Section 9.6

The Chain Rule

Part 1: Composition of functions

Example 1: If $f(x) = 3x^4$ and $g(x) = 3 - 2x$, find $h(x) = f(g(x))$.

Example 2: For the following functions h, find f and g such that $h(x) = f(g(x))$.

a.) $h(x) = (2x^4 - 5)^{25}$

b.) $h(x) = (3 - 2x)^{10}$

c.) $h(x) = \frac{2}{3}(x^6 + 3x^2 - 11)^8$

Example 3: For the following functions h, find f and g such that $h(x) = f(g(x))$.

a.) $h(x) = \frac{1}{3\,(3\,x^2+3\,x+5)^{3/4}}$

b.) $h(x) = \sqrt{x^2 + 3\,x}$

Part 2: The Chain Rule

Derivative Rule: The chain rule

If $h(x) = f(g(x))$, then $h'(x) = f'(g(x)) \cdot g'(x)$

This can be memorized as, $\frac{d}{d\,x} f(g(x)) = f'(g(x)) \cdot g'(x)$.

Example 2 revisited: For each example, find $h'(x)$.

a.) $h(x) = (2\,x^4 - 5)^{25}$

b.) $h(x) = (3 - 2\,x)^{10}$

Example 2 revisited (continued from previous page)

c.) $h(x) = \frac{2}{3}(x^6 + 3x^2 - 11)^8$

Example 3 revisited: For each example, find $h'(x)$.

a.) $h(x) = \frac{1}{3(3x^2+3x+5)^{3/4}}$

b.) $h(x) = \sqrt{x^2 + 3x}$

Example 4: Find the tangent line to $y = (x^2 + 1)^3$ at (2, 125).

Example 5 (for you): Find the tangent line to $y = \left(\frac{1}{x^3 - x}\right)^3$ at $x = 2$.

Example 6: Differentiate the following

a.) $y = \frac{5}{7}(2x^3 - x + 6)^{14}$

b.) $p = (q^3 + 1)^{-5}$

c.) $f(x) = \frac{1}{(x^2+2)^3}$

Example 7: Differentiate the following

a.) $g(x) = \frac{1}{(2x^3+3x+5)^{3/4}}$

b.) $y = \frac{(3x+1)^5 - 3x}{7}$

Example 8: $R(x) = 15(3x+1)^{-1} + 5x - 15$ gives the the dollars of revenue from the sale of x items. Find and interpret $\overline{\text{MR}}(4)$.

Quiz - Just for you

a.) Write down the product rule

b.) Write down the quotient rule

c.) Write down the chain rule

Example 9: The daily sales S attributed to an advertising campaign are given by the function $S(t) = 1 + \frac{3}{t+3} - \frac{18}{(t+3)^2}$ where t is the number of weeks the advertisement runs.

a.) Find the ROC when $t = 8$.

b.) Find the ROC when $t = 10$.

c.) Should the campaign continue after the tenth week?