Section 12.02

The Power Rule

Part 1: Differentials

The Differential: If $\frac{dy}{dx} = f'(x)$, then the differential $dy = f'(x)\,dx$.

Example 1: Find the differential dy if:

a.) $y = x^7 + 3x^2 + 2$

b.) $y = x^3 e^x$

Part 2: The Power Rule

Recall the power rule for derivatives: $\frac{d}{dx}[u(x)]^n = n[u(x)]^{n-1} u'(x)$. This leads to the power rule for integration where $\int n[u(x)]^{n-1} u'(x)\,dx = [u(x)]^n + C$.

Power rule for integration: Assuming that $n \neq -1$,

$$\int [u(x)]^n u'(x)\, dx = \frac{[u(x)]^{n+1}}{n+1} + C$$

or if $u = u(x)$, then

$$\int u^n \, du = \frac{u^{n+1}}{n+1} + C$$

Example 2: $\int (3x^3 + 1)^4 \, 9x^2 \, dx$

Example 3: $\int (3x^2 - 4)^6 \, x \, dx$

Example 4: $\int \frac{x\,dx}{(x^2+1)^3}$

Example 5: $\int 7\,x^3\,\sqrt{x^4+6}\;dx$

Example 6: $\int (x^2+1)^2\,dx$

Example 7: $\int \frac{5\,x\,dx}{(x^2-1)^{13}}$

Example 8: $\int \frac{x^3-1}{(x^4-4\,x)^3}\,dx$

Example 9: $\int \frac{x^2+1}{\sqrt{x^3+3\,x+10}}\,dx$

Part 3: Applications (time permitting)

Example 10: A new firm predicts that the number of franchises will grow at a rate $\frac{dn}{dt} = 9\sqrt{t+1}$ where t is in years, $0 \le t \le 10$. If there are presently three franchises (after zero years), how many franchises can be expected in eight years?