Section 11.05

Applications

Part 1: Elasticity

Elasticity measures the responsiveness of demand to price changes. High elasticity means responsiveness is high while low elasticity means that demand is relatively unchanged by changes in price.

On a scale of 1 to 10, how elastic do you think demand is in the following markets?

- chocolate 8 (elastic).
- insulin (medicine for diabetics) 1 (inelastic).
- clothing 7
- water 3
- cocaine 2
- automobiles 7
- cigerettes 2
- steel 5 ?
- text books 4

Do you think that elasticity is constant for a given product? That is, do you think that the responsiveness of demand to price changes might depend upon the price in any way?

> Elasticity of Demand: $\eta = -\frac{p}{q} \cdot \frac{dq}{dp} = -\frac{p}{q(p)} \cdot q'(p)$.
>
> Note that η is the Greek letter "eta."

We have skipped over a topic called implicit differentiation. For that reason, skip over example 2 in the text.

Note: p & q are related thru the demand.

Example 1: Find the elasticity of demand if demand is modeled by $2p+3q=150$ when $p=15,\ 37.5,\ \text{and}\ 45$.

(1) Solve for q: $q=\dfrac{150-2p}{3}$ $\quad q'=-\dfrac{2}{3}$

(2) $\eta = -\dfrac{p}{\frac{150-2p}{\cancel{3}}}\cdot\left(-\dfrac{2}{\cancel{3}}\right)$ $\quad \eta=\dfrac{p}{50-\frac{2}{3}p}\cdot\dfrac{2}{3}$

$=\dfrac{\frac{2}{3}p}{50-\frac{2}{3}p}$

$=\dfrac{2p}{150-2p}.$

(3) At $p=15$ $\quad \eta=0.25$ (inelastic)

$p=37.5$ $\quad \eta=1$ (unitary elastic)

$p=45$ $\quad \eta=1.5$ (elastic)

Vocabulary related to elasticity of demand:

If $\eta>1$, we say that demand is elastic

If $\eta<1$, we say that demand is inelastic

If $\eta=1$, we say that demand is unitary elastic

Example 2: Demand is given by $p = \frac{1000}{(q+1)^2}$. Find η when $q = 19$.

(1) solve for q: $(q+1)^2 = \frac{1000}{p}$

$$\Rightarrow q+1 = \pm\sqrt{\frac{1000}{p}}$$

$$\Rightarrow q = \sqrt{\frac{1000}{p}} - 1$$

(2) $$\eta = -\frac{p}{\sqrt{\frac{1000}{p}} - 1} \cdot \frac{1}{2}\left(\frac{1000}{p}\right)^{-\frac{1}{2}} \cdot (-1000p^{-2})$$

$$= \frac{500\cancel{p}}{\sqrt{\frac{1000}{p}}\left(\sqrt{\frac{1000}{p}} - 1\right)p^{\cancel{2}}}$$

(3) At $q = 19 \rightarrow p = 2.5$ AND $\eta = 0.53$
(inelastic)

Elasticity and Reve-
nue:

$$R(p) = p \cdot q(p)$$

where $q(p)$ is the quantity demanded at a price p.

$$\begin{aligned} R'(p) &= p \cdot q'(p) + q(p) \\ &= q(p) \cdot \tfrac{p}{q(p)} \cdot q'(p) + q(p) \\ &= q(p)(-\eta) + q(p) \\ &= q(p)(1 - \eta) \end{aligned}$$

critical value w/ $\eta = 1$.

Summary:

If $\eta > 1$, then $R' < 0$ and a price increase will result in a revenue decrease and visa versa

If $\eta < 1$, then $R' > 0$ and a price increase will result in a revenue increase and visa versa

If $\eta = 1$, then $R' = 0$ and an increase in price will not result in a change in revenue. Revenue is optimized at this point.

Example 3: Given the demand function $p = 120\sqrt[3]{125 - q}$, answer the following.

$$\Rightarrow \frac{p}{120} = \sqrt[3]{125-q} \Rightarrow \left(\frac{p}{120}\right)^3 = 125-q$$

a.) Find $\eta(p)$

$$\eta = -\frac{p}{125 - \left(\frac{p}{120}\right)^3} \cdot \frac{-3p^2}{120^3} \qquad \Rightarrow q = 125 - \left(\frac{p}{120}\right)^3$$

$$= \frac{+3p^3}{120^3\left(125 - \left(\frac{p}{120}\right)^3\right)}$$

b.) Find the point (q, p) where $\eta = 1$

$$\Rightarrow \eta = 1: \quad 120^3\left(125 - \left(\frac{p}{120}\right)^3\right) = +3p^3$$

$$\Rightarrow 120^3 \cdot 125 - p^3 = +3p^3$$

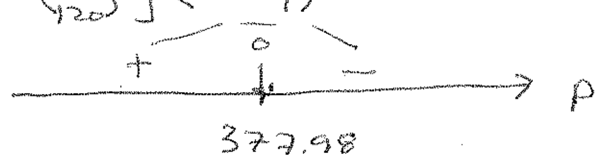
$$\Rightarrow \frac{120^3 \cdot 125}{4} = p^3$$

