-6\\0&1&-3\\0&-6&h+12\end{bmatrix}R_3+6R_2\to R_3\begin{bmatrix}1&4&-6\\0&1&-3\\0&0&h-6\end{bmatrix}$$

consistent and hence $\vec{y}$ in the span when $h=6$.

5.) (4 pts) Write the system of equations $\begin{matrix}x_1+2x_2=3\\4x_1+5x_2=6\end{matrix}$ as a vector equation (an equation that includes a linear combination of vectors).

$$x_1\begin{bmatrix}1\\4\end{bmatrix}+x_2\begin{bmatrix}2\\5\end{bmatrix}=\begin{bmatrix}3\\6\end{bmatrix}$$

6.) (4 pts) Complete the following proof.

Claim: If $\mathbf{u}\in\mathbb{R}^n$ and c,d are scalars, then $(c+d)\mathbf{u}=c\mathbf{u}+d\mathbf{u}$.

Proof.

Let $\mathbf{u}\in\mathbb{R}^n$ and scalars c,d be given.

$$\Rightarrow (c+d)\vec{u}=(c+d)\begin{bmatrix}u_1\\\vdots\\u_n\end{bmatrix}$$
$$=\begin{bmatrix}(c+d)u_1\\\vdots\\(c+d)u_n\end{bmatrix}$$
$$=\begin{bmatrix}cu_1+du_1\\\vdots\\cu_n+du_n\end{bmatrix}$$
$$=c\begin{bmatrix}u_1\\\vdots\\u_n\end{bmatrix}+d\begin{bmatrix}u_1\\\vdots\\u_n\end{bmatrix}$$
$$=c\vec{u}+d\vec{u}$$

$\therefore (c+d)\vec{u}=c\vec{u}+d\vec{u}$.