Section 9.3

Rates of Change: The Derivative

Part 1: Average Rates of Change (ROC)

Example 1: Find the average ROC of $f(x) = x^2$ on the intervals

a.) [2, 5]

b.) [2, 4]

c.) [2, 3]

d.) [2, 2+h]

Definition*: Average Rate of Change (ROC)*

The average ROC of $f(x)$ *on* $[a,\ b]$ *provided that f is continuous on the interval is:*

As the interval $[a, b]$ becomes smaller, (that is, as $(b - a) \to 0$), the average ROC approaches the instantaneous ROC.

Part 2: Instantaneous Rates of Change

Example 2: Find the instantaneous ROC of $f(x) = x^2$ at $x = 2$. Let h represent the change in x.
1st: Average ROC (and picture)

Definition*: The Difference Quotient (I call it "DQ")*

$$DQ = \frac{f(x+h) - f(x)}{h}$$

Example 2 continued:

2nd: Find $f(2 + h) - f(2)$

3rd: Form the difference quotient. (I have this as a seperate step because I find students struggle with step 2).

4th: Find the instantaneous ROC of f at $x = 2$ by evaluating the limit of the DQ.

Part 3: The Derivative

Definition*: The Derivative*
If f is a function defined by $y = f(x)$, then the derivative of $f(x)$ at any value x, denoted $f'(x)$, is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

if the limit exists. If $f'(c)$ exists, we say that f is differentiable at c.

Example 3: Find $f'(x)$ if $f(x) = 3x^2 - x$.

Example 4 (for groups): Find $g'(x)$ for $g(x) =$ ___ ___ ___ ___

The derivative gives the slope at a point (remember, the average ROC gave the slope of a secant line and the instantaneous ROC will give the slope at a point).

For some problems, it can be beneficial to recall the point-slope form for the equation of a line with slope m and that includes the point (x_1, y_1). The formula is: $y - y_1 = m(x - x_1)$.

Example 5: Find the tangent line to $f(x) = 3x^2 - x$ when $x = -3$ (use the result of example 3).

In chapter 1, marginal cost $\overline{\text{MC}}$ was defined as the slope of the cost function. marginal revenue and profit were similarly defined.

Example 6: Suppose the cost to produce x items is modeled by $C(x) = 25x + 1000$.

Review: What does the "+1000" represent in the context of the problem?

a.) Find the marginal cost.

b.) Interpret the result from part (a.).

Whereas in chapter 1, it was simple to find the slope because we need only determine the slope of a line, now we will find the slope at a point using calculus and the derivative.

Example 7: Suppose the profit from the sale of x cars (in $1,000's) is given by $P(x) = 500\,x - x^2 - 100$

a.) Find $\overline{\text{MP}}$

b.) Find and interpret $\overline{\text{MP}}(20)$

c.) Find and interpret $\overline{\text{MP}}(300)$

Part 4: Graphical Derivative

Remember that the derivative at a point represents the slope of the tangent at that point.

Example 8: For the given graph, order $f'(a)$, $f'(b)$, ...

Example 9: For the given graph, what can be said about g' at a, b, ...?