$$\Rightarrow p = \sqrt[3]{\frac{120^3 \cdot 125}{4}} = 377.98$$

and $q = 93.75$

c.) Construct a sign diagram of R'

$$R'(p) = \left[125 - \left(\frac{p}{120}\right)^3\right](1 - \eta)$$



d.) Find the maximimum revenue

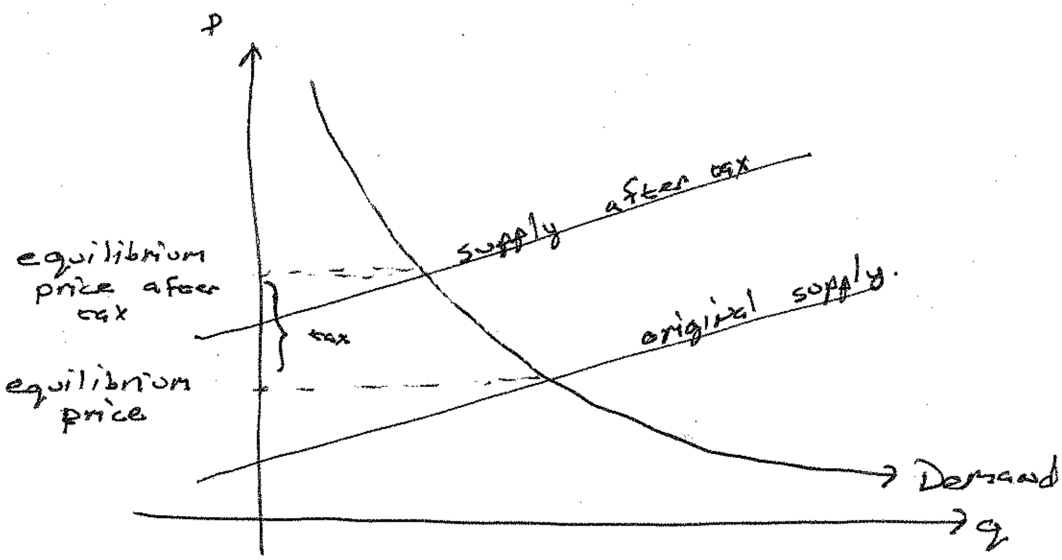
max revenue is.

$$377.98\,(93.75) = \$35{,}435.63$$

Part 2: Taxation in a competitive market

The goal of this section is to determine the level of taxation that will maximize tax revenue.

The picture:



So, rather than working with the supply function $S: \ p(q)$, work with the supply function after taxation $S: \ p(q)+t$. Also remember the simple formula that total tax is equal to $T = t \cdot q$.

Example 4: If the weekly demand function is $D: p = 200 - 2q^2$ and the supply function before taxation is $S: p = 20 + 3q$, what tax per item will maximize the total tax revenue and what is the maximum tax revenue that can be generated?

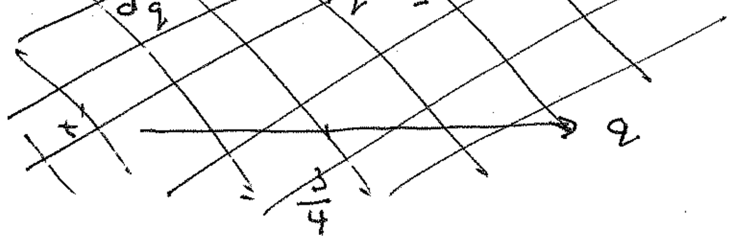
State gas tax
\$0.375
Federal
\$0.184

Cigs.
\$3.025/pack (state)
\$1.0066/pack (fed)

solve: $\underbrace{200 - 2q^2}_{\text{Demand}} = \underbrace{20 + 3q + t}_{\text{Supply.}}$

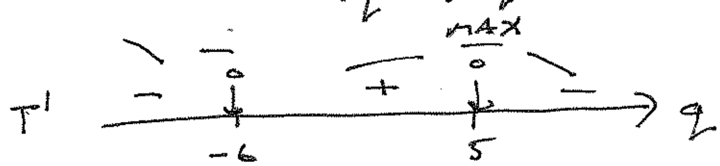
$\Rightarrow t = -2q^2 - 3q + 180$ (individual item tax).

$\cancel{\Rightarrow \frac{dt}{dq} = -4q - 3}$



$\Rightarrow T = q(-2q^2 - 3q + 180)$ (total tax revenue)
$= -2q^3 - 3q^2 + 180q$

$\Rightarrow T' = -6q^2 - 6q + 180$
$= -6(q^2 + q - 30)$
$= -6(q+6)(q-5)$



Tax revenue is maxed when a tax of \$115/item is carded.

Ethical question: Is it important for a government to generate the maximum possible tax revenue? Why or why not?