Applications of the Indefinite Integral

Part 1: Applications to Business

Example 1: A firm has marginal costs of $\overline{MC} = 6x + 60$. Its marginal revenue is $\overline{MR} = 180 - 2x$. The fixed cost of production are $100.

a.) Find the optimal level of production

Solve $\overline{MC} = \overline{MR}$

$\Rightarrow \quad 6x + 60 = 180 - 2x$

$\Rightarrow \quad 8x = 120$

$\Rightarrow \quad x = 15$

b.) Find the profit function

$\overline{MP} = \overline{MR} - \overline{MC}$

$= (180 - 2x) - (6x + 60)$

$= -8x + 120$

$P(x) = \int (-8x + 120)\,dx$

$P = -4x^2 + 120x + C$

$P(0) = -100 \Rightarrow C = -100$

$P(x) = -4x^2 + 120x - 100$

c.) Find the optimal profit (loss)

$P(15) = 800$

The max profit is \$800 when 15 units are produced/sold.

d.) Should production be continued in the short run and/or the long run?

In the long run, the company loses money & so it should only continue in the short run.

Example 2: A firm has marginal costs of $\overline{MC} = 3x + 20$. Its marginal revenue is $\overline{MR} = 44 - 5x$. The total cost of producing 10 units is \$11,400.

a.) Find the optimal level of production

$$\overline{MP} = \overline{MR} - \overline{MC}$$

$$= (44 - 5x) - (3x + 20)$$

$$= -8x + 24$$

Solve $\overline{MP} = 0$

$$\Rightarrow 8x = 24 \longrightarrow \quad \Rightarrow x = 3.$$

b.) Find the profit function

$$P = \int (-8x + 24)\, dx$$

$$= -4x^2 + 24x + c$$

~~$P(10) = -400 + 240 + C = 11900$~~

~~$\Rightarrow C = 11560$~~

$P(0) = -\text{Fixed cost} = C$.

$$\Rightarrow P(x) = -4x^2 + 24x - 11050$$

$$C = \int (3x + 20)\, dx$$

$$= \frac{3}{2}x^2 + 20x + C$$

$$C(10) = 150 + 200 + C = \boxed{11400}$$

$$\Rightarrow C = 11050$$

(fixed cost)

c.) Find the optimal profit (loss)

$$P(3) = -11014.$$

close shop! the best you can do is lose. \$11,014.

Part 2: National Consumption and Savings

If C represents the national consumption (in billions of dollars) and y represents the nations disposable income, then we will consider C to be a function of y: $C = f(y)$.

Question: What is national consumption?

The amount we spend/consume.

Question: What is disposable income?

The part of income you spend @ will

Question: What is discretionary spending?

Similar to disposable income ... but this is just the spending.

Example 3: If the national consumption as a function of disposable income can be represented by $C(y) = 0.95\, y + 12$, find and interpret $C(0)$ and the rate of change of C. (in billions).

$C(0) = 12$.   We will consume/spend $12 billion when we have no disposable income.

$c'(y) = 0.95$

For each additional billion of disposable income, we consume $950,000,000

Definition: The marginal propensity to consume is $\frac{dC}{dy}$ or C'.

Example 4: If consumption is $5.8 billion when disposable income is zero, and if the marginal propensity to consume is given

by:

$$\frac{dC}{dy} = \frac{1}{\sqrt{2y+9}} + 0.8 \text{ (in billions of dollars)}$$

find the national consumption function.

$$C(y) = \int \left(\frac{1}{\sqrt{2y+9}} + 0.8\right) dy$$

$$= 0.8y + \frac{1}{2} \int (2y+9)^{-1/2} \, 2 \, dy$$

Let $u = 2y+9$
$du = 2dy$.

$$= 0.8y + \frac{1}{2} \int u^{-1/2} \, du$$

$$= 0.8y + \frac{1}{2} \frac{u^{1/2}}{1/2} + k$$

$$= 0.8y + \sqrt{2y+9} + k.$$

AND $C(0) = 3 + k = 5.8$

$$\Rightarrow k = 2.8$$

so $C(y) = 0.8y + \sqrt{2y+9} + 2.8$

Definition: The marginal propensity to save is $\frac{dS}{dy} = 1 - \frac{dC}{dy}$ or $S' = 1 - C'$.

Example 5: Suppose that the marginal propensity to save is $\frac{dS}{dy} = 0.15$ and that consumption is $5.15 billion when disposable income is zero. Find the national consumption function.

$$\frac{dC}{dy} = 1 - \frac{dS}{dy} = 0.85$$

$$C(y) = \int 0.85 \, dy$$

$$= 0.85y + k$$

AND $C(0) = k = 5.15$

So $C(y) = 0.85y + 5.15$

Part 3: Background: National Consumption and Savings

Answer: What is national consumption?

In Keynesian economics consumption refers to personal consumption expenditure, i.e., the purchase of currently produced goods and services out of income, out of savings (net worth), or from borrowed funds. It refers to that part of disposable income (income after taxes paid and transfer payments received) that does not go to saving. (wikipedia.org)

Answer: What is disposable income?

