

Definition: A complex rational expression contains \_\_\_\_\_ within its numerator and/or denominator.

We will use two different methods for simplifying complex rational expressions. **DO NOT** mix the methods.

Example 1: Simplify

$$\text{a.) } \frac{\frac{2}{r} - \frac{3}{t}}{\frac{4}{r} - \frac{5}{t}}$$

$$\text{b.) } \frac{\frac{1}{a^3b} + \frac{1}{b}}{\frac{1}{a^2b^2} - \frac{1}{b^2}}$$

Method: Using multiplication by 1 to simplify a complex rational expression.

1. Find the LCD of all rational expressions within the complex rational expression.
2. Multiply the complex rational expression by 1, writing 1 as the LCD divided by itself.
3. Distribute and simplify so that the numerator and the denominator of the complex rational expression are polynomials.
4. Factor and, if possible, simplify.

Example 2: Simplify

$$\text{a.) } \frac{\frac{3}{a^2-9} + \frac{2}{a+3}}{\frac{4}{a^2-9} + \frac{1}{a+3}}$$

$$\text{b.) } \frac{\frac{3x}{y} - x}{2y - \frac{y}{x}}$$

Method: Using division to simplify a complex rational expression.

- 1.) Add or subtract, as necessary, to get one rational expression in the numerator.
- 2.) Add or subtract, as necessary, to get one rational expression in the denominator.
- 3.) Perform the indicated division (invert the divisor and multiply).
- 4.) Simplify, if possible, by removing any factors that equal 1.

Example 3: Simplify

a.) 
$$\frac{\frac{3x}{y} - x}{2y - \frac{y}{x}}$$

b.) 
$$\frac{\frac{x^2 - x - 12}{x^2 - 2x - 15}}{\frac{x^2 + 8x + 12}{x^2 - 5x - 14}}$$

$$\text{c.) } \frac{\frac{a^{-1} + b^{-1}}{a^2 - b^2}}{ab}$$

$$\text{d.) } \frac{\frac{y}{y^2 - 1} - \frac{3y}{y^2 + 5y + 4}}{\frac{3y}{y^2 - 1} - \frac{y}{y^2 - 4y + 3}}$$