Section 12.01

Indefinite Integrals

Part 1: Overview

Question: The derivative of what function (or functions) is $f'(x) = 3x^2$?

$x^3 + c$

The process of finding antiderivatives (as we did above) is called integration.

We would use the following notation to in the integration process:

Example 1: $\int 4x^3\,dx \;=\; x^4 + c.$

Example 2: $\int 1\, dx$, $\int x\, dx$, $\int x^2\, dx$, $\int x^3\, dx$, ...

$$\int 1\, dx = x + c$$

$$\int x\, dx = \frac{x^2}{2} + c$$

$$\int x^2\, dx = \frac{x^3}{3} + c$$

$$\int x^3\, dx = \frac{x^4}{4} + c$$

Formula for antiderivatives: $\int x^n\, dx$

$$= \frac{x^{n+1}}{n+1} + c$$

Part 2: Examples (ad nausium)

Example 3: $\int 16\, x^9\, dx$ $= \frac{16}{10} x^{10} + c$

$= \frac{8}{5} x^{10} + c$

Example 4: $\int (x^4 - 9\, x^2 + 3)\, dx$

$= x^5 - 3x^3 + 3x + c$

Example 5: $\int \left(17 + \sqrt[3]{x}\right) dx$ $= \int (17 + x^{1/3})\, dx$

$= 17x + \frac{3}{4} x^{4/3} + c$

Example 6: $\int \frac{6\,dx}{x^3}$ $= \int 6x^{-3}\,dx$

$$= \frac{6}{-2}x^{-2} + c$$

$$= \frac{-3}{x^2} + c$$

Example 7: $\int \left(3x^8 + \frac{4}{x^8} - \frac{5}{\sqrt[5]{x}}\right) dx$ $= \int 3x^8 + 4x^{-8} - 5x^{-1/5}\,dx$

$$= \frac{3}{9}x^9 + \frac{4}{-7}x^{-7} - \frac{5}{4/5}x^{4/5} + c$$

$$= \frac{1}{3}x^9 - \frac{4}{7}x^{-7} - \frac{25}{4}x^{4/5} + c$$

Part 2: Applications (time permitting)

Example 8: If the marginal revenue ($/unit) for a month is given by $\overline{MR} = -0.3\,x + 450$, what is the total revenue from the production and sale of 50 units?

$$R(x) = \int(-0.3x + 450)\,dx$$

$$= -\frac{0.3}{2}x^2 + 450x + C$$

Find C.

Example 9: Suppose a projectile is launched straight up with velocity given by $v(t) = 320 - 32\,t$ (measured in feet per second). After 10 seconds the projectile is ~~1000~~ 1800 feet above the launch point. When does the object hit the ground?

position is the antiderivative of velocity (except possibly differing by a const.).

(1) $p(t) = \int v(t)\,dt$

$= \int (320 - 32t)\,dt$

$= 320t - 16t^2 + C$

(2) Find C.

$P(10) = 1800 = 3200 - 1600 + C$

$\Rightarrow C = 200$

(3) So $p(t) = -16t^2 + 320t + 200$

using the quadratic formula...

$\underbrace{t = -0.607}_{\text{unreasonable}}$ or $t = 20.607$

(4) The ~~ball~~ object hits after 20.607 seconds.