

Give me [a lever long enough and] a place to stand and
I will move the earth.

Archimedes
287 - 212 BC (Italian mathematician)

No work = no credit
Non CAS Calculators allowed

Warm-ups (1 pt each): $1+1 = 2$ $A \cdot A^{-1} = I$

1.) (1 pt) What is one thing you are thankful for today?

There are many things to be thankful for including family, health, students and a God who loves me.

2.) (4 pts) If $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ and $\bar{b} = \begin{bmatrix} 4 \\ 2 \\ 1 \\ 3 \end{bmatrix}$, describe all least-squares solutions to $A\bar{x} = \bar{b}$. Find

the least-squares error. Please explain work done on the calculator.

$$\text{ref}([A^T A | A^T \bar{b}]) = \left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right] \Rightarrow \hat{x} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

$$A\hat{x} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 2 \\ 2 \end{bmatrix}$$

$$\text{error} = \left\| \begin{bmatrix} 4 \\ 2 \\ 1 \\ 3 \end{bmatrix} - \begin{bmatrix} 3 \\ 3 \\ 2 \\ 2 \end{bmatrix} \right\| = \left\| \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} \right\| = \sqrt{1+1+1+1} = 2$$

3.) (4 pts) Find the equation $y = \beta_0 + \beta_1 x$ of the least-squares line that best fits the points (2,3), (3,2), (5,1), and (6,0). Use the methods developed in this class making sure to explain work done on the calculator.

$$\begin{aligned} 3 &= \beta_0 + 2\beta_1 \\ 2 &= \beta_0 + 3\beta_1 \\ 1 &= \beta_0 + 5\beta_1 \\ 0 &= \beta_0 + 6\beta_1 \end{aligned} \Rightarrow \begin{matrix} A & \vec{x} & \vec{b} \\ \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 5 \\ 1 & 6 \end{bmatrix} & \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} & \begin{bmatrix} 3 \\ 2 \\ 1 \\ 0 \end{bmatrix} \end{matrix} \Rightarrow \hat{x} = (A^T A)^{-1} (A^T \vec{b}) = \begin{bmatrix} 4.3 \\ -0.7 \end{bmatrix}$$

The least-squares model is: $y = 4.3 - 0.7x$