Example 1: Complete the square to solve $ax^2+bx+c=0$

$$ax^2+bx+c=0 \qquad \left(\frac{\frac{b}{a}}{2}\right)^2 = \left(\frac{b}{2a}\right)^2$$

$$\Rightarrow ax^2+bx=-c$$

$$\Rightarrow x^2+\frac{b}{a}x=-\frac{c}{a}$$

$$\Rightarrow x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2=\frac{-c}{a}+\left(\frac{b}{2a}\right)^2$$

$$\Rightarrow \left(x+\frac{b}{2a}\right)^2=\frac{-c}{a}+\frac{b^2}{4a^2}$$

$$=\frac{b^2-4ac}{4a^2}$$

$$\Rightarrow x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$=\pm\frac{\sqrt{b^2-4ac}}{2|a|}$$

$$=\pm\frac{\sqrt{b^2-4ac}}{2a}$$

abs not needed because of $\pm$.

$$\Rightarrow x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$ Quadratic Formula.

Formula: The Quadratic Formula (MEMORIZE)

a.) The solutions to $ax^2+bx+c=0$ for $a\neq 0$ are given by $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Example 2: Solve using the quadratic formula

a.) $3u^2 = 8u - 5$ $\Rightarrow$ $3u^2 - 8u + 5 = 0$

$a = 3$

$b = -8$

$c = 5$

$$\Rightarrow u = \frac{8 \pm \sqrt{64 - 4(3)(5)}}{2(3)}$$

$$= \frac{8 \pm \sqrt{4}}{6}$$

$$= \frac{8 \pm 2}{6}$$

$$\Rightarrow u = \frac{5}{3} \text{ or } u = 1$$

b.) $x^2 + 6x + 4 = 0$

$a = 1$

$b = 6$

$c = 4$

$$x = \frac{-6 \pm \sqrt{36 - 4(1)(4)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{20}}{2}$$

$$= \frac{-6 \pm 2\sqrt{5}}{2}$$

$$x = -3 \pm \sqrt{5}$$

c.) $v^2 + 13 = 6v$

$$\Rightarrow v^2 - 6v + 13 = 0$$

$a = 1$, $b = -6$, $c = 13$

$$\Rightarrow v = \frac{6 \pm \sqrt{36 - 4(1)(13)}}{2(1)}$$

$$= \frac{6 \pm \sqrt{-16}}{2}$$

$$= \frac{6 \pm 4i}{2}$$

$$v = 3 \pm 2i$$

Method: To solve a quadratic equation

a.) If the equation can be easily written in the form $ax^2 = p$ or $(x+k)^2 = d$, use the principle of square roots as in Section 8.1.
b.) If Step (a.) does not apply, write the equation in the standard form $ax^2 + bx + c = 0$.
c.) Try factoring and using the principle of zero products.
d.) If factoring seems to be difficult or impossible, use the quadratic formula. Completing the square can also be used, but is usually slower.

Note: The solutions of a quadratic equation can always be found using the quadratic formula. They cannot always be found by factoring.

$ax^2 + bx + c = 0, \quad a \neq 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example 3: Solve

a.) $4x + x(x-3) = 5$

$\Rightarrow 4x + x^2 - 3x = 5$

$\Rightarrow x^2 + x - 5 = 0 \qquad a = 1,\ b = 1,\ c = -5$

$\Rightarrow x = \frac{-1 \pm \sqrt{1 - 4(1)(-5)}}{2(1)}$ ← discriminant

$= \frac{-1 \pm \sqrt{21}}{2}$

b.) $25x = 3x^2 + 28$

$\Rightarrow 0 = 3x^2 - 25x + 28$ * $\qquad a = 3,\ b = -25,\ c = 28$

$\Rightarrow x = \frac{25 \pm \sqrt{625 - 4(3)(28)}}{2(3)}$

$= \frac{25 \pm \sqrt{289}}{6}$

$= \frac{25 \pm 17}{6}$

$\Rightarrow x = 7$ or $x = \frac{4}{3}$

* factored.
$0 = (x-7)(3x-4)$

Example 4: Solve $x^3+1=0$ (find all solutions).

$$\Rightarrow (x+1)(x^2-x+1)=0$$

$$\Rightarrow x+1=0 \quad \text{or} \quad x^2-x+1=0 \qquad a=1,\ b=-1,\ c=1$$

$$\Rightarrow x=-1 \quad \text{or} \quad x=\frac{1\pm\sqrt{(-1)^2-4(1)(1)}}{2(1)}$$

$$=\frac{1\pm\sqrt{-3}}{2}$$

$$=\frac{1\pm i\sqrt{3}}{2}$$

$$\Rightarrow x=-1 \quad \text{or} \quad x=\frac{1+i\sqrt{3}}{2} \quad \text{or} \quad x=\frac{1-i\sqrt{3}}{2}$$

real complex

Example 5: Let $f(x)=\dfrac{3-x}{4}$ and $g(x)=\dfrac{1}{4x}$. Find all *x* for which $f(x)=g(x)$.

solve $\dfrac{3-x}{4}=\dfrac{1}{4x}$

$$\Rightarrow 4x(3-x)=1\cdot 4$$

$$\Rightarrow \frac{12x-4x^2}{-4}=\frac{4}{-4}$$

$$\Rightarrow -3x+x^2=-1$$

$$\Rightarrow x^2-3x+1=0$$

$$\Rightarrow x=\frac{3\pm\sqrt{5}}{2(1)}$$

$$=\frac{3\pm\sqrt{5}}{2}$$

Discriminant: b^2-4ac

$9-4(1)(1)=5$