Current income received by persons less their contributions for social insurance, personal tax, and nontax payments. It is the income available to persons for spending and saving. Nontax payments include passport fees, fines and penalties, donations, and tuitions and fees paid to schools and hospitals operated mainly by the government. (nces.ed.gov)

Answer: What is discretionary spending?

Discretionary spending is a category of federal spending subject to the annual appropriations process. It includes such things as funding for transportation, environmental programs, job training, and education programs. (nvfc.org) Thus, discretionary spending does not (as an economic term, apply to this situation.

Americans save a decreasing amount of their disposable income.

% saved of disposable income
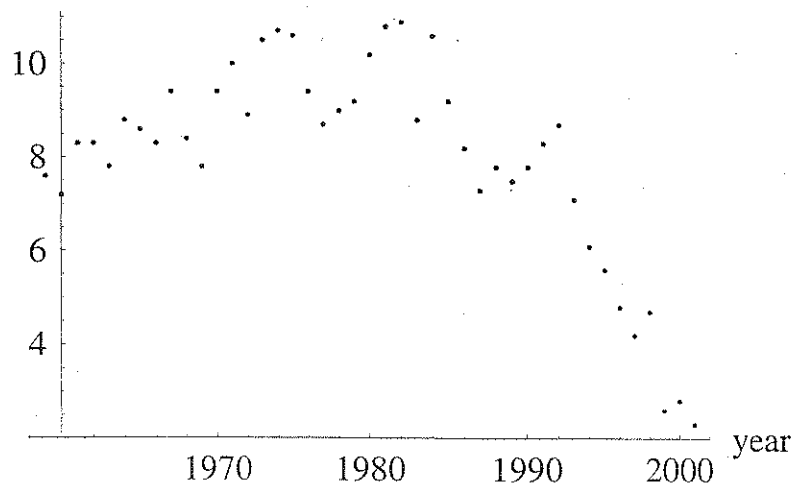
Answers to Important Questions About How Americans Give, 2000 (fidelity.com)

1.   How much money do Americans give to charity?

- More than two in three (69%) U.S. households gave at least $25 in charitable donations in 1999.

- One in five (20%) gave more than $500 and about one in ten (11%) gave more than $1,000.

2.   How many charities do they support?

- The average number of charitable organizations supported by Americans who give to charity is three

- One out of four households (24%) support more than five organizations, but few (11%) support more than ten organizations.

- Fourteen percent of U.S. households report that all of their financial gifts in 1999 went to a single organization.

3.   Does age affect how Americans give?

- Young, affluent givers (age 21-30 with at least $50K-$75K annual income) want to be very involved in their giving:

- Most (61%) give large donations to a few charities, rather than small donations to many.

- More than half (55%) want to know more about organizations before they give.

- These young givers also tend to be reactive:

- More than one-third (34%) are likely to contribute in response to a news story or crisis.

- Less than half (48%) give to an organization regularly, on a weekly or monthly basis.
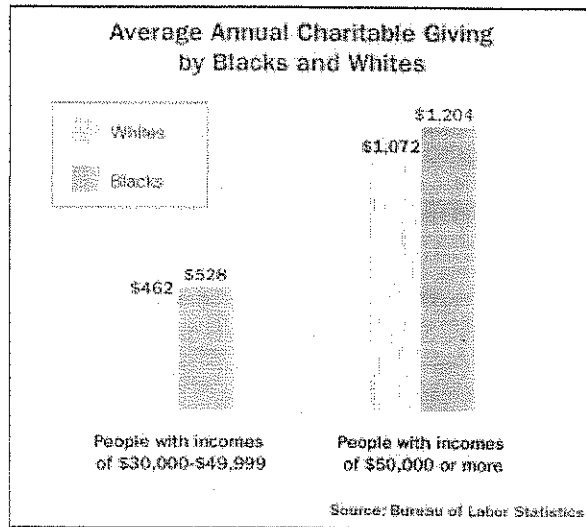
4.   Do Americans budget for or plan their charitable giving?

- Less than half (42%) of Americans have a household financial budget, but the majority (61%) of those who do budget, include charitable giving as part of that budget.

- Charitable giving is a top priority for these households, running third behind entertainment expenses (70%) and retirement savings (66%), but winning out over vacation expenses (53%).

- One-third (33%) of American households report setting aside time at least once a year to plan how much they intend to give, which groups to support and how often to give.

5.   Does planning their charitable giving make a difference?

- These "planned givers" tend to give more dollars to charity: 41% gave $500- $2,499 and exactly half (50%) gave more than $2,500 in gifts.

- One-third (33%) of Americans report that when viewing their charitable giving records, they are sometimes surprised by the total amount their household gave. Most (78%) gave more than they thought.

Giving by race
(philanthropy.com)

Average Annual Charitable Giving by Blacks and Whites

Part 4: Scratch